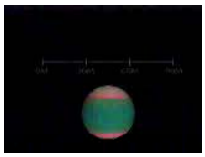
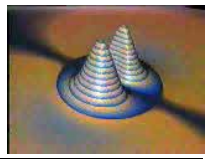
Today in Astronomy 102: gravitational radiation

- The Einstein field equation
- Light: more of its details
- Gravitational radiation: gravity's counterpart to light
- Experimental tests of general relativity
- The Hulse-Taylor pulsar and the discovery of gravitational radiation





Gravity waves from a pulsating black hole (Ed Seidel, NCSA, U. Illiniois).

Hieroglyphics: Einstein's field equation

The field equation is the ultimate mathematical expression of Einstein's general theory of relativity.

Astronomy 102 version:

"Spacetime, with its curvature, tells masses how to move; masses tell spacetime how to curve."

Physics 413 - Astronomy 554 version:

$$\frac{1}{2}\left[\frac{\partial^2 g^{\lambda}_{\lambda}}{\partial x^{\kappa}\partial x^{\mu}} - \frac{\partial^2 g_{\mu\lambda}}{\partial x^{\kappa}\partial x_{\lambda}} - \frac{\partial^2 g_{\lambda\kappa}}{\partial x_{\lambda}\partial x^{\mu}} + \frac{\partial^2 g_{\mu\kappa}}{\partial x^{\lambda}\partial x_{\lambda}}\right] + g_{\eta\sigma}\left[\frac{\partial x^{\eta}}{\partial \xi^{\alpha}}\frac{\partial^2 \xi^{\alpha}}{\partial x^{\lambda}\partial x_{\lambda}}\frac{\partial x^{\sigma}}{\partial \xi^{\alpha}}\frac{\partial^2 \xi^{\alpha}}{\partial x^{\mu}\partial x^{\kappa}} - \frac{\partial x^{\eta}}{\partial \xi^{\alpha}}\frac{\partial^2 \xi^{\alpha}}{\partial x^{\kappa}\partial x_{\lambda}}\frac{\partial x^{\sigma}}{\partial \xi^{\alpha}}\frac{\partial^2 \xi^{\alpha}}{\partial x^{\mu}\partial x^{\lambda}}\right]$$

$$-\frac{1}{2}g_{\mu\kappa}\left(\frac{1}{2}\left[2\frac{\partial^2 g^{\lambda}_{\lambda}}{\partial x^{\mu}\partial x_{\mu}} - \frac{\partial^2 g^{\mu}_{\lambda}}{\partial x^{\mu}\partial x_{\lambda}} - \frac{\partial^2 g_{\lambda\mu}}{\partial x_{\lambda}\partial x_{\mu}}\right] + g_{\eta\sigma}\left[\frac{\partial x^{\eta}}{\partial \xi^{\alpha}}\frac{\partial^2 \xi^{\alpha}}{\partial x^{\lambda}\partial x_{\lambda}}\frac{\partial x^{\sigma}}{\partial \xi^{\alpha}}\frac{\partial^2 \xi^{\alpha}}{\partial x_{\mu}\partial x^{\mu}} - \frac{\partial x^{\eta}}{\partial \xi^{\alpha}}\frac{\partial^2 \xi^{\alpha}}{\partial x^{\mu}\partial x_{\lambda}}\frac{\partial x^{\sigma}}{\partial \xi^{\alpha}}\frac{\partial^2 \xi^{\alpha}}{\partial x_{\mu}\partial x^{\lambda}}\right]\right)$$

$$= -8\pi G\sum_{n} p_{n\mu}\frac{dx_{n\kappa}}{dt}\delta^3(x - x_n)$$

What you get when you solve the field equation

In case you're interested (i.e. **not on the exam**):

- The solution to the field equation is a function called the **metric tensor** (g, in the field equation)**.** This function tells how much distance or time is displaced in each dimension, per unit displacement in a given dimension.
  - Thus the metric tensor describes the all the details curvature of spacetime that corresponds to the mass distribution entered on the right side of the equation.
- Accordingly, the metric tensor is related to the **absolute interval**. Each different solution to the field equation corresponds to a different absolute interval.

Gravitational radiation (a.k.a. gravity waves)

One of the first results Einstein obtained from his new general theory of relativity was that there should be a gravitational analogue of light.

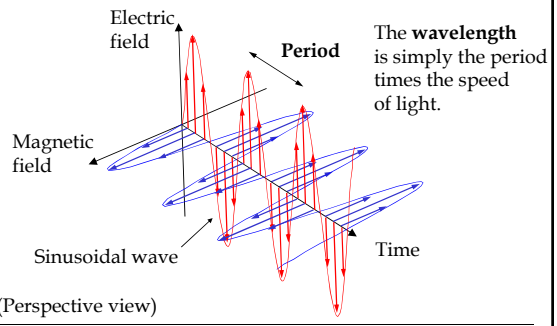
- By writing the field equation for spacetime that contains no masses, an equation is generated that has **waves** of curvature as its solution.
  - Specifically: the components of the metric tensor *g* vary in a periodic, repeating manner as the wave passes by a given point in space.
- These waves would propagate through empty spacetime at the same speed light does.
- Einstein noted that the effects of such a wave would be quite weak, though, and doubted that gravitational radiation would ever be observed.

Interlude: Light

Practically all of the information humanity has collected about celestial objects has arrived in the form of light.

- Light, like every other elementary form of energy, exhibits both wave and particle properties, depending upon what sort of experiment is being performed on it.
- In its **wave** guise, it consists of waves of electric and magnetic fields.
- This was first inferred by Maxwell in the 1860s:
  By writing the Maxwell equations for space that contains no electric charges or currents, and combining the results, equations are generated for the electric and magnetic field that have **sinusoidal** waves of electric and magnetic field as their solution.

A "plane wave" of light: electric and magnetic fields at one point in space, as functions of time.



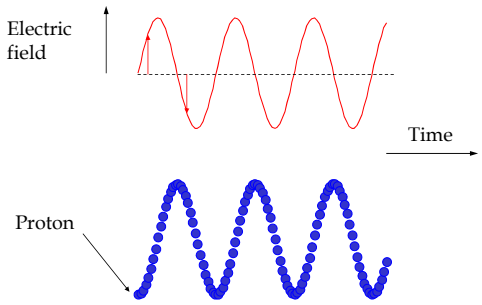
The **wavelength** is simply the period times the speed of light.

(Perspective view)

Some properties of light

- The ripples of electric and magnetic field that comprise light travel through empty space at the speed of light (of course).
- An electric field exerts a force on electric charges, in the direction of the field. A magnetic fields exerts a weaker force on a moving charge, in the direction perpendicular to both the field and the velocity.
  - Individual electric charges -- like protons or electrons -- will accelerate in response to a passing light wave.
- In turn, if charges are accelerated -- perhaps by some other force -- they emit light.
- Light represents the transport of electromagnetic energy through empty space, without involving the transport of electric charges or currents.

Snapshots of a proton's position when light is passing by

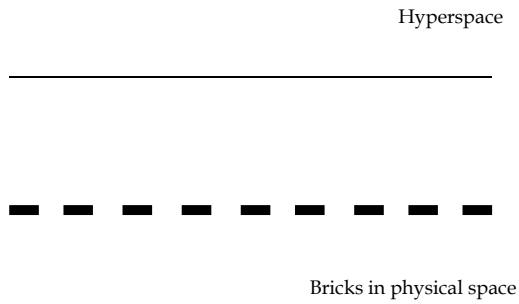


Some properties of gravitational radiation

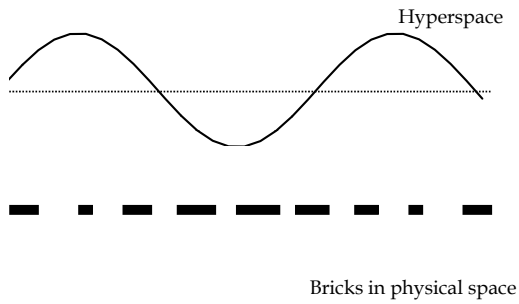
- A gravitational field exerts force on masses, in the direction of the field. Alternatively, one can think of this as changing curvature of spacetime, leading to motion of masses.
  - Spacetime will warp (mass will accelerate) in response to a passing gravity wave.
- In turn, if spacetime is warped (or masses are accelerated) gravitational radiation is produced.
- Gravitational radiation represents the transport of gravitational energy through empty space, without involving the transport of (rest) masses.

Note the direct analogy of gravity waves and light, and of masses and electric charges/currents.

No gravity wave, seen in physical space and hyperspace



Gravity wave, seen in physical space and hyperspace



Find the incorrect statement:

a. Gravity waves are solutions to the Einstein field equation, as light waves are to the Maxwell equations.
b. A gravity wave would make a mass bob up and down in physical space, as light would make an electric charge do.
c. Gravity waves travel through vacuum at speed *c*, just as light does.
d. Gravity waves are travelling bundles of gravitational field, as light waves are travelling bundles of electric and magnetic fields.

Direct detection of gravitational radiation

How could we detect gravity waves directly?

- **Bar detectors:** make very precise length measurements of a solid bar, which will stretch back and forth when a gravity wave passes by, as the bricks in our previous pictures do.
  (Obsolete, replaced by...)
- **Laser interferometers**: ultra-precise "bar-length" measurements, in principle capable of bypassing some of the limitations of the ordinary bar detectors.
  - LIGO (the laser interferometer gravity-wave observatory), is based upon this technology.

Direct detection of gravitational radiation (continued)

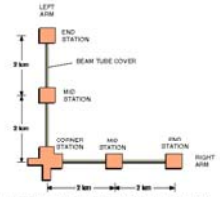


Fig. 1: Schematic layout of LIGO Site at Hanford, WA (Installation at Livingston, LA has no mid-stations)

LIGO Hanford, WA (Similar facility in Livingston, LA.)

Unfortunately, gravity waves from distant or ordinary processes *are* as weak as Einstein thought, so we are probably still years (decades?) away from the direct detection of gravity waves by instruments like LIGO.

Mid-lecture Break.

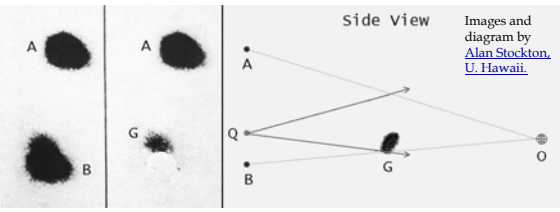
- Homework #2 is due tomorrow at 5:30 PM EST.
- Exam #1 will take place on in a week: 1 October 2009, your choice of any hour and fifteen minutes between 12 and 6 PM EST.
- It will be given on line, using WeBWorK. You may take it from anywhere.
- WeBWorK will also provide you a Practice Exam. It will appear on the system soon after the due date/time of Homework #2 has passed.
- A review session will be given right here in Hoyt, starting at 7 PM Wednesday evening, 30 September 2009, by Jae Song.

Experimental tests of general relativity

Until the discovery of black holes, general relativity was only tested with rather weak gravitational fields, but the variety of validations possible have been impressive. These have been the most important experiments

- Precession of the "perihelion" of Mercury's orbit: matter following gravity-induced curves in spacetime (Einstein, 1915, explaining the observation first made by Le Verrier, 1859).
- Gravitational lensing: light following gravity-induced curves in spacetime.
  - Stars visible during a solar eclipse that should be behind the Sun (Eddington, 1919).
  - Light from distant quasars deflected by galaxies (Walsh, Carswell & Weymann, 1979).

Gravitationally-lensed quasars

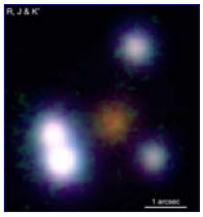


Images and diagram by Alan Stockton, U. Hawaii.

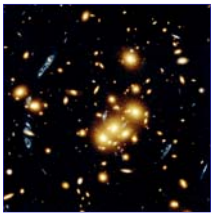
"Double quasar:" mirror image of A subtracted from B leaves a faint galaxy.

Light from distant quasar Q follows warped space around galaxy G; we (at O) see images of Q in two different places on the sky. (Vertical scale greatly exaggerated.)

Gravitational lenses



PG 1115+080: gravity of the nucleus of an unusually massive galaxy (red) produces four images (blue) of a much more distant quasar. (CISCO, Subaru Telescope, NAOJ)



GC 0024+1654: gravity of a galaxy cluster (orange-ish) produces several images of a more distant, ring-shaped galaxy (blue). (W.N. Colley et al., HST/NASA/STScI)

Experimental tests of general relativity (continued)

- Gravitational redshifts in the spectrum of stars (Adams, 1925) and on Earth (Pound & Rebka 1959): direct observation of gravitational time dilation.
- The "geodetic effect:" precession of a gyroscope in orbit (NASA LAGEOS and Gravity Probe B satellites, 1995-2005).

Of special importance among the weak-field validations of general relativity, though, is the

- Discovery of gravitational radiation (Hulse & Taylor 1975).

which is therefore worth illustrating in a little more detail.

PRSs, still.

Is it possible for scientific theories to be proven wrong?

A. Yes, by good experimental results that contradict their predictions.
B. Yes, by consensus of the best workers in the field.
C. No, as it is possible that they will eventually agree with experiment.
D. No, as long as any reputable researcher believes in them ("veto power").

Discovery of gravity waves: the Hulse-Taylor binary pulsar

In 1974, Princeton professor Joe Taylor and his graduate student Russell Hulse discovered and observed extensively a binary pulsar, now known as PSR 1913+16.

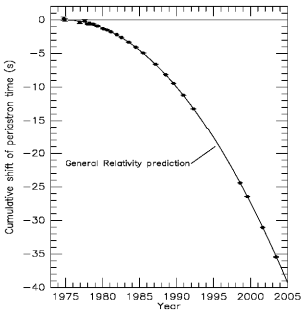
- The binary pulsar, as its name implies, consists of two **neutron stars** revolving around each other, one of which is a **pulsar**. (We will be studying neutron stars in a few weeks.)
- Pulse arrivals can be timed with exquisite accuracy. The pulse arrival times in PSR 1913+16 exhibit a periodic delay/advance resulting from the orbital motion.
- With high-precision pulse timing, Hulse and Taylor were able to derive the size of the orbit, the masses of the stars, and their velocities very accurately. By watching for a long time, they observed that the orbit is **shrinking**.

Discovery of gravity waves: the Hulse-Taylor binary pulsar (continued)

Because the orbit is shrinking, the binary system is losing energy somehow. Hulse and Taylor realized that this loss could be gravitational radiation: the neutron stars accelerate as they orbit.

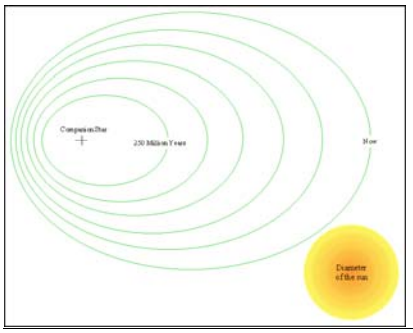
- So they calculated the gravitational-radiation loss expected from general relativity, for the stellar masses, orbital size and speed.
- The GR result is in precise agreement with their measurements.
- This observation therefore constitutes the **discovery of gravitational radiation**, and an important experimental verification of general relativity.
  The 1993 Nobel Prize in Physics went to Hulse and Taylor for this work.

Discovery of gravity waves: the Hulse-Taylor binary pulsar (continued)



Graph: the shift in "periastron time," an indicator of the distance of closest approach of the two neutron stars in PSR 1913+16, as a function of time. From Weisberg and Taylor 2005.

Discovery of gravity waves: the Hulse-Taylor binary pulsar (continued)



Projected size of the orbit, as a function of time. The two neutron stars will coalesce in about 300 million years. (From Weisberg, Taylor and Fowler 1981).

Experimental tests of general relativity (continued)

Results of experiments:

- All reproducible experiments to date have confirmed the predictions of Einstein's relativity theories.
- Few scientific theories are so well-supported by experiment, in fact.
- We keep using the theory to predict new effects. Those effects involving conditions within those for which the theory has been tested are very likely to be real.
- Experimental tests of these newly-predicted effects are in many cases even sterner tests of the theories.

Black holes were among the first of these "new effects" predicted by the general theory of relativity, though this was not recognized at the time.