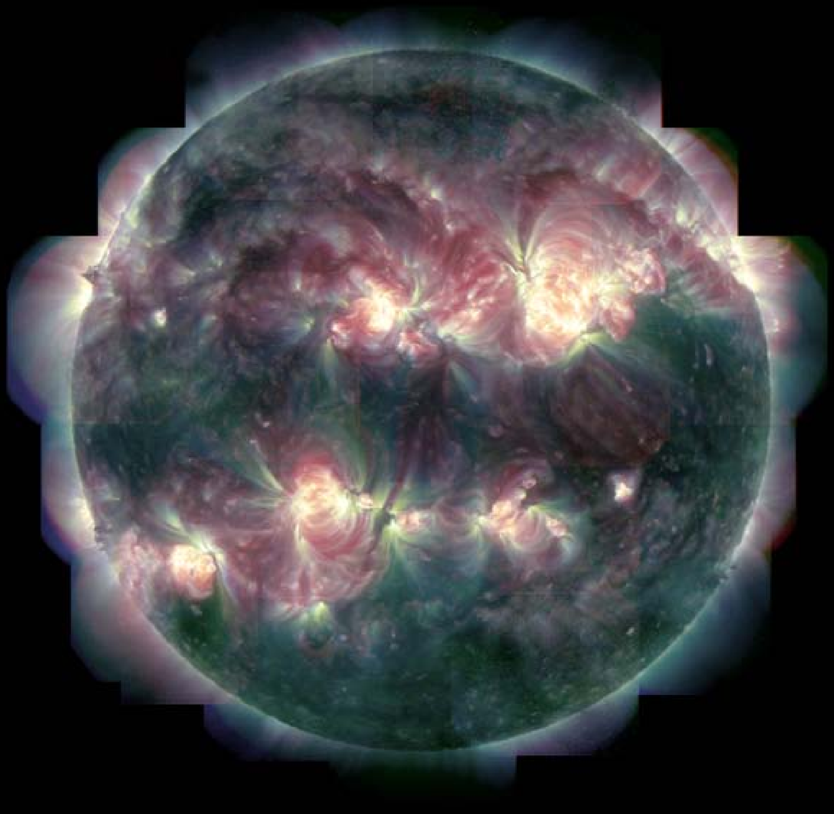
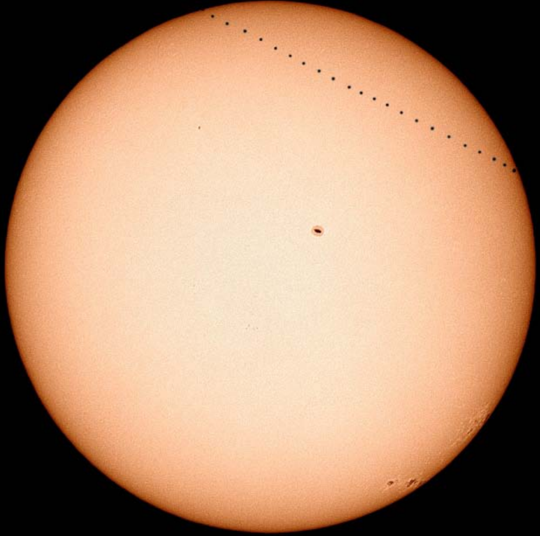
Today in Astronomy 111: the Sun and other blackbodies

- ☐ A few salient facts about the Sun
- ☐ Nucleosynthesis
- ☐ Blackbody radiation and temperatures of stars
- ☐ The spectrum of blackbodies, and solid angle
- ☐ Wien's Law



Three-color X-ray mosaic image of the Sun, made by the NASA TRACE satellite (J. Covington, LMMS)

The Sun: basic facts



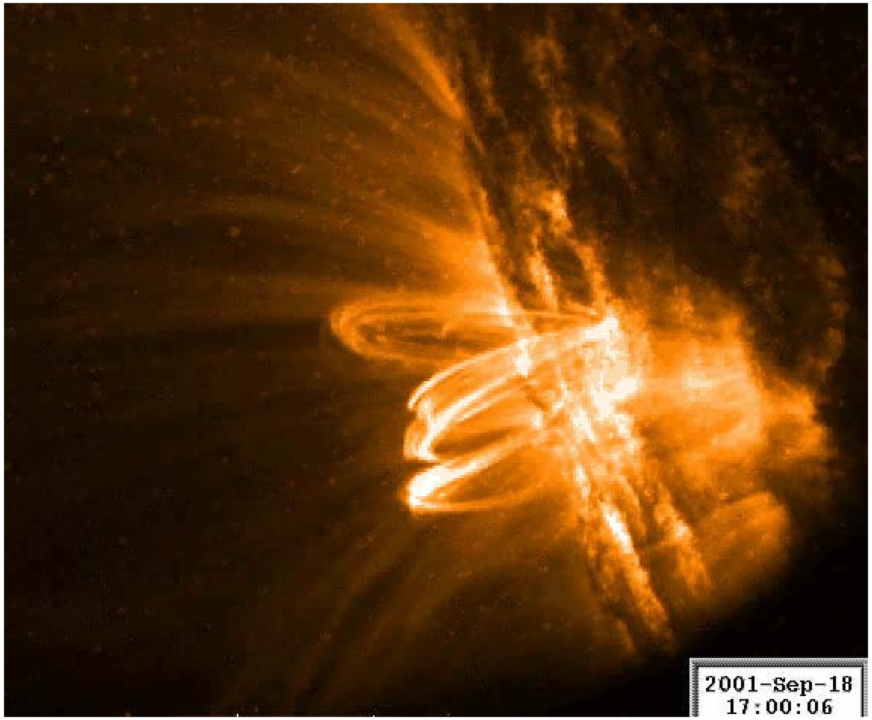
Sun, sunspots, and Mercury transit, May 2003, by Dominique Dierick)

Mass	1.98892×10^{33} grams
Luminosity	3.826×10^{33} erg sec^{-1}
Radius	6.96×10^{10} cm
Surface rotation period, sidereal, at equator	2.193×10^{6} sec (25.38 days)
Surface temperature	5800 K
Surface pressure	10 dyne cm^{-2} $\left(10^{-5} \text{ atmospheres}\right)$
Central temperature	1.58×10^{7} K
Central pressure	2.50×10^{17} dyne cm^{-2}
Surface composition, by mass	H (70.4%), He (28.0%), O (0.76%), C (0.28%), Ne (0.17%), N (0.08%), others (0.32%)

Much more later

One learns a lot more about the Sun and other stars, including the reasons why the Sun has the set of properties that it does, in AST 142 and AST 241.

For our purposes it will suffice to discuss only three of its properties now: its composition, its surface temperature, and its luminosity.



X-ray movie of loop prominences on the Sun's surface, from the NASA [TRACE] satellite.

The Sun's composition, and nucleosynthesis

The Sun is a sphere of hot ionized gas, obviously boiling at the surface, so one might think it's well mixed. It isn't.

- ☐ The Sun's size is determined by the balance of its weight and its internal pressure.
- ☐ The heat that provides the pressure would leak away (in the form of sunshine) within a few million years if it were not replenished somehow.
- ☐ The replenishment of heat is provided by **nuclear fusion** reactions taking place at the very center.
  - When atomic nuclei fuse to produce a nucleus lighter than that of iron (Fe, $Z = 26$, $A = 56$), kinetic energy is liberated (the reaction is **exothermic**), and added as heat to the energy of the surroundings.

Nucleosynthesis (continued)

- ❑ By the same token, this means that the contents of stars continuously transmute themselves from light elements to heavier elements.
  - For instance: models of the Sun's interior indicate that the composition at the very center is 33.6% H by mass, and 64.3% He. Much of the H there has been burned to He, mostly due to the **proton-proton** reaction chains.
  - In hotter, denser stellar cores, fusion of He (by the "triple alpha" reaction), C, O, Ne, and even heavier elements can take place at respectable rates…
  - …and elements heavier than Fe can be made in small quantities in such stars, by the **endothermic** (opposite of exothermic) **s-process** reactions.

And that's a Good Thing, for us

The Universe was born (in a fireball we can see, which we call the Big Bang) with hardly any elements heavier than helium.

- ❑ This is because fusion of two normal helium nuclei ($^4_2\mathrm{He}$) produces an isotope of beryllium ($^8_4\mathrm{Be}$) that is quite unstable, dissolving back into helium within 10^{-16} sec.
- ❑ Fusion of *three* normal helium nuclei produces a stable result:

$$3\left(^4_2\mathrm{He}\right) \rightarrow {}^{12}_{6}\mathrm{C} + \gamma$$

Triple-α process

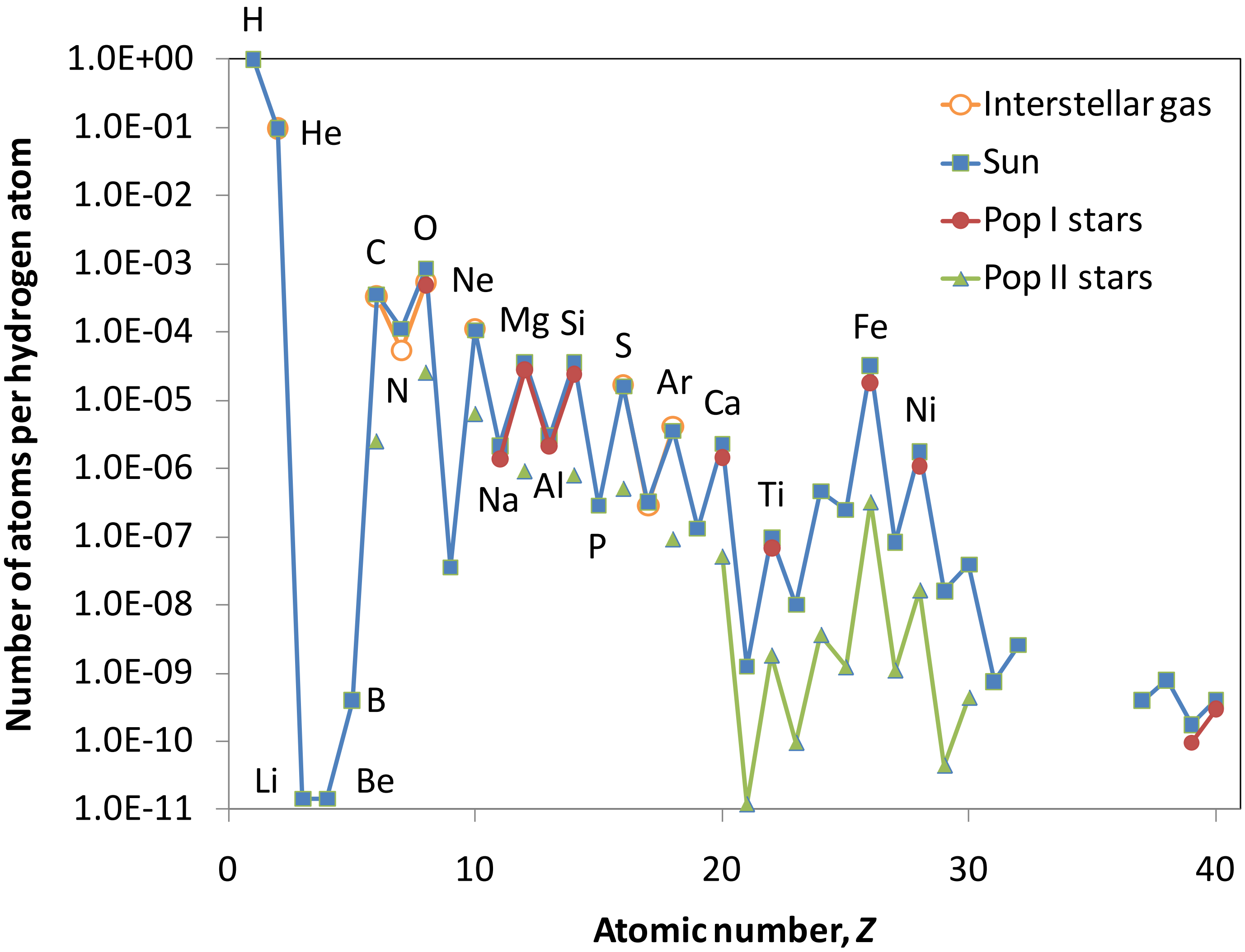
But very high density *and* temperature are necessary in order to make three He nuclei collide simultaneously, energetically enough to fuse. And by the time the Universe cooled enough for nuclei to condense, the density was too low.

Good Thing (continued)

In the cores of some stars, however, one can find temperature and density both large enough for the triple-α process to proceed.

- And the carbon produced thereby can react to form other heavier nuclei.
- Thus the heavy elements that the planets – and you – are made of were synthesized by nuclear reactions in the cores of stars that lived and died billions of years ago.
- Stars die before they burn more than about 10% of their total H into He, so the Universe will take a very long time to run out of hydrogen.

The nuclear-chemical evolution of the Milky Way



New stars

Younger stars

Older stars

The temperature of the Sun's surface

The Sun is a ball of hot gas; it has no sharp, solid surface. So what do we mean by the "surface of the Sun"?

- ☐ Usually we mean the deepest place we can *see* in the Sun's atmosphere; this is called the **photosphere**.
- ☐ The absorption of light in the Sun varies with wavelength, though, so the position of the photosphere is different for different parts of the spectrum.
- ☐ The temperature of course varies with depth within the Sun's atmosphere and interior, so the physical temperature of the surface that we can see is different for different wavelengths.

The temperature of the Sun's surface (continued)

So it is more convenient to characterize the Sun's "surface" temperature in a way that depends upon the luminosity (total power output), which of course is wavelength-independent.

- ☐ Thus is defined the **effective temperature** of the Sun: it's the temperature that would produce the Sun's observed luminosity, if the Sun were a spherical **blackbody** with the same radius.
- ☐ One learns all about blackbodies in PHY 143/123 and PHY 227. We will only list the properties and equations pertaining to blackbodies that we'll need to use this semester, and defer the detailed explanations to those courses.

Blackbodies and blackbody radiation

A blackbody is a body that is perfectly opaque: it absorbs all of the light incident on it.

- ❑ If such a body has a constant temperature, it must therefore also *emit* light, at exactly the same rate as it absorbs light. This is called **blackbody radiation.**
- ❑ The **flux** – total power emitted into all directions by a blackbody at temperature *T*, per unit area of the blackbody – of a blackbody is given by

$$f = \sigma T^4 \quad \text{, where}$$

Stefan's Law

$$\sigma = 5.67051 \times 10^{-5} \text{ erg sec}^{-1} \text{ cm}^{-2} \text{ K}^{-4}$$
$$= 5.67051 \times 10^{-8} \text{ joule sec}^{-1} \text{ m}^{-2} \text{ K}^{-4}$$

Stefan-Boltzmann constant

Definitions: luminosity and flux

Luminosity is the total power output (in the form of light) emitted in all directions by an object. Units = erg sec^{-1}.

- ☐ The Sun's luminosity, as we listed above, is

$$L_\odot = 3.83 \times 10^{33} \text{ erg sec}^{-1}$$

Flux is total power (in the form of light) per unit area that passes through a (sometimes imaginary) surface.
Units = erg $\text{sec}^{-1}\text{cm}^{-2}$.

- ☐ The flux of sunlight at the Sun's surface is

$$f = L_\odot \big/ 4\pi R_\odot^2 = 6.29 \times 10^{10} \text{ erg sec}^{-1} \text{ cm}^{-2}$$

- ☐ The flux of sunlight at Earth's orbit (the **solar constant**) is

$$f = L_\odot \big/ 4\pi (1 \text{ AU})^2 = 1.36 \times 10^{6} \text{ erg sec}^{-1} \text{ cm}^{-2}$$

The Sun's effective temperature

So if the Sun were a blackbody with radius equal to that of its visible photosphere ($R_\odot = 6.96\times 10^{10}$ cm) and luminosity as observed ($L_\odot = 3.83\times 10^{33}$ erg sec^{-1}), then its (effective) temperature T_e would be given by

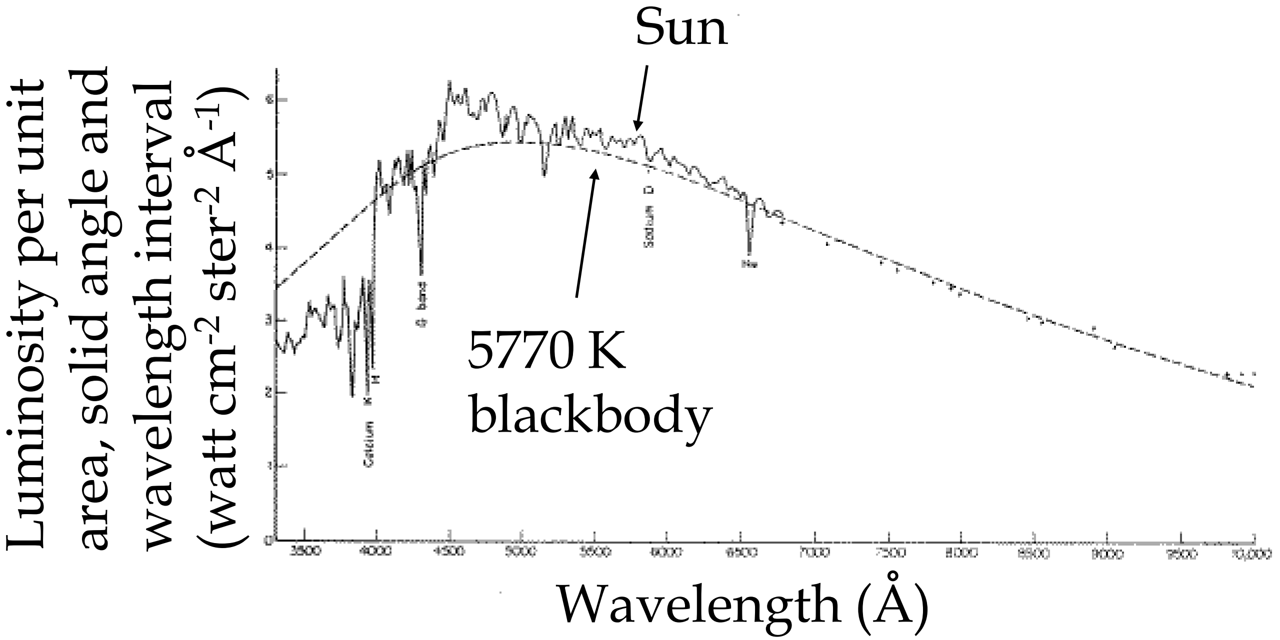
$$L_\odot = fA = \sigma T_e^4 \, 4\pi R_\odot^2 \quad , \text{ or}$$

$$T_e = \left(\frac{L_\odot}{4\pi\sigma R_\odot^2}\right)^{1/4} = \left(\frac{3.83\times 10^{33}}{4\pi\times\left(5.67\times 10^{-5}\right)\times\left(6.96\times 10^{10}\right)^2}\right)^{1/4} \text{ K} = 5770 \text{ K.}$$

Hence the number given above for the surface temperature. This turns out not to be far off the physical temperature typical of the photospheric region.

The spectrum of a blackbody

One reason the effective temperature comes close to the relevant physical temperature is that the spectrum of the Sun resembles the spectrum of a blackbody.



(from Carroll and Ostlie, *Modern astrophysics*)

The spectrum of a blackbody (continued)

The spectrum of blackbody radiation is given by the Planck function:

$$u_\lambda = \frac{2hc^2}{\lambda^5}\frac{1}{\exp\left(\dfrac{hc}{\lambda kT}\right)-1}\quad , \text{ where}$$

$$h = 6.626\times 10^{-27}\ \text{erg sec}$$ **Planck's constant**

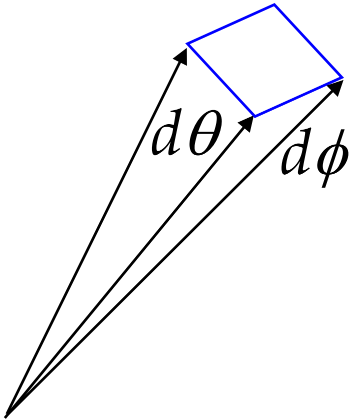
$$k = 1.381\times 10^{-16}\ \text{erg K}^{-1}$$ **Boltzmann's constant**

☐ The dimensions of the Planck function may seem confusing at first: they are power per unit area, per unit solid angle, per unit wavelength interval; for example, erg sec^{-1} cm^{-2} ster^{-1} μm^{-1}.

Solid angle?

Solid angle is to area what angle is to length. It's usually defined by a differential element, in spherical coordinates:

$$d\Omega = \sin\theta d\theta d\phi$$

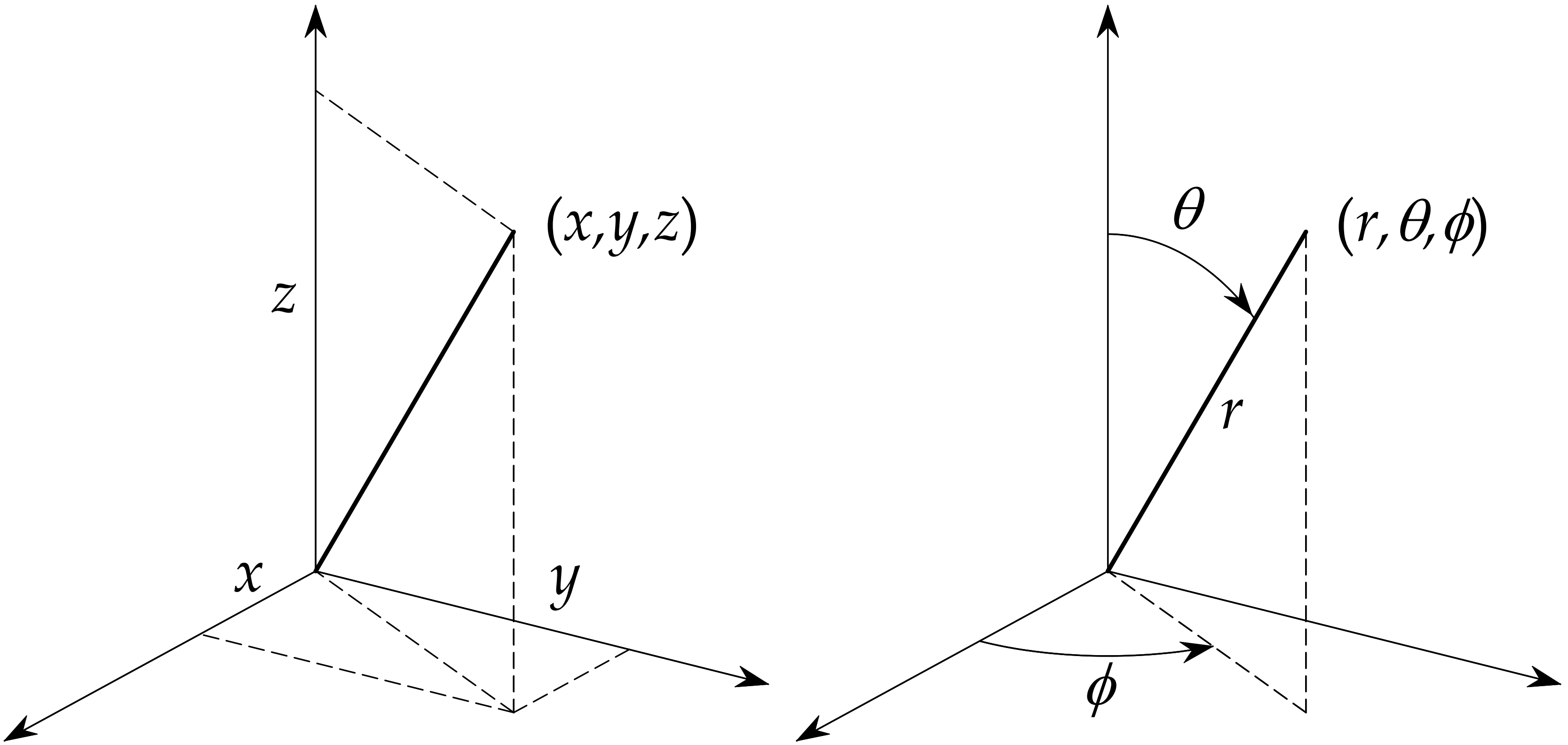


Angles in radians
Unit of solid angle = **steradian**

- For small angles, the solid angle is calculated from the angular widths of the "patch" in the same manner as plane-geometrical areas. For a rectangle the two angles,

$$\Delta\Omega = \Delta\phi \sin\theta \times \Delta\theta$$

Reminder: the spherical coordinates r, θ, ϕ



$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

$$r = \sqrt{x^2+y^2+z^2} \quad \theta = \arctan\left(\sqrt{x^2+y^2}\,/z\right)$$

$$\phi = \arctan\left(y/x\right)$$

Simple example

What is the solid angle of a square patch of sky 4.2 arcminutes on a side? (This is the solid angle covered by the CCD camera you will use on the Mees 24-inch telescope.)

4.2 arcminutes is a small angle (<< 1 radian). Since I didn't specify where the *z* axis is, we're free to point the it through the square's center, so $\sin\theta = 1$:

$$\Delta\Omega = \Delta\theta \times \Delta\phi = \left(4.2' \times \frac{1^\circ}{60'} \times \frac{2\pi \text{ radians}}{360^\circ}\right)^2$$

$$= 1.5 \times 10^{-6} \text{ steradians}$$

Computing solid angles

When the angles are not small, one must integrate the differential element over the range of θ and ϕ. You won't be doing that in AST 111, but just so you can see how it works…

The solid angle of the entire sky:

$$\Omega = \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta = -2\pi \times \cos\theta\Big]_0^{\pi} = -2\pi \times \left(-2\right)$$

$$\Omega = 4\pi \quad \text{steradians}$$

The solid angle of a cone with an angular radius of $\pi/8$ radians:

$$\Omega = \int_0^{2\pi} d\phi \int_0^{\pi/8} \sin\theta d\theta = -2\pi \times \cos\theta\Big]_0^{\pi/8} = 2\pi \times (1 - 0.924)$$

$$\Omega = 0.478 \quad \text{steradians}$$

The spectrum of a blackbody (continued)

- The reason for the funny dimensions of the Planck function is that it has to be integrated over wavelength, solid angle and area in order to produce a luminosity:

$$f = \int_{\Omega} d\Omega \int_{0}^{\infty} d\lambda u_{\lambda} \qquad L = \int_{A} dA \int_{\Omega} d\Omega \int_{0}^{\infty} d\lambda u_{\lambda}$$

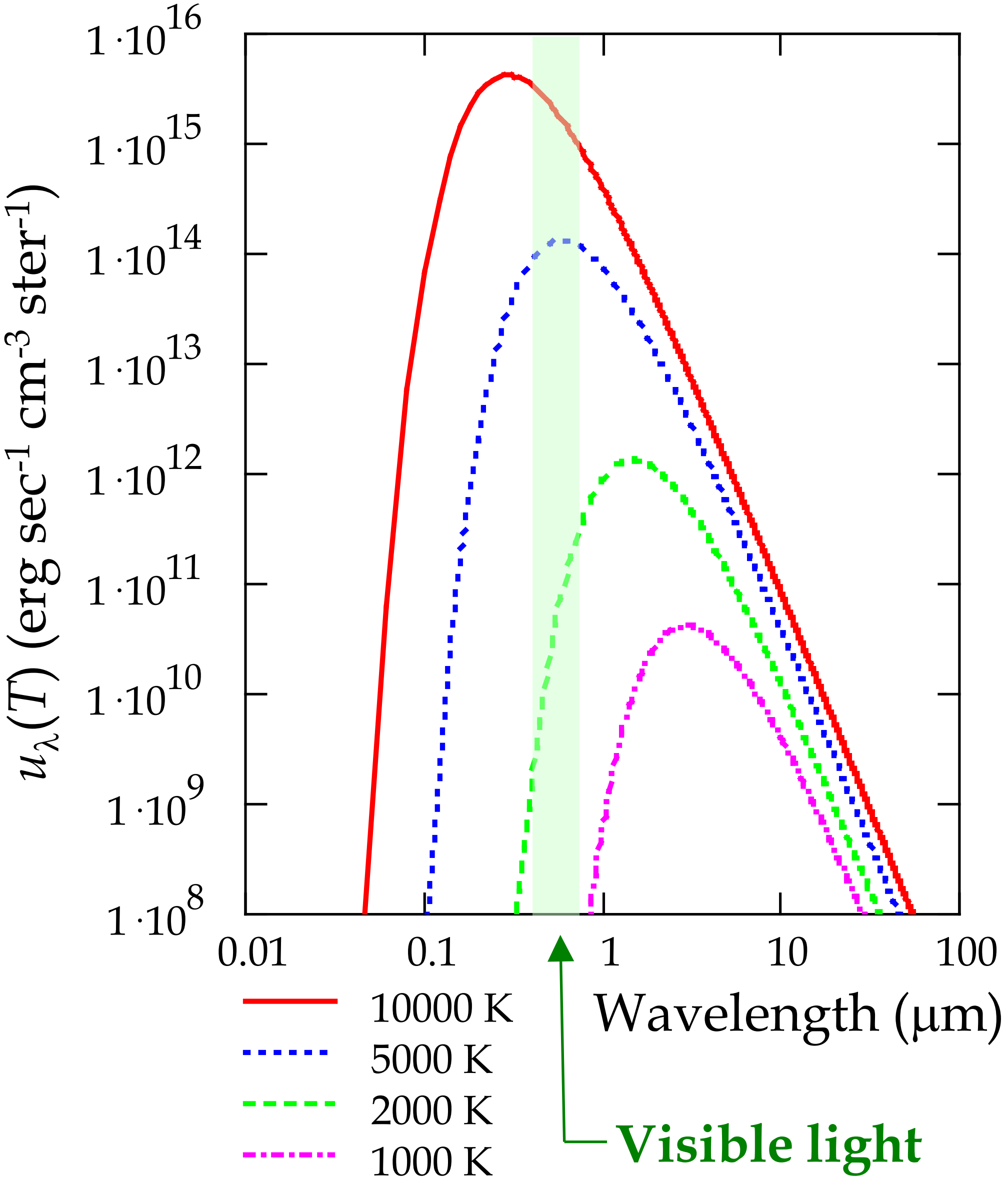
In PHY 227 or AST 241, one learns how to prove that the power per unit area emitted into all directions by a blackbody is

$$f = \int_{0}^{2\pi} d\phi \int_{0}^{\pi/2} d\theta \cos\theta \int_{0}^{\infty} d\lambda u_{\lambda} = \frac{2\pi^5 k^4}{15c^2h^3} T^4 \equiv \sigma T^4$$

that is, Stefan’s Law follows from the Planck function.

The Planck function

- ❑ Note that the higher the temperature, the shorter the wavelength at which the peak occurs.
- ❑ Note also that visible wavelengths don't include much of the luminosity, whatever the temperature.



Don't worry.

We won't be using the Planck function much, and we certainly won't be integrating it. It appears now because of a useful consequence of the *shape* of the function.

- ☐ It's fairly easy to convince one's self – as you will, in Homework #1 – that there's a simple relation between the temperature of a blackbody, and the wavelength at which it's brightest:

$$\lambda_{\max} T = 0.2897756 \text{ cm K}$$
$$= 2.897756 \times 10^{-3} \text{ m K}$$

Wien's law

- ☐ Thus **the shape of the spectrum tells one what the effective temperature is**. This shape is easier to measure accurately than luminosity is.

Examples

At what wavelength do you (T =98.6 F = 37 C = 310 K) emit the most blackbody radiation?

$$\lambda_{\max} = \frac{0.29 \text{ cm K}}{T} = \frac{0.29 \text{ cm K}}{310 \text{ K}}$$

$$= 9.4 \times 10^{-4} \text{ cm} = 9.4\ \mu\text{m}$$

Infrared light

Gamma-ray bursters emit most of their light at a wavelength of about 10^{-10} cm. If this emission were blackbody radiation, what would be the temperature of the burster?

$$T = \frac{0.29 \text{ cm K}}{\lambda_{\max}} = \frac{0.29 \text{ cm K}}{10^{-10} \text{ cm}} = 2.9 \times 10^{9} \text{ K}$$

Pretty hot