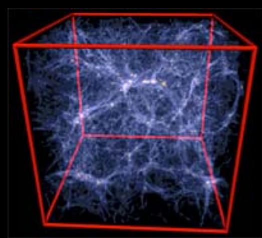
Today in Astronomy 102: Big Bang cosmology

- Expansion rates, ages and fates of GR universes in Big Bang cosmology.
- Inflation: how vacuum fluctuations might have kick-started the Universe.
- Matter-dominated universes, and measurements of the mass density and age of the Universe in which we live: an open Universe?



Simulation of structure in a universe dominated by cold dark matter and dark energy (Springel and Hernquist 2003).

Big bang cosmological models: finer points of the hieroglyphics

We won't be solving the Einstein field equation for the Universe, or use it on homework or exams, but to understand the differences between various universe models it is useful to look more carefully at the Hieroglyphics.

Universal expansion rate

Mass density (mass per unit volume)

Space (i.e. not spacetime) curvature

$$\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8\pi G}{3}\rho - \frac{c^2}{3}\Lambda = -c^2\frac{k}{R^2}$$

Typical distance between galaxies

Cosmological constant. Added by Einstein to allow static (constant in time) solutions.

Big bang cosmological models (continued)

$$\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8\pi G}{3}\rho - \frac{c^2}{3}\Lambda = -c^2\frac{k}{R^2}$$

This equation is a mathematical machine that can provide answers for all the terms in the equation (R, ρ, k, etc.) at all values of Time (past present and future), if it is given "initial" conditions: the values of these quantities at any time during the Universe's history.

- Usually, of course, we provide it measured values for the terms, at the *present* time.

Big bang cosmological models (continued)

It is popular to define a **critical mass density**, $\rho_0 = 3H_0^2 / 8\pi G$
a **normalized** mass density, $\Omega_M = \rho / \rho_0$
and a normalized cosmological constant, $\Omega_\Lambda = c^2 \Lambda / 3H_0^2$
in terms of which the field equation is

$$\left(\frac{1}{H_0 R}\frac{dR}{dt}\right)^2 - \Omega_M - \Omega_\Lambda = -\frac{c^2}{\left(H_0 R\right)^2} k$$

The critical mass density comes out to

$$\rho_0 = \frac{3H_0^2}{8\pi G} = 7.9 \times 10^{-30} \text{ gm cm}^{-3}$$

which is pretty small by Earthly standards.

Big bang cosmological models (continued)

Since Ω_M is a ratio of mass densities, it may be useful to think of Ω_Λ as a ratio of densities too. We often therefore define

$$\rho_\Lambda = \frac{c^2 \Lambda}{8\pi G} \qquad \text{so that} \qquad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_0}.$$

- And since ρ_Λ (and Ω_Λ) are expressed in the same units and terms as mass densities but are not densities of matter or radiation or anything related (like ρ, ρ_0 and Ω_M are), we need new words to name them.
  Currently the most popular name for the "substance" that corresponds to ρ_Λ and Ω_Λ is **dark energy**. $\rho_\Lambda c^2$ can be thought of as a dark energy density.

Big bang cosmological models (continued)

$$\left(\frac{1}{H_0 R}\frac{dR}{dt}\right)^2 - \Omega_M - \Omega_\Lambda = -\frac{c^2}{\left(H_0 R\right)^2} k$$

It looks at first as if Ω_M and Ω_Λ should have the same effect on how R and k come out in the solutions, but they don't.

- Since mass is conserved, the normalized mass density Ω_M decreases as the universe expands.
- Ω_Λ, related as it is to the cosmological constant, stays the same as the Universe expands.
- As we will see, this property makes positive values of Ω_Λ lead inexorably to expansion, no matter what the value of Ω_M may be. (And to inexorable collapse, for negative values.)

Big bang cosmological models (continued)

In these terms we will now discuss how the GR Big Bang model,

$$\left(\frac{1}{H_0 R}\frac{dR}{dt}\right)^2 - \Omega_M - \Omega_\Lambda = -\frac{c^2}{\left(H_0 R\right)^2}k$$

applies, and how it is constrained by measurements of some of the quantities like R, k, and Ω_M :

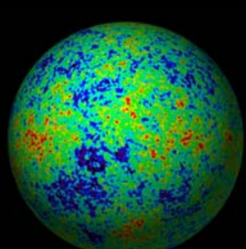
- The very early universe and **inflation**.
- Matter-dominated universes, like the one we used to think we live in, and **dark matter**.
- The new flat Universe, and **dark energy**.

Inflation: the cosmic microwave background is almost *too* isotropic.

No part of the cosmic microwave background differs in brightness from the average by more than 0.001%. It is hard to make gases, or the light they emit, that smooth or uniform. (Consider sunspots!)

- To do so would usually require that all parts of the gas be interacting with each other strongly, or that the gas be extremely well mixed.
- This would not seem possible for different parts of the decoupling surface. We were once part of that surface, and the parts of it that we see today have been out of contact with us (and each other) since the Big Bang, since we're only now receiving light from these parts and no signal or interaction can travel faster than light.

Inflation (continued)



Big Bang's visible surface: temperature varies by about 0.001% of the average. (WMAP/NASA)

Sun's visible surface: temperature varies by 10-20% of the average. (By Robert Gendler)

Inflation (continued)

One theoretically-popular way out of this problem is to postulate a brief period of **inflation** early in the Universe's history. Briefly, this is thought to happen as follows.

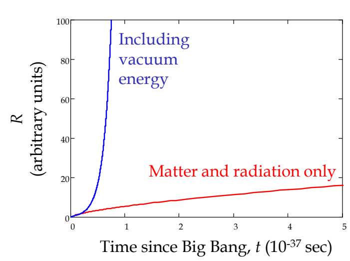
- Shortly after the Big Bang, the **vacuum** could have had a much larger energy density, in the form of virtual pairs, than it does today. This possibility is allowed under certain theoretical models of numbers and interactions of elementary particles.
- At some time during the expansion, the vacuum underwent a **phase transition** (like freezing or condensing) to produce the lower-energy version we have today, presumably driven by the changes in spacetime curvature.

Inflation (continued)

- While the vacuum was in its high-energy-density state, it gave a large additional impulse to Universal expansion.
  - Vacuum fluctuations fill whatever volume the Universe has, independent of how much real matter it contains.
  - Thus the high-energy vacuum acts like dark energy, i.e. like a cosmological constant.
- Accounting for the vacuum's influence in general relativity leads to a **very much smoother and faster expansion**. During this period, spacetime's radius of curvature increases more like a bubble blowing up, than like a blast wave - hence the name **inflation** for the process.

Inflation (continued)

Solutions to the field equation for the early Universe:



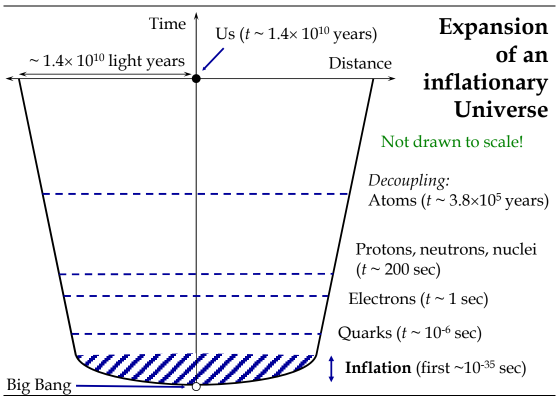
Inflation (continued)

The inflationary era would have been relatively brief, much shorter than the time between Big Bang and decoupling.

- If it lasted through 100 doublings of the Universe's size, that would do it, and this takes only about 10^{-35} seconds.
- During the remaining "normal" expansion between the end of inflation (decay of the vacuum to its low energy density state) and decoupling, the bumps and wiggles normally present in blast waves still wouldn't have had enough time to develop.

We know of course that the Universe has become much less smooth since decoupling. The seeds for inhomogeneities like galaxies, stars and people were not sown before decoupling, however.

Expansion of an inflationary Universe

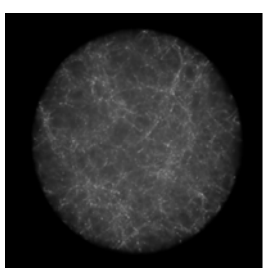


Mid-lecture Break

Exam #3 (the last one) will be a week from today, on WeBWorK.

- As usual, there will be a WeBWorK practice exam.
- Also as usual, there will be a review session, 7PM Wednesday, hosted by Brian DiCesare.

And Homework #6 is due Monday evening, 5:30 PM.



Cold-dark-matter Early Universe model (Institute for Computational Cosmology).

Matter-dominated universes

After inflation has come and gone, and decoupling has already happened, the energy density of everything we know about in the Universe (that is, taking $\Omega_\Lambda = 0$) is dominated by the rest mass of matter. Such a universe is called **matter-dominated.** Matter-dominated universes have three interesting properties:

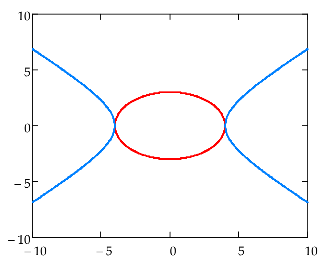
- The present matter density relative to the critical density, ρ_0, uniquely determines what the value of the curvature k is:

  If $\Omega_M = \dfrac{\rho}{\rho_0} > 1$ then $k = 1$ (positively-curved space)

  $\Omega_M = 1$ then $k = 0$ (flat space)

  $\Omega_M < 1$ then $k = -1$ (negatively curved space)

2-D reminder of positive and negative curvature



The **ellipse** is positively curved, and is a closed path.
The **hyperbola** is negatively curved and reaches to infinity.

Matter-dominated universes (continued)

- The curvature, in turn, determines the boundedness of the universe: whether the universe is open or closed. If it is open, it is possible for paths to extend to infinity, like the hyperbola; if closed, all paths eventually return to the starting point, like the ellipse.
- And further, the matter density uniquely determines the age and fate of the universe.

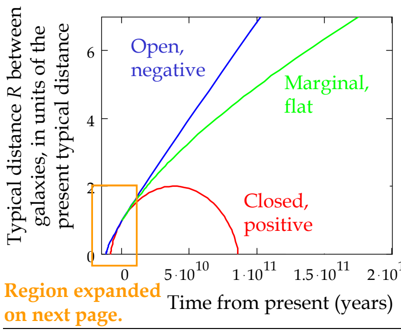
  If $\Omega_M = \dfrac{\rho}{\rho_0} > 1$ then the universe is gravitationally bound.

  $\Omega_M < 1$ then the universe is **not** gravitationally bound.

  $\Omega_M = 1$ is called a critical, or marginal, universe.

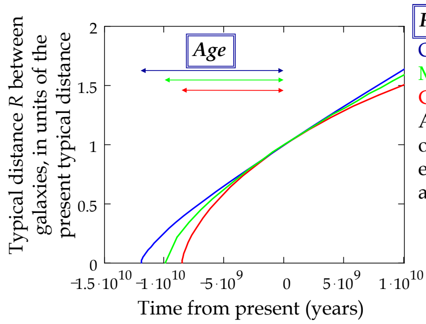
If bound, the expansion will reverse and it will re-collapse; if not, the expansion will continue forever.

Matter-dominated universes (continued)



Here are some results of such calculations, for matter-dominated universes with three different present-day densities. Labels indicate boundedness and the sign of the spacetime curvature k.

Matter-dominated universes (continued)



Fate

Open

Marginal

Closed

All matched to observed expansion rate at present time.

How can we tell which "universe" is our Universe?

Assuming we live in a matter dominated Universe there are three "simple" measurements we can make to determine which model applies:

1. Measure the **density** directly, using observations of the motions of galaxies to determine how much gravity they experience. (Much like our way of measuring black-hole masses by seeing the orbital motion of companion stars.)
2. Measure the **ages of the oldest objects** in the Universe.
3. Measure the **acceleration or deceleration of galaxies**: the rate of change of the Hubble "constant."

The first two ways are least difficult and provide most of our data. In order…

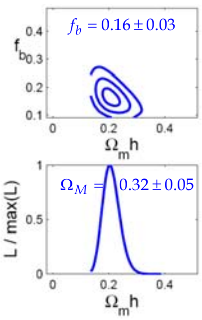
The mass density of the Universe

Astronomers have been perfecting measurements of this quantity for decades.

- Observational bounds on Ω_M, made from "nearby" galaxy redshift surveys over the past twenty years, consistently indicate

$$\Omega_M = 0.3 \pm 0.1$$

and that **only about 16% of this is normal matter**. We will take $\Omega_M = 1/3$.



Likelihood L of various values of $\Omega_M h$, where h = 0.65 (the Hubble constant in units of 100 km/sec/Mpc), and the resulting probabilities for the fraction of the mass that's in the form of normal ("baryonic") matter. From the Sloan Digital Sky Survey (Pope et al. 2004); see www.sdss.org.

Dark matter

You heard that right: we detect lots of matter, through that matter's gravity and its influence on the motions of galaxies – a third of the amount it would take to close the Universe – but only a small fraction of this mass, 0.16 of it (16%) exists in the form of normal matter (i.e. atoms).

- The rest (84%!) is called **dark matter** because it signals its existence only by its gravity (so far), not by emitting light.
- We don't know what it is made of; all we know is that it can't contain protons and neutrons. (It can't be photons or neutrinos or electrons either.)
- Thus we search for its nature among the zoo of elementary particles that can be produced and detected in high-energy physics experiments.

The mass density of the Universe (continued)

- So if the Universe is matter-dominated, its curvature is negative, it is open, and it will continue to expand.
- It is, however, a strong theoretical prediction of many models of elementary particles and of the early Universe, especially those involving **inflation**, that **Ω_M should be exactly 1**, and that for unknown reasons the present measurements of Ω_M are faulty. Observers and theoreticians used to argue incessantly about this.
- There are **no good experimental results or theoretical arguments to suggest that the universe is matter-dominated and closed**. We don't think our Universe is a black hole.

Age of matter-dominated universes

A general result of the solutions for matter-dominated universes is that the age is always given, in terms of the present value of the Hubble "constant", as

$$t = A\frac{1}{H_0}$$

where the value of the factor A depends on Ω_M, but is less than or equal to 1.

- The factor A is equal to 1 if Ω_M is very small compared to 1. The larger the value of Ω_M, the smaller the value of A. Open universes have values of A between 2/3 and 1, and closed universes have values of A smaller than 2/3.
- Jargon: $t = 1/H_0$ is often called "one **Hubble time**."

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Age of matter-dominated universes (continued)

- If Ω_M is assumed to be much smaller than 1, the age would be

$$t = \frac{1}{H_0} = \frac{\text{sec Mly}}{20\text{ km}} = \frac{\text{sec}\times\left(10^6\times 3\times 10^5\text{ km sec}^{-1}\times\text{year}\right)}{20\text{ km}}$$

$$= 1.5\times 10^{10}\text{ years.}\quad \text{(as we saw a few lectures back)}$$

- If Ω_M is assumed to be 1, the factor A turns out to be exactly 2/3, and the age is

$$t = \frac{2}{3}\frac{1}{H_0} = 1.0\times 10^{10}\text{ years}$$

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Age of matter-dominated universes (continued)

For the best experimental value, $\Omega_M = 1/3$, we get

$$t = 1.2\times 10^{10}\text{ years}$$

Other constraints on the Universe's age, independent of density determinations:

- We know that the Universe must be older than the solar system, which is 4.6×10^9 years old, so an age of 1.2×10^{10} years would be OK on this score.
- The ages of white dwarf stars and globular star clusters turn out to be accurately measurable; the oldest of these are 1.2×10^{10} years old (± about 0.1×10^{10} years).

This agrees with Ω_M = 1/3 (though smaller would be more comfortable)**, and is in conflict with Ω_M = 1.**

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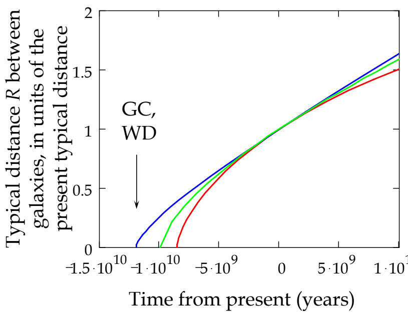
Age of matter-dominated universes (concluded)



$\Omega_M = 1/3$
$\Omega_M = 1$
$\Omega_M = 2$

The arrow marks the age of the oldest globular clusters and white dwarfs in the Milky Way.

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