# Astronomy 111 Recitation #1

1-2 September 2011

**Workshop problems**

**Warning!** The workshop problems you will do in groups in Recitation are a crucial part of the process of building up your command of the concepts important in AST 111 and subsequent courses. Do not, therefore, do your work on scratch paper and discard it. Better for each of you to keep your own account of each problem, in some sort of bound notebook.

1. *Calculate and illustrate with a drawing.* The radius of the Earth is $R_\oplus = 6.4 \times 10^8$ cm. How long (in cm) is the equator? How long (in cm) is the $45°$ parallel? (*Hint*: what is the radius of the $45°$ parallel?) What is the distance (in cm) between two points on the equator that lie 0.01 radian of longitude apart? Between two points at latitude $45°$?

2. Generalize the previous result: what is the distance, $d$, between two points at latitude $\lambda$, that differ in longitude by $\Delta L$?

3. Adapt the previous result: in the view of an observer at the Earth's center, what is the angle $\Delta\theta$ (in radians) between two points on the Earth's surface at latitude $\lambda$, that differ in longitude by a small amount $\Delta L$?

4. And generalize *that* result. The celestial coordinate system which astronomers use to describe the location of objects on the sky is the projection of the lines of latitude and longitude onto the *celestial sphere*. The angle corresponding to longitude is called *right ascension* ("RA," usually given the symbol $\alpha$) and *declination* ("Dec", or $\delta$). The zero point of RA is one of the two places at which the celestial equator (the projection of the Earth's equator on the celestial sphere) intersects the plane of the solar system (the zodiac), and increases toward the east. See Figure 1.

   So in the view of an observer on Earth, what is the angle $\Delta\theta$ (in radians) between two points at declination $\delta$ on the celestial sphere, that differ in right ascension by the small amount $\Delta\alpha$?

**Learn your way around the sky, lesson 1.** (An *exclusive* feature of AST 111 recitations.) Use the lab's celestial globes, TheSky running on the lab computers, SIMBAD at http://simbad.harvard.edu/simbad/sim-fid, and any other resources you would like to use, to answer these questions about the celestial sphere, the constellations, and paths of the planets through the sky.

5. You are in Rochester staring at a field of stars that lies on the celestial equator, and envisioning their celestial coordinates. Simple question: do the stars' right ascensions increase from left to right, or right to left, as one moves across the starfield? Check this by consulting the celestial globes at your table.

6. Normally, astronomers report right ascension in *hours* instead of degrees, because the rotation of the Earth makes the sky march across one's view so that the whole sky is covered in one rotation.

   How many degrees are there in one hour of right ascension?

7. In a week or two you will derive (here in workshop) this approximate formula for the angle between two celestial coordinates, $(\alpha_1, \delta_1)$ and $(\alpha_2, \delta_2)$:

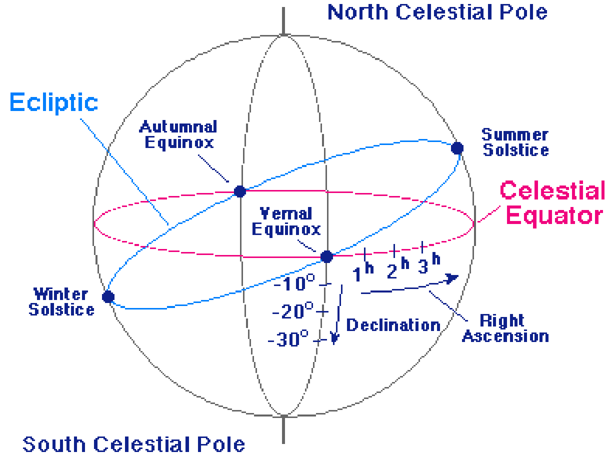



Figure 1: the celestial coordinate system. The intersections between the celestial equator and the ecliptic (a.k.a. zodiac) are called the *equinoctes*. One of these, the *vernal equinox*, is the zero point for right ascension. The vernal equinox is where the Sun is on the first day of Spring.

$$\Delta\theta = \left[ (\delta_1 - \delta_2)^2 + (\alpha_1 - \alpha_2)^2 \cos\delta_1 \cos\delta_2 \right]^{1/2}$$

Use this to calculate the angular length of the base of the Big Dipper, defined by the stars Merak and Phecda. Look up their coordinates in SIMBAD:

http://simbad.harvard.edu/simbad/sim-fid

8. *Discussion question.* You are Christopher Columbus. You want to get rich by finding a short cut to the Indies, China and Japan. All you know about those places is that Japan is supposed to be at a latitude similar to Spain's, that China is much bigger and stretches south from Japan, and the Indies are a collection of large and small islands east of a jungle-covered peninsula running NW-SE (the Malay peninsula) that stretches down from China. You have seen many estimates of the size of Asia and the circumference of the Earth, and have noted that if one takes the largest estimate for Asia and the smallest for Earth that these places could be as close to Spain as 3000 miles. You are also a good sailor, and know well the pattern of winds in the eastern Atlantic. So you set off on your journey. Here's what you see:

   a. There are indeed islands – large ones mixed with small ones – only a few thousand miles due west of the Canary and Cape Verde Islands (that is, well south of Spain). Remember that distances E-W were hard to measure in Columbus's day.

   b. These islands seem to bound a mirror image of the winds in the east: a circulation pattern whose N-S extent is a few thousand miles -- consistent with the dead-reckoning estimate of the E-W distance.

   c. Travelling further west one finds an unbroken, jungle-covered coastline running roughly NW-SE.

   d. The islands and the unbroken coast are inhabited by people who resemble the oriental people as described in Marco Polo's book.

e. Further south – past all the islands – another unbroken coast is seen, from which drain rivers sufficiently large that this land must be a continent, not an island.

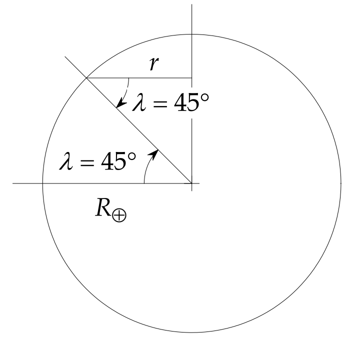
Columbus concluded from these observations that he was in the Indies. Now we know he wasn't, of course, but, thinking as a scientist: what was wrong with his reasoning, and what clues should he have interpreted differently that would have put him on the right track?

9. *Observation planning*. When is the next night that is likely to be clear at Mees Observatory? Try

www.weather.com or
http://cleardarksky.com/cgi-bin/find_chart.py?keys=mees&type=text&Mn=telescope

Who is available to go observing on that night? Collect at least one group of no more than four people who can observe that night, each of whom should read the "Stargazing" portion of the Project manual, choose a non-planetary celestial object to take pictures of, and provide their contact information to the instructors.

**Problem solutions**




1. The radius of the equator is the same as the radius of the Earth, so of course its length is $2\pi R_\oplus = 4\times10^9$ cm. The radius of the 45th parallel is $r = R_\oplus \cos 45°$ (see the sketch at right), so its length is $2\pi r = 2\pi R_\oplus \cos 45° = 2.8\times10^9$ cm. Since the parallels all cover $360°$ of longitude, the length of 0.01 rad of longitude on the $45°$ parallel is $0.01/2\pi$ of this, or $4.5\times10^6$ cm.

2. Evidently, then, the circumference of the parallel at latitude $\lambda$ is $2\pi R_\oplus \cos\lambda$, and $d$ comprises only $\Delta L$ of the parallel's $2\pi$ radians, so

$$d = \frac{\Delta L}{2\pi} 2\pi R_\oplus \cos\lambda = \Delta L R_\oplus \cos\lambda \qquad (\Delta L \text{ in radians}) .$$

3. Angle is subtense divided by radius:

$$\Delta\theta = \frac{d}{R_\oplus} = \Delta L \cos\lambda \qquad \text{(angles still in radians).}$$

4. And RA is analogous to longitude, and Dec to latitude, so

$$\Delta\theta = \Delta\alpha \cos\delta \qquad \text{(angles still in radians).}$$

5. RA increases west to east. (See Figure 1.) As you see it, looking up at the sky – and noting that you have to look toward the south to see the celestial equator – east is to your left and west to your right. So RA increases right to left. (Remember this view. Newbies are always confused when they see astronomical images – in which north is up but east is left instead of right – because they forget that they're looking up at the sky, not looking down on the ground, map-wise.)

6. A rotation is 24 hours or 360 degrees, so there are 15 degrees in an hour.

7. The coordinates are

$$\begin{aligned} \alpha_1 &= 11\text{h } 01\text{m } 50.477\text{s} = 11.031\text{h} & \alpha_2 &= 11\text{h } 53\text{m } 49.847\text{s} = 11.897\text{h} \\ \delta_1 &= +56°22'56.736'' = +56.382° & \delta_2 &= +53°41'41.136'' = +53.695° \end{aligned}$$

for Merak and Phecda respectively, so we get

$$\begin{aligned} \Delta\theta &\cong \sqrt{\left(\frac{15°}{\text{h}}\right)^2 \left(11.897\text{h} - 11.031\text{h}\right)^2 \cos(53.695°)\cos\left(56.382°\right) + \left(53.695° - 56.382°\right)^2} \\ &= 7.92° \end{aligned}$$

8. Any answer along these lines would be acceptable:

By and large, his observations were good and his reasoning was sound: his observations matched previous descriptions of the Indies and the Malay peninsula. One could complain that he cherry-picked his data on the sizes of Asia and the Earth's circumference, but he did check that with the size of the trade-wind circulation and got good agreement. Columbus is often slammed for not recognizing that South America was a significant part of the story, but after all it is well south of anywhere for which he had a description, and his main interest was getting to China. And he did recognize that what we now call South America is a continent.

One could fault him for not trying to sail around the end of the Malay peninsula, I suppose.

The big problem: most of the "theory" to which he was matching his measurements was pretty vague and suspicious, and the distances he was using turned out to be all wrong. But he wasn't doing anything stupid. If you had a theory for which the first several predictions were borne out by observations, what would you have thought?

9. The rest of this week, and all of Labor Day Weekend, look pretty grim weather-wise. The next two or three days after that, however, are quite promising and should definitely be used for observing.