Today in Astronomy 111: Venus

- Inertial moments for spherical bodies: detecting differentiation.
- Venus as a twin of Earth: rock composition, tectonics, volcanism, and differentiation.
- Melting the insides of infant terrestrial planets with radioactivity.
- Loss of water and organic molecules from hot infant planets

End of the 2004 solar transit of Venus, from the Swedish 1 m solar telescope (D. Kiselman, Institute for Solar Physics)
Moment of inertia

Angular momentum $L$ and moment of inertia $I$ of a point mass $m$ revolving in a circle with radius $r_{\perp}$:

$$L = mvr_{\perp} = \left( mr_{\perp}^2 \right) \left( \frac{v}{r_{\perp}} \right) = I\omega$$

$$I = mr_{\perp}^2 \quad \text{Units are gm cm}^2$$

$$\omega = \frac{v}{r_{\perp}} \quad \text{Units are rad sec}^{-1}$$

In general for a point mass $m$ with velocity $v$ a displacement $r$ from the coordinate origin:

$$L = r \times p = r \times mv$$

$$= (mvr \sin \alpha) \hat{n} = mvr_{\perp} \hat{n}$$
Breaking a sphere into manageable bits

For non-point masses, e.g. planets, one finds the moment of inertia by decomposing the mass into chunks small enough to treat as point masses (infinitesimal elements: \( dm = \rho dV \)), and then add up (integrate) their moments of inertia.

In Cartesian coordinates:

\[
dV = dx dy dz
\]

In spherical coordinates:

\[
dV = (dr)(rd\theta)(r \sin \theta d\phi)
\]

\[
= r^2 \sin \theta dr d\theta d\phi
\]
Reminder: the spherical coordinates $r, \theta, \phi$

\[
x = r \sin \theta \cos \phi \\
y = r \sin \theta \sin \phi \\
z = r \cos \theta
\]

\[
r = \sqrt{x^2 + y^2 + z^2} \\
\theta = \arctan\left(\frac{\sqrt{x^2 + y^2}}{z}\right) \\
\phi = \arctan\left(\frac{y}{x}\right)
\]
Fundamental theorem of calculus:

\[
\text{if } f(x) = \frac{d}{dx} F(x), \text{ then } \int_{a}^{b} f(x) \, dx = F(b) - F(a).
\]

\[
\frac{d}{dx} x^n = nx^{n-1} \quad \Leftrightarrow \quad \int_{a}^{b} x^n \, dx = \frac{x^{n+1}}{n+1} \bigg|_{a}^{b} = \frac{1}{n+1} \left( b^{n+1} - a^{n+1} \right)
\]

\[
\frac{d}{dx} e^{cx} = ce^{cx} \quad \Leftrightarrow \quad \int_{a}^{b} e^{cx} \, dx = \frac{1}{c} \left( e^{cb} - e^{ca} \right)
\]

\[
\frac{d}{dx} \sin cx = c \cos cx \quad \Leftrightarrow \quad \int_{a}^{b} \cos cx \, dx = \frac{1}{c} \left( \sin cb - \sin ca \right)
\]

\[
\frac{d}{dx} \cos cx = -c \sin cx \quad \Leftrightarrow \quad \int_{a}^{b} \sin cx \, dx = -\frac{1}{c} \left( \cos cb - \cos ca \right)
\]
Moment of inertia of an undifferentiated sphere, about an axis through its center

Suppose the sphere has mass $M$, radius $R$, and uniform density $\rho$, by which we mean that density does not depend on $r$, $\theta$, or $\phi$. Then we know the relation of $M$ and $\rho$ without doing an integral:

$$\rho = \frac{M}{V} = \frac{3M}{4\pi R^3}$$

Within the sphere each bit of mass $dm$ follows a circle with radius $r_\perp = r \sin \theta$:

$$dI = r_\perp^2 dm = (r \sin \theta)^2 dm$$

$$dm = \rho dV = \rho r^2 \sin \theta dr d\theta d\phi$$
Moment of inertia of an undifferentiated sphere (continued)

Thus \( I = \int dI = \int r_{\perp}^2 dm \)

\[
= \int \int \int (r \sin \theta)^2 \rho r^2 \sin \theta dr d\theta d\phi \\
\quad \int_0^{2\pi} \int_0^{\pi} \int_0^R \sin^3 \theta d\theta \int_0^R r^4 dr : 
\]

three one-variable integrals we can take one at a time:

\[
\int_0^{2\pi} d\phi = 2\pi \\
\int_0^R r^4 dr = \frac{1}{5} R^5
\]
Moment of inertia of an undifferentiated sphere (continued)

The third one takes a few more steps:

\[
\int_0^{\pi} \sin^3 \theta d\theta = \int_0^{\pi} \left(1 - \cos^2 \theta\right) \sin \theta d\theta = \int_0^{\pi} \sin \theta d\theta + \int_0^{\pi} \cos^2 \theta (-\sin \theta d\theta)
\]

Substitute: \( u = \cos \theta, \ du = -\sin \theta d\theta \)

As \( \theta = 0 \rightarrow \pi, \ u = 1 \rightarrow -1, \) so

\[
\int_0^{\pi} \sin^3 \theta d\theta = -\cos \theta \bigg|_0^\pi + \int_{-1}^1 u^2 du = 1 + 1 + \frac{u^3}{3} \bigg|_{-1}^1 = 2 - \frac{1}{3} - \frac{1}{3} = \frac{4}{3}
\]

\[
I = \rho 2\pi \frac{4}{3} \frac{R^5}{5} = \frac{3M}{4\pi R^3} 2\pi \frac{4}{3} \frac{R^5}{5} = \frac{2}{5} MR^2
\]
as previously advertised.
Moment of inertia of a differentiated sphere

For example: density decreasing linearly from a central value of $\rho_0$ to zero at the surface:

$$\rho = \rho_0 \left(1 - \frac{r}{R}\right)$$

This time we have to do an integral to get $M$ and $\rho_0$ in terms of each other:

$$dm = \rho(r) r^2 \sin \theta dr d\theta d\phi,$$

$$M = \int dm = \rho_0 \int_0^{2\pi} d\phi \int_0^{\pi} \sin \theta d\theta \int_0^R r^2 \left(1 - \frac{r}{R}\right) dr$$

$$= \rho_0 \cdot 2\pi \cdot 2 \cdot \left(\frac{R^3}{3} - \frac{R^3}{4}\right) = \frac{\pi}{3} R^3 \rho_0 \Rightarrow \rho_0 = \frac{3M}{\pi R^3}$$
Moment of inertia of a differentiated sphere (continued)

\[
I = \int r^2 dm \\
= \int \int \int (r \sin \theta)^2 \rho_0 \left(1 - \frac{r}{R}\right) r^2 \sin \theta drd\theta d\phi \\
= \rho_0 \int d\phi \int \sin^3 \theta d\theta \int r^4 \left(1 - \frac{r}{R}\right) dr \\
= \frac{8\pi}{3} \rho_0 \left[ \frac{R^5}{5} - \frac{R^5}{6} \right] = \frac{8\pi}{3} \rho_0 \frac{R^5}{30} \\
= \frac{4}{15} MR^2 = 0.267 MR^2
\]

The integrals over \( \phi \) and \( \theta \) are the same as before \((2\pi \text{ and } 4/3)\), and will be as long as density doesn't depend on the angles.

also as previously advertised.
### Venus' vital statistics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>$4.8685 \times 10^{27}$ gm ($0.815M_\oplus$)</td>
</tr>
<tr>
<td>Equatorial radius</td>
<td>$6.0518 \times 10^8$ cm ($0.949R_\oplus$)</td>
</tr>
<tr>
<td>Average density</td>
<td>5.243 gm cm$^{-3}$</td>
</tr>
<tr>
<td>Moment of inertia</td>
<td>$0.33MR^2$</td>
</tr>
<tr>
<td>Albedo</td>
<td>0.67</td>
</tr>
<tr>
<td>Orbital semimajor axis</td>
<td>$1.0821 \times 10^{13}$ cm (0.72333 AU)</td>
</tr>
<tr>
<td>Orbital eccentricity</td>
<td>0.00677323</td>
</tr>
<tr>
<td>Sidereal revolution period</td>
<td>224.701 days</td>
</tr>
<tr>
<td>Sidereal rotation period</td>
<td>-243.686 days</td>
</tr>
<tr>
<td>Length of day</td>
<td>116.75 days</td>
</tr>
<tr>
<td>Magnetic field</td>
<td>zero</td>
</tr>
</tbody>
</table>

C. Hamilton

(Galileo/JPL/NASA)
Venus’ atmosphere’s vital statistics

- Surface pressure: 92 earth atmospheres
- Average temperature near surface: 737 K (464 C)
  - Compare to $T = 327$ K expected for its orbit
- Diurnal (day-night) temperature range: ~ 0
- Surface wind speeds: 0.3 - 1.0 m/s
- Atmospheric composition (near surface, by volume):
  - 96.5% CO$_2$, 3.5% N$_2$, 0.015% SO$_2$, 0.007% Ar, 0.002% H$_2$O

Ultraviolet image of Venus’ cloud tops by the Pioneer 1 Venus Orbiter (NASA)
Phases of Venus

Venus gets at most 47° from the Sun – as noted by the ancients – and displays phases, as first noted by Galileo (1610). These facts show directly that Venus orbits the Sun, not the Earth.

Photos by Wah!
Visits to Venus

Venus was the first planet to receive an Earthly spacecraft on its surface: the Soviet *Venera 7*, in 1970, lasted 23 minutes and made the first *in situ* report of temperature and pressure (748 K, 90 atmospheres).

- Including missions with other final destinations, there have been **25 successful visits** between *Mariner 2* (1962) and the current ESA *Venus Express* (2006-), highlighted by many Soviet *Venera* landers.

- Also a large number of failures, mainly in the Soviet *Sputnik*, *Zond*, and *Cosmos* series.

*Venera 9* landing site: the first image retrieved from the surface of a planet besides the Earth-Moon system (*Ted Stryk*).
Visits to Venus (continued)

Failures are dominated by early Soviet losses at launch or en route. The success rate of landers is good, considering the high degree of difficulty of descending through the dense atmosphere and operating in extreme heat.

<table>
<thead>
<tr>
<th>Agency</th>
<th>Successful Venus missions</th>
<th>Unsuccessful Venus missions</th>
</tr>
</thead>
<tbody>
<tr>
<td>NASA (USA)</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>USSR</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>ESA (EU)</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>JAXA (Japan)</td>
<td>0</td>
<td>(1)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Agency</th>
<th>Successes in retrieving data from the surface</th>
<th>Failures to retrieve data from the surface</th>
</tr>
</thead>
<tbody>
<tr>
<td>Venera and Vega (USSR)</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>NASA (USA)</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>
The composition of Venus' surface

Five of the *Venera* landers made composition measurements on soil and/or rock.

- *Venera* 8 reported a composition similar to granitic (felsic) rocks; all the others obtained a mafic composition.

- The best measurements come from *Venerae* 13 and 14, which landed in the highlands (13) and lowlands (14) of Beta Regio, and drilled into solid rocks.

- Both measured compositions closely similar to alkaline basalts, similar to those retrieved from mid-ocean ridges on Earth, and distinctly different from Lunar mare basalts.

X-ray spectra of Venereal rocks (solid curves) by *Venerae* 13 (top) and 14 (bottom) compared to ocean-floor basalts (points) (*Surkov et al. 1984*).
The structure of Venus’ surface

- Most mapping of Venus has been made by (cloud-penetrating) radar maps, such as those by the NASA Magellan and ESA Venus Express orbiters.
- Venus Express also includes high-resolution infrared cameras capable of thermal imaging, and shows that the warmest (coolest) parts of the surface lie at the lowest (highest) elevations.

VIRTIS/Venus Express (ESA)
The structure of Venus’ surface (continued)

The radar maps reveal many of the features of plate tectonics and volcanism that we know on Earth:

- many deep, asymmetrical troughs that resemble subduction zones, and long ridges that resemble mid-ocean ridges.
- some large shield volcanoes, and unique lava vents such as coronae and “pancake domes.”
  - Lava flows are seen to stretch many hundreds of km from such volcanoes.
- hardly any impact craters.

(Magellan/JPL/NASA)
Most of this activity is extinct, though.

- The tectonic-like features don’t link up to make a global plate-boundary system as on Earth.
- Still has active volcanoes, but not very many.
- Overall, there hasn’t been much tectonic activity over the past few hundred million years, although there was some long ago.

(Magellan/JPL/NASA)
Venus’ internal structure

Like Mercury and Earth, Venus is differentiated:

- It has a large bulk (average) density $(5.243 \text{ gm cm}^{-3})$ but its surface is covered in mafic silicate rocks $(3.2 \text{ gm cm}^{-3})$.

- It has a rather small moment of inertia $(I = 0.33MR^2)$, too small to represent uniform density, for which $I = 0.40MR^2$.

- So as one might suspect a priori, Venus’s internal structure appears similar to Earth’s, which we think includes a liquid iron-nickel core, plastic mantle, and crust.

Diagram by UCAR (U. Michigan)
Why new planets are molten, and thus become differentiated quickly

Brand-new terrestrial planets are usually very hot, hot enough for most or all of the volume to be molten.

- If the interior is molten, buoyancy rules: dense metals and minerals can sink freely to the center, and lighter ones can float to the surface.

There are two reasons why the interiors are so hot.

- **Accretion.** The gravitational potential energy of a planet’s ingredients decreases from zero to very large negative values in the act of forming, and since energy is conserved this leads to very large positive energy in the form of heat.

- **Radioactivity.** Just as in nuclear reactors, it converts nuclear electrostatic potential energy to kinetic energy of the products, which heat the medium. And planetary ingredients were a lot more radioactive then than now.
Why new planets are molten (continued)

We won’t be discussing accretion, heat transport, and the temperatures of planetary interiors til next month. But one can get an appreciation for the temperatures using what we know now about surface temperatures and radioactivity:

- Radioactivity occurs throughout the interior, and its heat input can be calculated just from the mass if the composition is typical of the materials out of which the planets were built: \( P_{\text{rad}} = M \Lambda_{\text{rad}}(t) \).
  - The heating rate per gram, \( \Lambda_{\text{rad}} \), decreases with time, as there’s no means to replenish decayed radionuclides with fresh interstellar-medium material.

- In turn this heat input can just be added to that due to sunlight, to work out the temperature of the surface.
An update to the textbook’s Table 5.3: radioactive heating rates for primitive (carbonaceous chondrite) meteoritic material, currently and at the birth of the Solar system.

<table>
<thead>
<tr>
<th>Radionuclide</th>
<th>Daughters</th>
<th>Halflife, Myr</th>
<th>Heating today, $10^{-8}$ erg/sec/gm</th>
<th>Heating 4567 Myr ago, $10^{-8}$ erg/sec/gm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{40}$K</td>
<td>$^{40}$Ar, $^{40}$Ca</td>
<td>1248</td>
<td>2.91</td>
<td>36.8</td>
</tr>
<tr>
<td>$^{232}$Th</td>
<td>$^{208}$Pb</td>
<td>14050</td>
<td>1.04</td>
<td>1.30</td>
</tr>
<tr>
<td>$^{235}$U</td>
<td>$^{207}$Pb</td>
<td>704</td>
<td>0.0500</td>
<td>4.60</td>
</tr>
<tr>
<td>$^{238}$U</td>
<td>$^{206}$Pb</td>
<td>4468</td>
<td>1.52</td>
<td>3.13</td>
</tr>
<tr>
<td>$^{26}$Al</td>
<td>$^{26}$Mg</td>
<td>0.717</td>
<td>0</td>
<td>25400</td>
</tr>
<tr>
<td>$^{36}$Cl</td>
<td>$^{36}$S, $^{36}$Ar</td>
<td>0.301</td>
<td>0</td>
<td>601000</td>
</tr>
<tr>
<td>$^{60}$Fe</td>
<td>$^{60}$Ni</td>
<td>2.60</td>
<td>0</td>
<td>4640000</td>
</tr>
</tbody>
</table>

$\Lambda_{\text{rad}}(t)$ for primitive meteorites, erg/sec/gm

2.60 $\times 10^{-8}$

$\Lambda_{\text{rad}}(t)$ for primitive meteorites, erg/sec/gm

5.52 $\times 10^{-1}$

5.27 $\times 10^{-2}$
Radioactive heating (continued)

So consider the **surface** temperature $T_s$ for a spherical planet (mass $M$, radius $R$) of solar-system composition, a distance $r$ from a star with luminosity $L$.

\[
P_{\text{in}} \text{ (sunlight and radioactive heating)} = P_{\text{sunlight}} + P_{\text{rad}} = P_{\text{out}} \text{ (blackbody radiation)}
\]

\[
\frac{L}{4\pi r^2} \pi R^2 + M \Lambda_{\text{rad}}(t) = 4\pi R^2 \sigma T_s^4
\]

\[
T_s = \left( \frac{LR^2 + 4M\Lambda_{\text{rad}}(t)r^2}{16\pi\sigma r^2 R^2} \right)^{1/4}
\]

The Sun’s luminosity was larger by a factor of about 2.5 back then, but $\Lambda_{\text{rad}}$ was larger by a factor of almost a million.
Radioactive heating (continued)

Results, assuming that each body lacks an atmosphere and formed in a time short compared to the halflife of $^{60}$Fe.

- Compare to the melting points of planetary ingredients: 1000-1500 K for silicate rocks, 1800 K for iron.

- And, though we haven’t learned how to calculate interior temperatures yet, most of us will realize that each body is hotter inside, if heated from within.

- So Venus was almost certainly molten all the way through when new, and probably Mercury too.
Loss of volatile molecules from newborn planets

Venus’s original ingredients were no doubt similar to the small primitive Solar-system bodies we find as meteorites, and were rich in organic molecules and ices.

- These molecules contain nearly all of the carbon and nitrogen and hydrogen in meteorites. A little oxygen too.
- When Venus was molten these molecules were all evaporated into a temporary atmosphere.
- Ultraviolet sunlight quickly breaks such molecules up into atoms, which are all light enough to escape the gravity of the planet.
- Thus Venus, like every other terrestrial planet, is very poor in carbon, nitrogen and hydrogen, compared to other Solar system bodies.
Loss of volatile molecules (continued)

- The carbon, nitrogen and hydrogen currently possessed by the terrestrial planets was added after their surfaces cooled, by captured asteroids and comets.
- This includes the hydrogen in water: 90% from asteroids, 10% from comets.

<table>
<thead>
<tr>
<th>Element</th>
<th>Earth</th>
<th>Sun</th>
<th>Comets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen</td>
<td>147058.8</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>Carbon</td>
<td>&lt; 0.1</td>
<td>44.1</td>
<td>44.1</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>13.4</td>
<td>13.4</td>
<td>13.4</td>
</tr>
<tr>
<td>Oxygen</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>Sodium</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Magnesium</td>
<td>30.4</td>
<td>4.3</td>
<td>4.3</td>
</tr>
<tr>
<td>Aluminum</td>
<td>2.8</td>
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<td>0.3</td>
</tr>
<tr>
<td>Silicon</td>
<td>28.6</td>
<td>3.7</td>
<td>3.7</td>
</tr>
<tr>
<td>Phosphorus</td>
<td>0.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sulfur</td>
<td>4.8</td>
<td>1.4</td>
<td>1.4</td>
</tr>
<tr>
<td>Potassium</td>
<td>0.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calcium</td>
<td>2.0</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Iron</td>
<td>30.6</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>Nickel</td>
<td>1.6</td>
<td>0.2</td>
<td>0.2</td>
</tr>
</tbody>
</table>