Astronomy 111 Recitation #2

8-9 September 2011

Formulas to remember

Blackbody radiation:

Stefan's law: $f = \sigma T^4$, where $\sigma = 5.67051 \times 10^{-5}$ erg sec^{-1} cm^{-2} K^{-4}

Planck function:

$$u_\lambda = u_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda kT}\right) - 1} \quad ,$$

where $h = 6.626 \times 10^{-27}$ erg sec, $k = 1.381 \times 10^{-16}$ erg K^{-1}.

Dimensions of the Planck function are power per unit area, per unit solid angle, per unit wavelength interval.

Wien's law: $\lambda_{max} T =$ constant = 0.2897756 cm K.

Cartesian, spherical and celestial coordinates (see Figure 1):

$$x = r \sin\theta \cos\phi \qquad r = \sqrt{x^2 + y^2 + z^2}$$
$$y = r \sin\theta \sin\phi \qquad \theta = \arctan\left(\sqrt{x^2 + y^2}/z\right)$$
$$z = r \cos\theta \qquad \phi = \arctan(y/x)$$

If the x axis points toward the Vernal equinox and the z axis toward the north celestial pole, then

$$\alpha = \phi \quad , \quad \delta = 90° - \theta \quad .$$

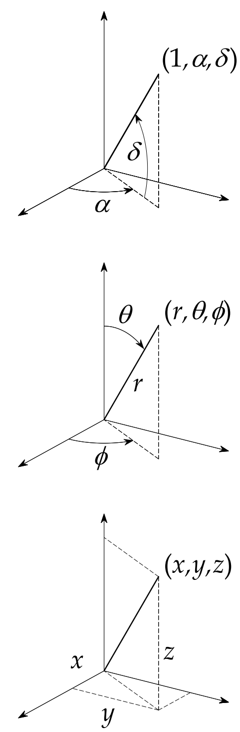

Figure 1: Celestial (top), spherical (middle), and Cartesian (bottom) coordinate systems.

Workshop problems

Warning! The workshop problems you will do in groups in Recitation are a crucial part of the process of building up your command of the concepts important in AST 111 and subsequent courses. Do not, therefore, do your work on scratch paper and discard it. Better for each of you to keep your own account of each problem, in some sort of bound notebook.

1. Estimate your body's surface area A (in cm^2), your skin temperature (in K, assumed uniform over your body), and the room's temperature (also in K).

2. How much power, P_{em} (in erg sec^{-1}), does your body emit, in blackbody radiation?

3. How much power, P_{abs} (in erg sec^{-1}), does your body *absorb*, in the room's blackbody radiation?

4. Suppose the balance between these two powers has to be made up by chemical energy released in your body, ultimately from the digestion of food. How much power, P_{food} (in erg sec^{-1}), does your body need to produce? How many calories per hour is that? (Note: a *dietary* calorie, which is what we mean here, is what chemists call a kilocalorie, and is equal to 4.18×10^{10} erg .)

Learn your way around the sky, lesson 2. (An *exclusive* feature of AST 111 recitations.) Use the lab's celestial globes, TheSky running on the lab computers, SIMBAD at http://simbad.harvard.edu/simbad/sim-fid, and any other resources you would like to use, to answer these questions about the celestial sphere, the constellations, and paths of the planets through the sky.

a. *Celestial, spherical and Cartesian coordinates*. A star has right ascension and declination α and δ. Draw a Cartesian coordinate system with the *x*-axis pointing through $\alpha = \delta = 0$ and the *z*-axis pointing toward $\delta = 90°$. Write an expression for a unit vector pointing at the star, in terms of the Cartesian unit vectors $\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}$ and $\hat{\boldsymbol{z}}$.

b. Recall that the inner ("dot") product of two vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ can be written in these two ways:

$$\begin{aligned} \boldsymbol{A} \cdot \boldsymbol{B} &= AB\cos\psi \\ &= A_x B_x + A_y B_y + A_z B_z \quad , \end{aligned}$$

where A and B are the magnitudes of the two vectors, ψ is the angle between them, and the subscripts *x*-*y*-*z* indicate the components of the vectors along those Cartesian axes.

Two stars have celestial coordinates α_1, δ_1 and α_2, δ_2. Use your result from part a to obtain an *exact* expression for the angle between (the directions toward) these stars. Remember this resulting equation: it is one of the two equations that are fundamental for understanding angular distances along the celestial sphere.

(Hint: for simplification, recall the trigonometric identity $\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$.)

c. Straight over head at about 9PM these days, one finds a distinctive triangle of very bright stars, commonly called Vega, Deneb and Altair. This is the *summer triangle*, and is a useful landmark for finding other, fainter objects and constellations in the summer sky.

To what constellations do the three stars belong?

d. Look up the coordinates of these stars in SIMBAD (http://simbad.harvard.edu/simbad/sim-fid). What are the dimensions of the summer triangle – that is, the lengths of the legs of the triangle, in degrees? (Three significant-figure answers are good enough.)

e. What is Vega's main claim to fame?

f. In one of the most ambitious extrasolar-planet-hunting projects to date, the NASA *Kepler* satellite (http://kepler.nasa.gov/) is continuously monitoring the brightness of some 100000 nearby stars. The idea is to find a large number of stars in which periodic, very small decreases in brightness are seen. This would indicate "eclipses" of these stars by planets whose orbits are viewed edge-on. All of these 100000 stars lie in a small patch of the sky, at which the *Kepler* telescope stares continuously from the vantage of its spacecraft's Earth-trailing orbit around the Sun.

Starting from the Summer Triangle, describe how to find the patch of sky at which *Kepler* is staring. (This will be interesting to point out to your friends. The *Kepler* mission team has announced a list of 54 Earth-size planets in this little patch of sky which lie in the habitable zones of their host stars.)

Problem solutions

1. I estimate my height to be h =174 cm, and I am about p = 81.3 cm around. Approximating myself as a cylinder (don't forget the circular ends), I get an area of $A = ph + 2\pi\left(p/2\pi\right)^2 = 1.5\times10^4\ \mathrm{cm}^2$. I think the temperature of the room is about 12 C (285 K) and that of my skin is about 19 C (292 K).

2. Thus I emit

$$P_{\mathrm{em}} = f(T_{\mathrm{skin}})A = \sigma T_{\mathrm{skin}}^4 A = 6.26\times10^9\ \mathrm{erg\ sec^{-1}}.$$

3. And I absorb

$$P_{\mathrm{abs}} = f(T_{\mathrm{room}})A = \sigma T_{\mathrm{room}}^4 A = 5.68\times10^9\ \mathrm{erg\ sec^{-1}}.$$

4. And the difference is

$$\begin{aligned} P_{\mathrm{food}} &= P_{\mathrm{em}} - P_{\mathrm{abs}} = 0.58\times10^9\ \mathrm{erg\ sec^{-1}}\left(\frac{\mathrm{Cal}}{4.18\times10^{10}\ \mathrm{erg}}\right)\left(\frac{3600\ \mathrm{sec}}{\mathrm{hr}}\right) \\ &= 50\ \mathrm{Cal\ hr^{-1}}. \end{aligned}$$

   This overestimates the difference – more heat is *conducted* to your skin from your surroundings, than is radiated – but the comparison is still interesting.

5. All AST 111 students are assumed to be familiar with trigonometry, and to remember the simpler and more often-used trig identities. And by now, even those who are taking their first physics class this semester have already been exposed to vectors. Not that we'll be using vectors all that much in AST 111…

   a. With the axes arranged as indicated in Figure 1, right ascension (α) and declination (δ) correspond to the spherical-coordinate angles as given in the Formulas to Remember section above:

$$\phi = \alpha \qquad \theta = 90° - \delta$$

   One converts from spherical to Cartesian coordinates with

$$\begin{aligned} x &= r\sin\theta\cos\phi \\ y &= r\sin\theta\sin\phi \\ z &= r\cos\theta \quad , \end{aligned}$$

   so the components of the unit vector $\left(r=1\right)$ we are given are

$$\begin{aligned} x &= \sin\left(90° - \delta\right)\cos\alpha = \cos\delta\cos\alpha \\ y &= \sin\left(90° - \delta\right)\sin\alpha = \cos\delta\sin\alpha \\ z &= \cos\left(90° - \delta\right) = \sin\delta \quad ; \end{aligned}$$

   that is, the unit vector is

$$\hat{\boldsymbol{n}} = \cos\delta\cos\alpha\hat{\boldsymbol{x}} + \cos\delta\sin\alpha\hat{\boldsymbol{y}} + \sin\delta\hat{\boldsymbol{z}} \quad .$$

b. Write out the inner product of unit vectors pointing in those two directions, in those two ways, and solve for the angle ψ between the unit vectors:

$$\begin{aligned}
\hat{n}_1 \cdot \hat{n}_2 = \cos\psi &= n_{1x}n_{2x} + n_{1y}n_{2y} + n_{1z}n_{2z} \\
&= \cos\alpha_1\cos\alpha_2\cos\delta_1\cos\delta_2 + \sin\alpha_1\sin\alpha_2\cos\delta_1\cos\delta_2 + \sin\delta_1\sin\delta_2 \\
&= \cos(\alpha_1 - \alpha_2)\cos\delta_1\cos\delta_2 + \sin\delta_1\sin\delta_2 \\
\psi &= \arccos\left[\cos(\alpha_1 - \alpha_2)\cos\delta_1\cos\delta_2 + \sin\delta_1\sin\delta_2\right]
\end{aligned}$$

The expression for angular distance that you used in last week's Workshop Problems is an approximation to this one, valid for distances small compared to a radian. You will show how to obtain last week's equation from this one, in next week's recitation.

c. As you can see by finding these stars on the celestial globes or in TheSky, Vega, Deneb and Altair belong respectively to the constellations Lyra (the Harp), Cygnus (the Swan) and Aquila (the Eagle). They are also the brightest stars in these constellations – the "alpha" stars – and can thus be referred to as α Lyrae, α Cygni and α Aquilae. [1]

d. Recall that there are 15 degrees per hour of right ascension, as you showed in last week's recitation. The coordinates are

$$\begin{aligned}
\text{Vega}\ \alpha &= 18^{\text{h}}36^{\text{m}}56.3364^{\text{s}} = 18.61565^{\text{h}} = 279.2347° \\
\delta &= +38°47'01.291'' = 38.7837° \\
\text{Deneb}\ \alpha &= 20^{\text{h}}41^{\text{m}}25.9147^{\text{s}} = 20.69053^{\text{h}} = 310.3580° \\
\delta &= +45°16'49.217'' = 45.2803° \\
\text{Altair}\ \alpha &= 19^{\text{h}}50^{\text{m}}46.999^{\text{s}} = 19.84639^{\text{h}} = 297.69583° \\
\delta &= +8°52'05.959'' = 8.868322°
\end{aligned}$$

From part b, the angular distances between the stars are

$$\begin{aligned}
\psi &= \arccos\left[\cos(\alpha_1 - \alpha_2)\cos\delta_1\cos\delta_2 + \sin\delta_1\sin\delta_2\right] \\
\psi(\text{Vega-Deneb}) &= 23.8° \\
\psi(\text{Deneb-Altair}) &= 38.0° \\
\psi(\text{Altair-Vega}) &= 34.2°
\end{aligned}$$

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[1] Note that the endings of the constellation names change when they are incorporated into star names. The constellation names are all Latin, and the brightness designations (α, β, γ) are all Greek. It is a quaint custom of astronomers to give these Latin nouns their proper Latin declensions when used in sentences, no matter what language the rest of the sentence is written in. So the nominative or accusative forms (Lyra, Cygnus, Aquila) appear when discussing the constellations as subjects or predicates, and the genitive forms (e.g. α Lyrae = "the brightest star *of the Harp*") are used in stellar names.

e. There are several good answers to this question. Vega is the brightest star in the northern celestial hemisphere; it was the first mature star discovered to have a disk of planet-forming debris in orbit around it (in 1983); it is very close to having magnitude zero at all wavelengths; and it is on the path that the north celestial pole takes through the sky as the Earth's polar axis precesses, so it has been the North Star in the distant past, and will be again in the distant future. Take your pick.

f. It's a roundish patch lying halfway between Vega and Deneb, with diameter about half as long as the Vega-Deneb distance. So get used to pointing to the spot halfway between those stars; it contains the densest concentration of extrasolar planets known.