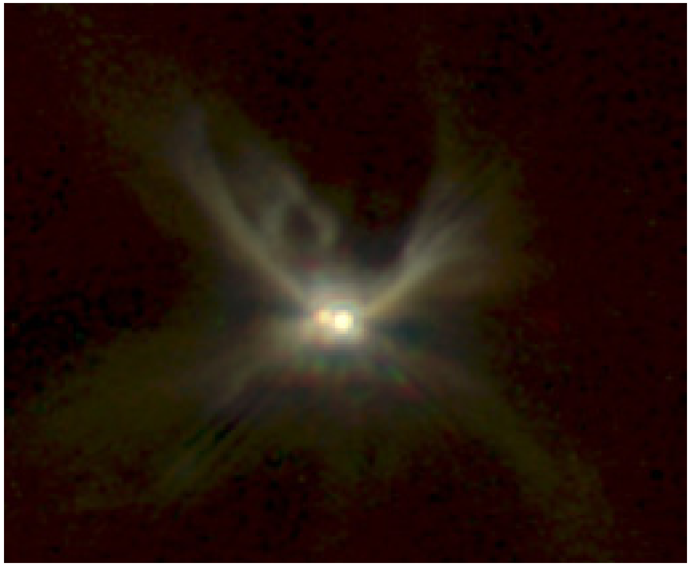
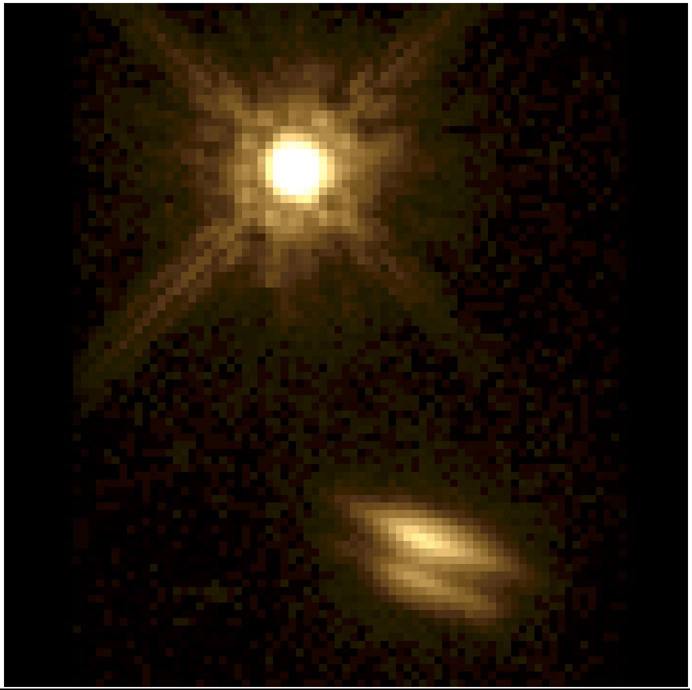
Today in Astronomy 142: binary stars

- Binary-star systems.
- Direct measurements of stellar mass, radius and temperature in eclipsing binary-star systems.





Two young binary star systems in the Taurus star-forming region, [CoKu Tau/1] (top) and [HK Tau/c] (bottom), by Deborah Padgett and Karl Stapelfeldt with the HST (STScI/NASA).

Measuring stellar mass, radius and temperature

Radius of isolated stars: Stars are so distant compared to their size that normal telescopes cannot make images of their surfaces or measurements of their sizes; stellar interferometry can be used in some of the larger, nearby cases.

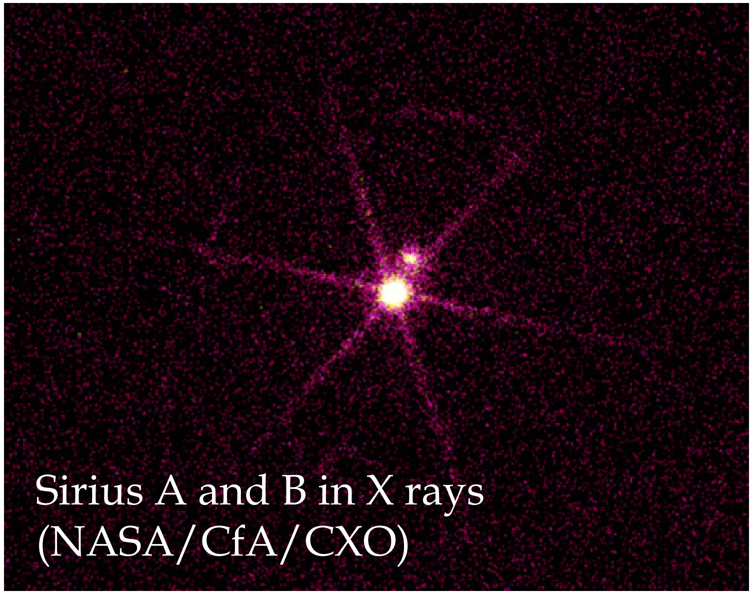
Mass: We can't measure the mass of an isolated star; we need a test particle in "gravitational contact" with it.

- Most helpful test particles: **binary star systems,** though in principle any multiple-star system could be probed for the radial velocities and periods required to measure masses.
- Observations of certain binary star systems can also help in the determination of **radius and temperature**.

There are enough nearby binary stars to do this for the full range of stellar types.

Binaries

Resolved visual binaries: see stars separately, measure orbital axes and radial velocities directly. There aren't very many of these. (Example: Sirius A and B.) The rest are **unresolved**.


Sirius A and B in X rays (NASA/CfA/CXO)

Astrometric binaries: only brighter member seen, with periodic wobble in the track of its proper motion.

Spectroscopic binaries: unresolved (relatively close) binaries told apart by periodically oscillating Doppler shifts in spectral lines. Periods = days to years.

- **Spectrum binaries**: orbital periods longer than period of known observations.
- **Eclipsing binaries**: orbits seen nearly edge on, so that the stars actually eclipse one another. (Most useful.)

Eclipsing binary stars and orientation

If the distance between members of binary systems is small compared to their radii (as it is, typically), then the orbital axis must be very close to 90°.

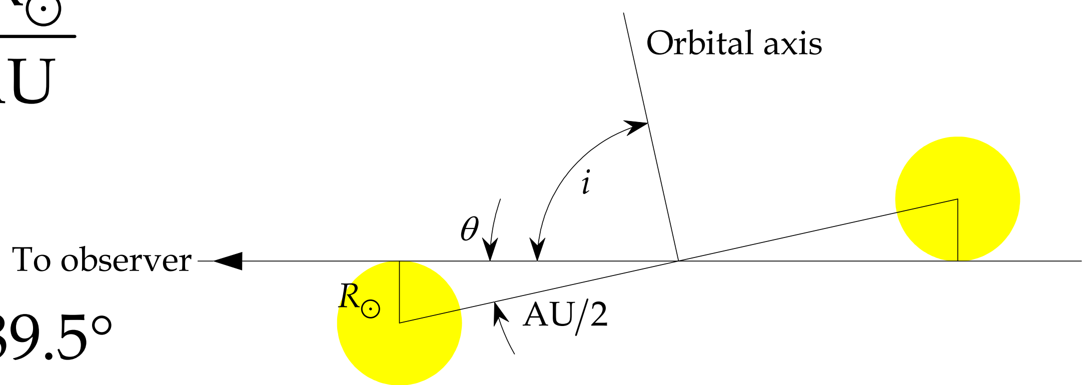
- Example: consider two Sun-like stars orbiting each other 1 AU apart, viewed so they just barely eclipse each other in the view of a distant observer:

$$\theta = \arcsin \frac{2R_\odot}{\mathrm{AU}} \cong \frac{2R_\odot}{\mathrm{AU}}$$

$$i = \frac{\pi}{2} - \frac{2R_\odot}{\mathrm{AU}}$$

$$= 1.561 \text{ radians} = 89.5°$$

$$\sin i = 0.99996$$



Binary-star radial velocities

Radial velocity v_r : the component of velocity along the line of sight.

Doppler effect: shift in wavelength of light due to motion of its source with respect to the observer.

$$\frac{\lambda_{\text{observed}} - \lambda_{\text{rest}}}{\lambda_{\text{rest}}} = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{v_r}{c}$$

- Positive (negative) radial velocity leads to longer (shorter) wavelength than the rest wavelength.
- To measure small radial velocities, a light source with a very narrow range of wavelengths, like a spectral line, must be used.

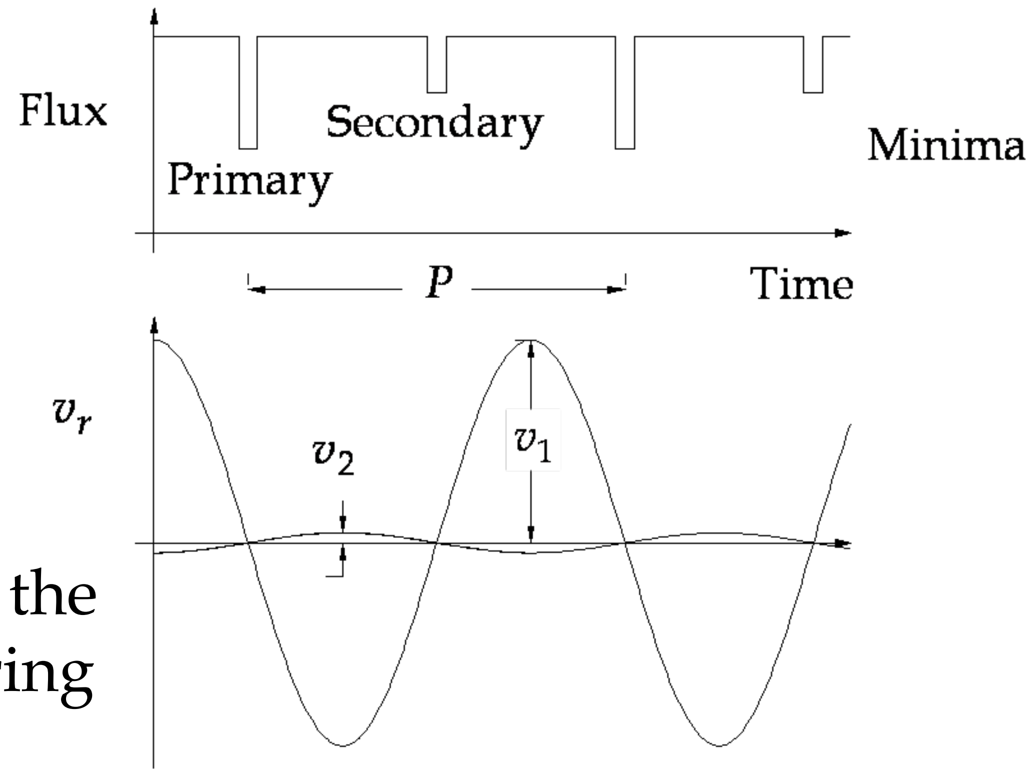
Binary-star radial velocities (continued)

- If their orbits are circular, the radial velocity of each component will be sinusoidal in time, since this velocity tracks only one component of the motion:

$$r(t) = r_0, \quad \phi(t) = \frac{vt}{r_0}$$

$$x(t) = r_0 \cos\frac{vt}{r_0}$$

$$y(t) = r_0 \sin\frac{vt}{r_0}$$



- The radial velocities of the two stars are equal during eclipses.

Measurement of binary-star masses

Use radial-velocity measurements in Kepler's Laws:

#1: all binary stellar orbits are coplanar ellipses, each with one focus at the center of mass.

- Most binary orbits turn out to have very low eccentricity (are nearly circular).

#2: the position vector from the center of mass to either star sweeps out equal areas in equal times.

#3: the square of the period is proportional to the cube of the sum of the orbit semimajor axes, and inversely proportional to the sum of the stellar masses:

$$P^2 = \frac{4\pi^2}{G\left(m_1 + m_2\right)}\left(a_1 + a_2\right)^3$$

Measurement of binary-star masses (continued)

If orbital major axes (relative to center of mass) or radial velocities known, so is the ratio of masses:

$$\frac{m_1}{m_2} = \frac{a_2}{a_1} = \frac{v_{2r}}{v_{1r}} = \frac{v_2}{v_1}$$

Furthermore, from Kepler's third law (cf. HW#1),

$$m_1 + m_2 = \frac{P}{2\pi G}\left(\frac{v_1 + v_2}{\sin i}\right)^3 \quad .$$

For **unresolved** binaries (the vast majority), we can measure only P and the two velocity amplitudes, so this is two equations in three unknowns m_1, m_2 and $\sin i$.

Measurement of binary-star masses (continued)

That's why eclipsing binaries are so important: if the system eclipses, we must be viewing the orbital plane very close to edge on: sin i is very close to 1, leaving us with two equations in two unknowns, m_1 and m_2.

- **Simulation** of binary star orbits, useful in visualizing the radial-velocity measurement situation:

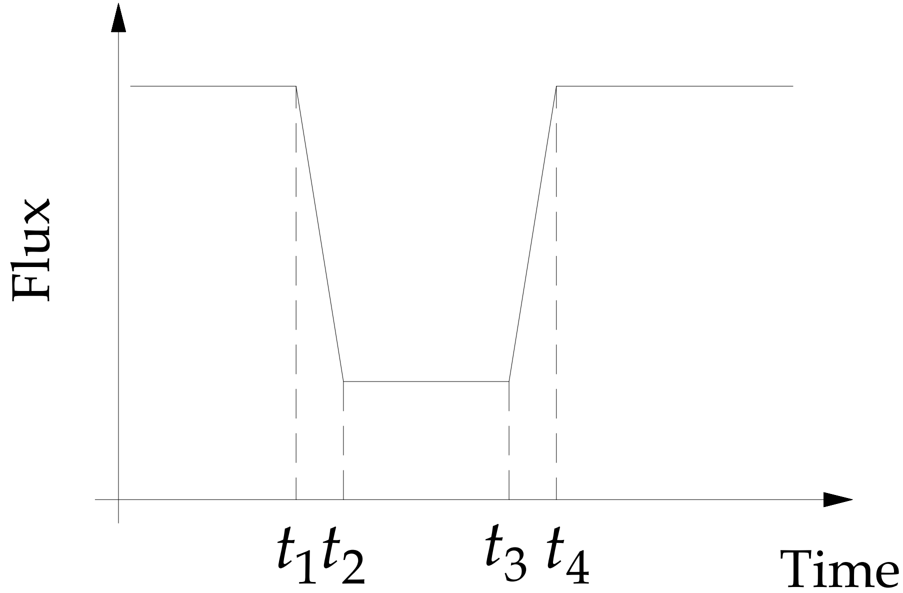
  http://www.pas.rochester.edu/~dmw/ast142/Java/binary.htm .

  Courtesy of Terry Herter (Cornell).

Measurement of stellar radius

Eclipses don't just wink off and on instantaneously: the transitions are gradual. The shape of the system's **light curve** is sensitive to the sizes of the stars.

- If one star can completely block the light of the other, the "bottom" of the eclipse light curve will be flat.
- The extent of the flat part of the curve turns out to be determined by the radius of the larger star.
- The slope of the transitions turns out to be determined by the radius of the smaller star.

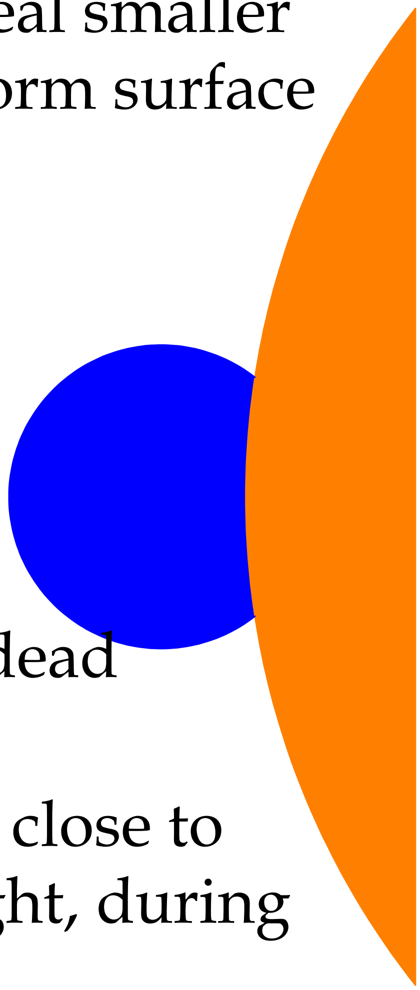


Measurement of stellar radius (continued)

- If the light curve never flattens out, then the eclipse isn’t “total:” neither star can completely block the other.
- Much can be made of the details of the shapes of the light curve at the onset (**ingress**) and end (**egress**) of the eclipse, to make out details in the atmosphere of each star, such as the uniformity of the brightness of each star’s disk.
  - And much of this can even be done analytically!
- The derivations can be very lengthy, though, so we will only illustrate with simple setups that yet describe a large fraction of the eclipsing binaries.

Measurement of stellar radius (continued)

- Assume that one star (radius R_S) is a good deal smaller than the other (R_L), and has a disk with uniform surface brightness.
- Then the light curve is that of a uniformly-bright circle with surface brightness I_0 being occulted by a straight edge, and can be calculated from the area of un-occulted star as a function of time.
- For simplicity we will take the eclipse to be dead center (i exactly 90°)…
- …and note that the velocities of the stars are close to constant, and perpendicular to the line of sight, during the eclipse.



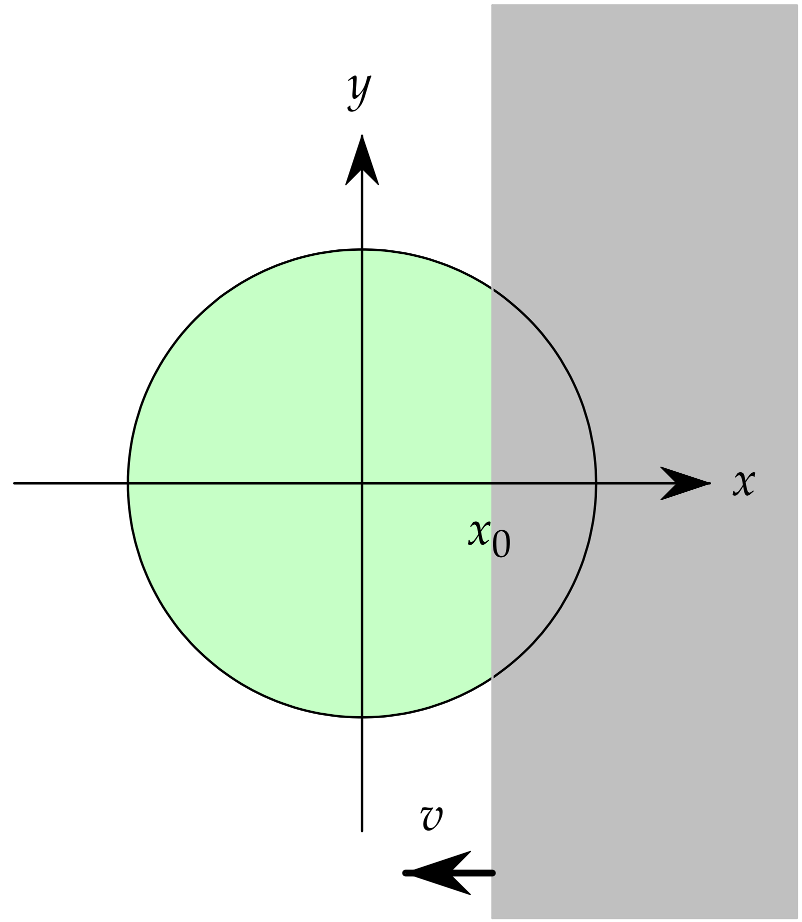
Measurement of stellar radius (continued)

The boundary of the star is

$$x^2 + y^2 = R_S^2$$

and the occulting edge is a vertical line at $x = x_0$. Thus the exposed area is obtained by integrating, $dA = dxdy$, first in y from one edge of the circle to the other, then in x from the left edge of the circle to the occulting edge:

$$A = \int_{-R}^{x_0} dx \int_{-\sqrt{R_S^2 - x^2}}^{\sqrt{R_S^2 - x^2}} dy$$



Measurement of stellar radius (continued)

Thus

$$f = I_0 A = \int_{-R}^{x_0} dx \int_{-\sqrt{R_S^2 - x^2}}^{\sqrt{R_S^2 - x^2}} dy = 2\int_{-R}^{x_0} dx \sqrt{R_S^2 - x^2} = 2R_S^2 \int_{-R}^{x_0} \frac{dx}{R_S}\sqrt{1 - \frac{x^2}{R^2}}$$

Substitute:

$$\sin\theta = x/R_S \text{ , } \cos\theta d\theta = dx/R_S \text{ ,}$$

$$-\pi/2 \le \theta \le \arcsin\left(x_0/R_S\right) = \theta_0 ,$$

$$f = 2I_0 R_S^2 \int_{-\pi/2}^{\theta_0} d\theta \cos^2\theta = R_S^2\left(\frac{\pi}{2} + \theta_0 + \sin\theta_0\cos\theta_0\right)$$

$$= I_0 R_S^2\left(\frac{\pi}{2} + \arcsin\frac{x_0}{R_S} + \frac{x_0}{R_S}\sqrt{1 - \frac{x_0^2}{R_S^2}}\right)$$

For your general math edification

As many of you know, that last integral is carried out by integrating by parts once and using a dirty trick. Recall:

Substitute $u = \cos\theta$, $du = -\sin\theta d\theta$,

$dv = \cos\theta d\theta$, $v = \sin\theta$: $\int u dv = uv - \int v du$ gives

$$\int_{-\pi/2}^{\theta_0} d\theta \cos^2\theta = \cos\theta \sin\theta \Big|_{-\pi/2}^{\theta_0} + \int_{-\pi/2}^{\theta_0} d\theta \sin^2\theta$$

But $\cos^2 x + \sin^2 x = 1$, so

$$\int_{-\pi/2}^{\theta_0} d\theta \cos^2\theta = \int_{-\pi/2}^{\theta_0} d\theta - \int_{-\pi/2}^{\theta_0} d\theta \sin^2\theta$$

Add these results

Continuing your general math edification

$$2\int_{-\pi/2}^{\theta_0} d\theta \cos^2\theta = \cos\theta\sin\theta\Big|_{-\pi/2}^{\theta_0} + \int_{-\pi/2}^{\theta_0} d\theta$$

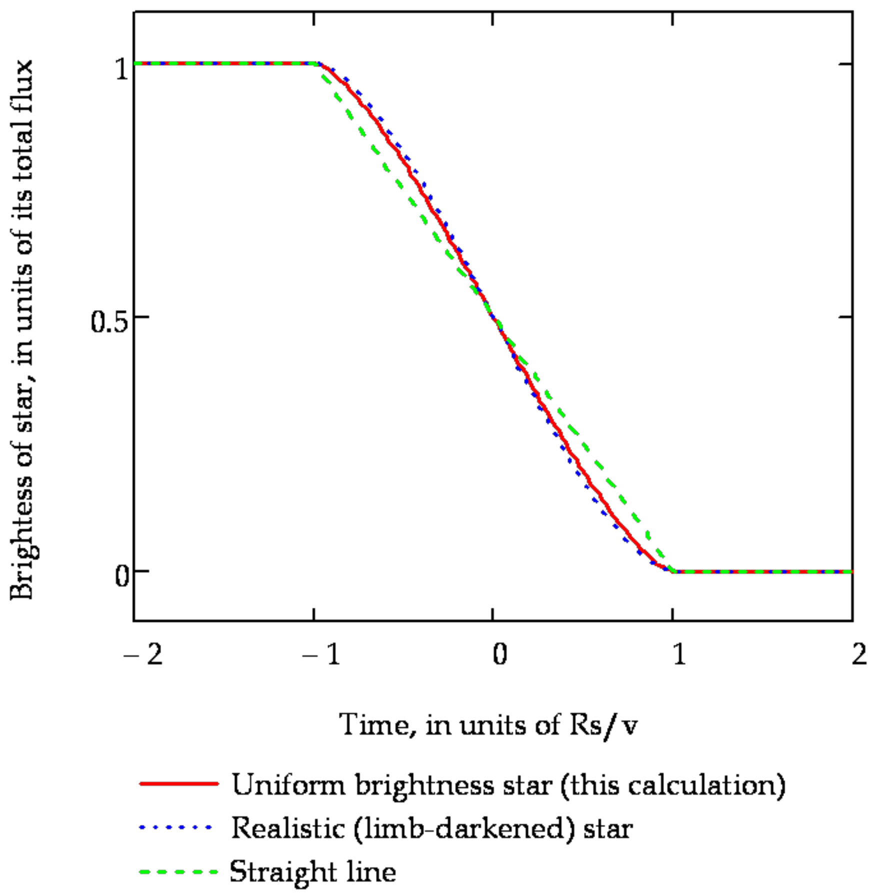
$$= \cos\theta_0 \sin\theta_0 + \theta_0 + \frac{\pi}{2}$$

$$\int_{-\pi/2}^{\theta_0} d\theta \cos^2\theta = \frac{1}{2}\left(\frac{\pi}{2} + \theta_0 + \cos\theta_0 \sin\theta_0\right)$$

Measurement of stellar radius (continued)

The result is that the light-curve shape on ingress and egress is reasonably close to a straight line, and does have well-defined beginning and ending points, precisely where the edges of the stars cross each other.

(This is why the light curves are drawn as straight lines in all the books.)



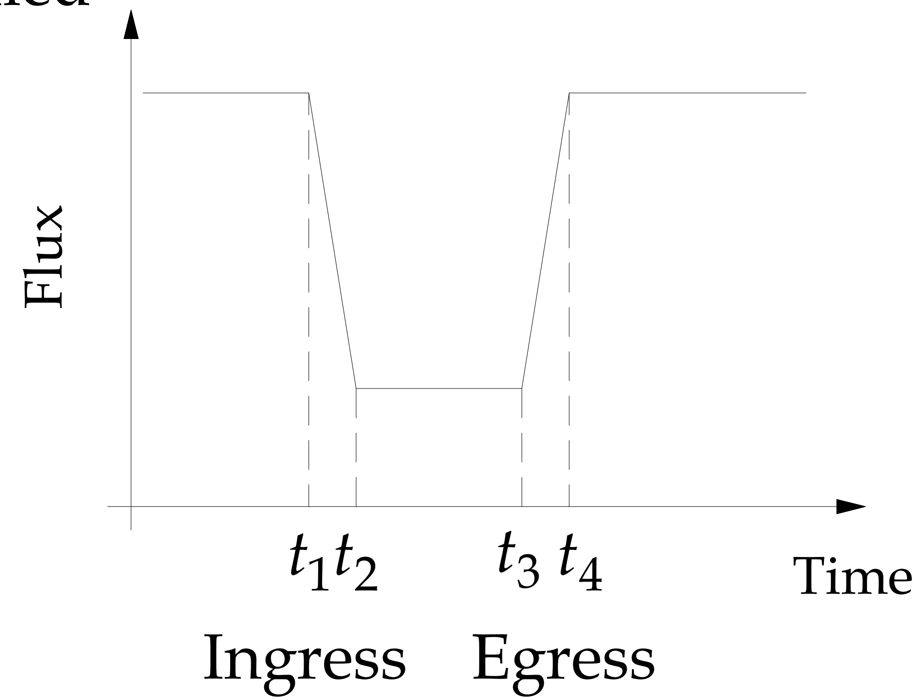
Measurement of stellar radius (continued)

The "corners" are usually labelled as indicated at right.

During an eclipse, the stars move at speeds v_1 and v_2, in opposite directions perpendicular to the line of sight, where v_1 and v_2 are the two speeds measured from the orbital periods and radial-velocity amplitudes. Thus

$$2R_S = (v_1 + v_2)(t_2 - t_1)$$

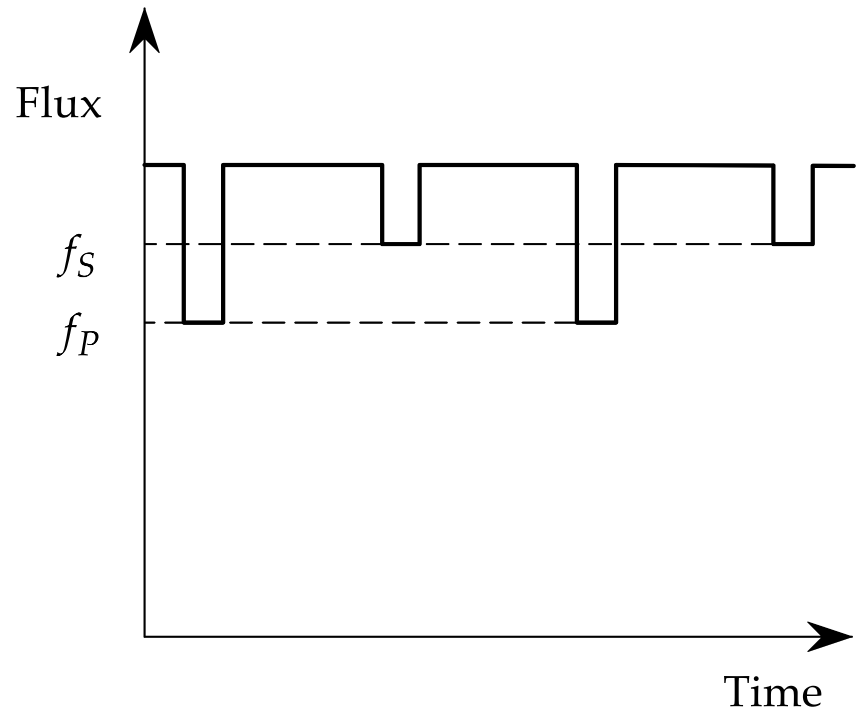
$$2R_L = (v_1 + v_2)(t_3 - t_1)$$



Measurement of stellar effective temperature

A crude but useful measurement of the ratio of the stars' T_e comes from the relative depths of the primary (deeper) and secondary eclipses. When the stars are not eclipsed, their total flux is

$$f = \pi R_S^2 I_{0S} + \pi R_L^2 I_{0L}$$



If the small star is hotter, the primary eclipse is when this star is behind the larger star:

$$f_P = \pi R_L^2 I_{0L}$$

Measurement of stellar effective temperature (continued)

Then the secondary eclipse is when the small star passes in front of the larger one:

$$f_S = \pi\left(R_L^2 - R_S^2\right)I_{0L} + \pi R_S^2 I_{0S}$$

Three equations, two unknowns. But usually these are used to construct one ratio, to remove uncertainties in distance *r*:

$$\frac{f - f_P}{f - f_S} = \frac{\pi R_S^2 I_{0S} + \pi R_L^2 I_{0L} - \pi R_L^2 I_{0L}}{\pi R_S^2 I_{0S} + \pi R_L^2 I_{0L} - \pi\left(R_L^2 - R_S^2\right)I_{0L} - \pi R_S^2 I_{0S}}$$

$$= \frac{\pi R_S^2 I_{0S}}{\pi R_L^2 I_{0L} - \pi\left(R_L^2 - R_S^2\right)I_{0L}} = \frac{\pi R_S^2 I_{0S}}{\pi R_S^2 I_{0L}} = \frac{I_{0S}}{I_{0L}}$$

Measurement of stellar effective temperature (continued)

Note that the way we have defined the I_0s,

$$f = \pi R^2 I_0 = \frac{L}{4\pi r^2} = \frac{4\pi R^2 \sigma T_e^4}{4\pi r^2} \quad \Rightarrow \quad I_0 = \frac{\sigma T_e^4}{\pi r^2}$$

Thus

$$\frac{f - f_P}{f - f_S} = \frac{I_{0S}}{I_{0L}} = \frac{T_{eS}^4}{T_{eL}^4}$$

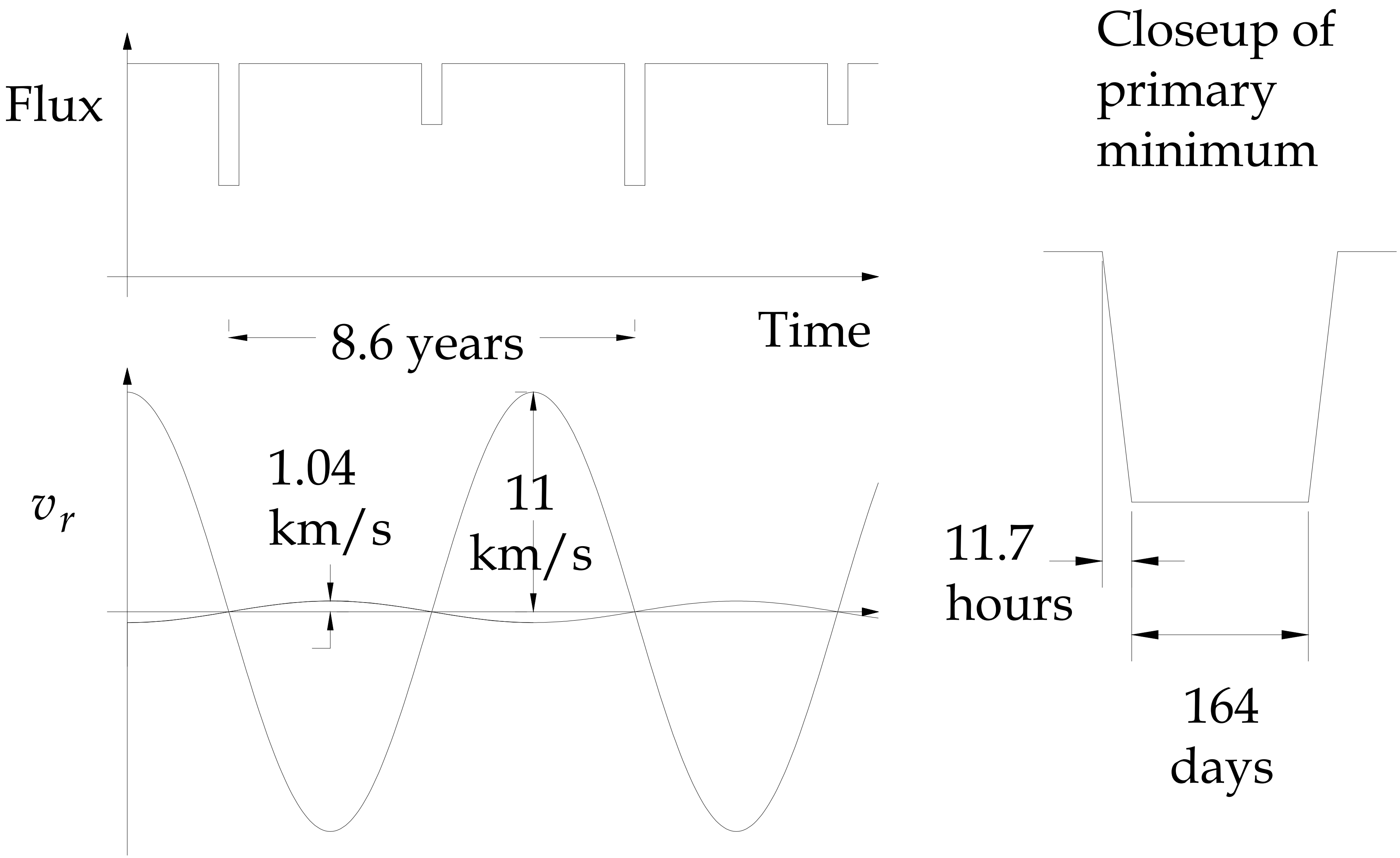
If r is known, the luminosity, and $T_e = \left(L \big/ 4\pi r^2 \sigma \right)^{1/4}$, of the brighter star is determined accurately from the primary minimum flux, and thus T_e of the fainter star from this ratio.

Example

An eclipsing binary is observed to have a period of 8.6 years. The two components have radial velocity amplitudes of 11.0 and 1.04 km/s and sinusoidal variation of radial velocity with time. The eclipse minima are flat-bottomed and 164 days long. It takes 11.7 hours for the ingress to progress from first contact to eclipse minimum.

- ❑ What is the orbital inclination?
- ❑ What are the orbital radii?
- ❑ What are the masses of the stars?
- ❑ What are the radii of the stars?

Example (continued)



Example (continued)

Answers

- Since it eclipses, the orbits must be observed nearly edge on; since the radial velocities are sinusoidal the orbits must be nearly circular.
- Orbital radii:

$$\frac{m_\ell}{m_s} = \frac{v_s}{v_\ell} = \frac{11}{1.04} = 10.6$$

$$r_s = v_s \frac{P}{2\pi} = 1.42\times10^{14}\ \mathrm{cm}$$
$$=9.5\ \mathrm{AU}$$

$$r_\ell = v_\ell \frac{P}{2\pi} = 1.34\times10^{13}\ \mathrm{cm}$$
$$=0.90\ \mathrm{AU}$$

$$r = 10.4\ \mathrm{AU}$$

Example (continued)

- Masses:

$$m_s + m_\ell = \frac{r^3}{P^2} = 15.2\ M_\odot \qquad \text{(Kepler's third law)}$$

$$m_s + 10.6 m_s = \qquad \text{(previous result)}$$

$$\Rightarrow m_s = 1.3\ M_\odot,\ m_\ell = 13.9\ M_\odot$$

- Stellar radii (note: solar radius = 6.96×10^{10} cm):

$$R_s = \frac{v_s + v_\ell}{2}\left(t_2 - t_1\right)$$

$$= \left(6.02 \text{ km s}^{-1}\right)\left(11.7 \text{ hr}\right)$$

$$= 7.6 \times 10^{10} \text{ cm} = 1.1\ R_\odot$$

$$R_\ell = \frac{v_s + v_\ell}{2}\left(t_3 - t_1\right)$$

$$= 369\ R_\odot$$

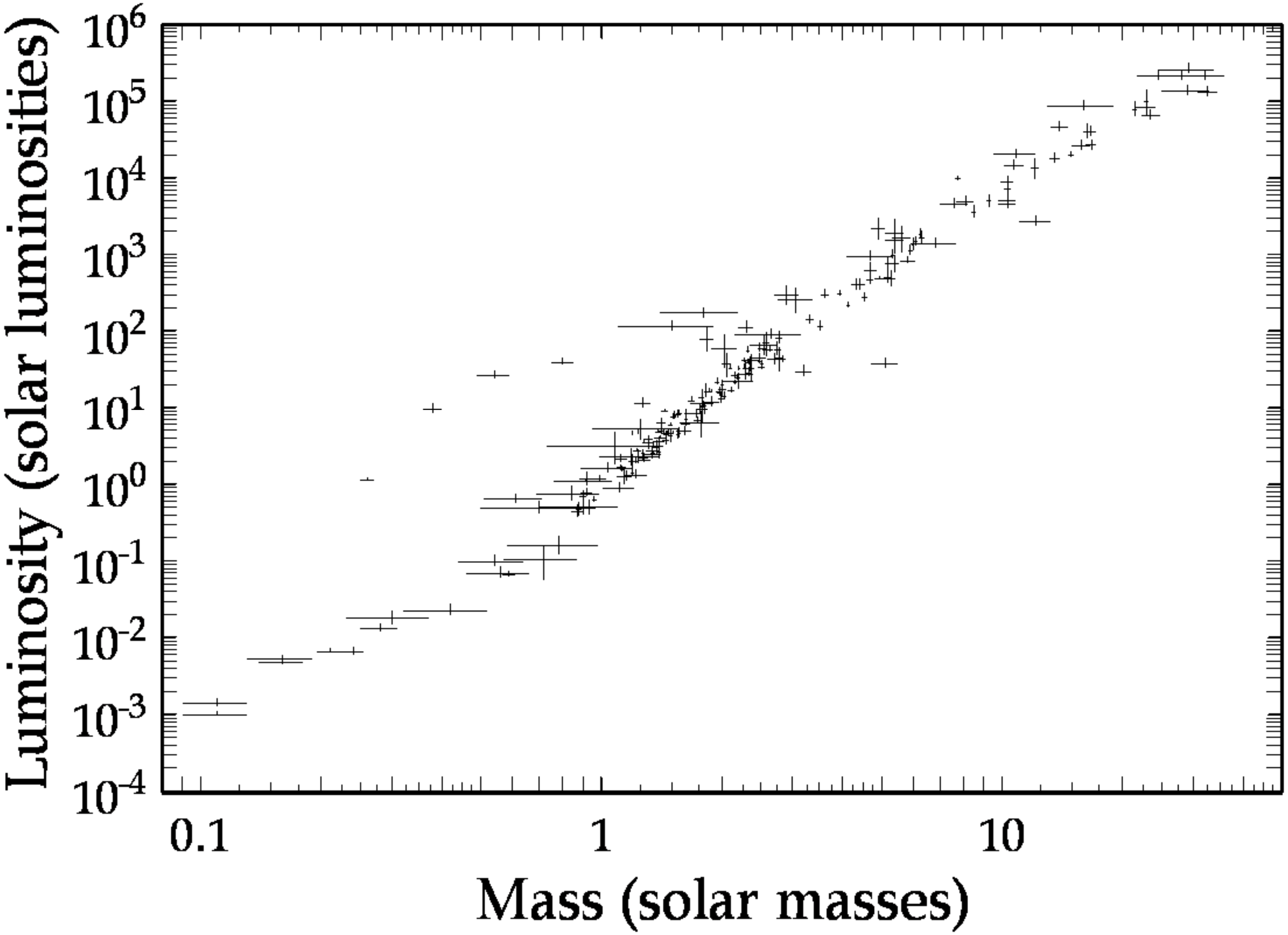
Data on eclipsing binary stars

The most useful, ongoing, big compendium of eclipsing binary data is by O. Malkov (1993), which is mostly based upon many decades of work by Dan Popper. See following slides, and see here for the data themselves.

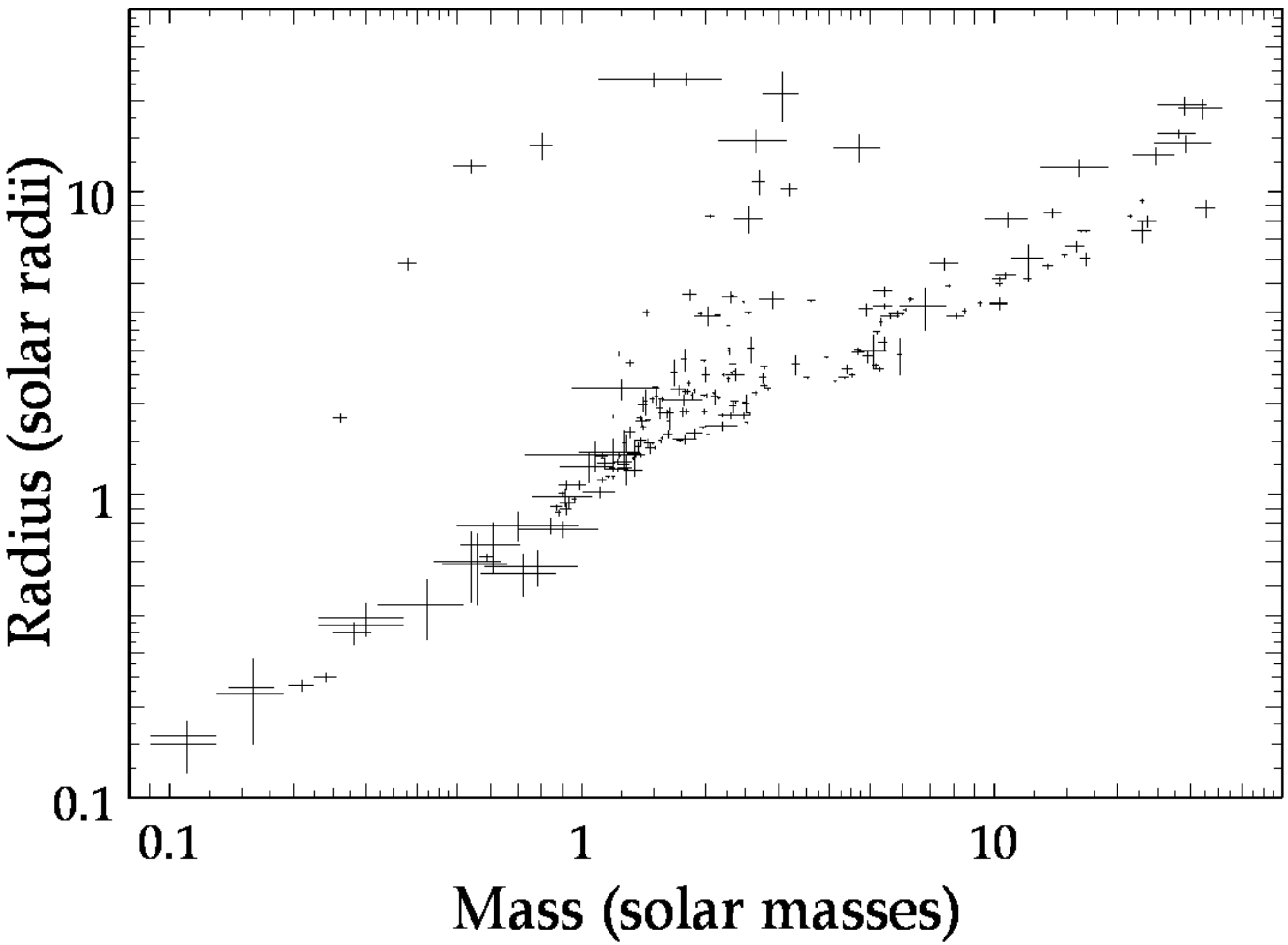
Why do the graphs appear as they do?

That's what we'll try to figure out, as we study stellar structure during the next few lectures.

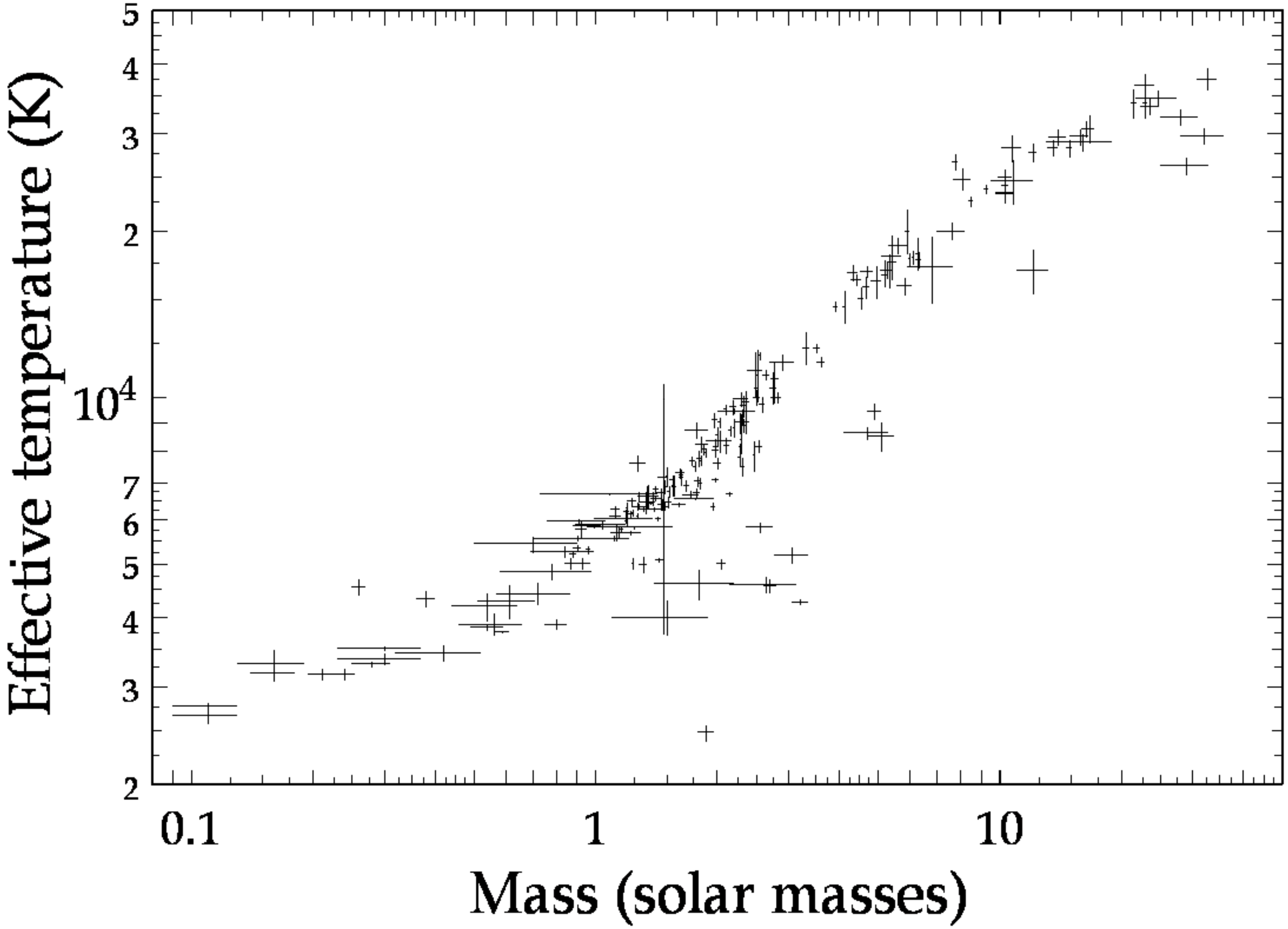
Luminosities of eclipsing binary stars



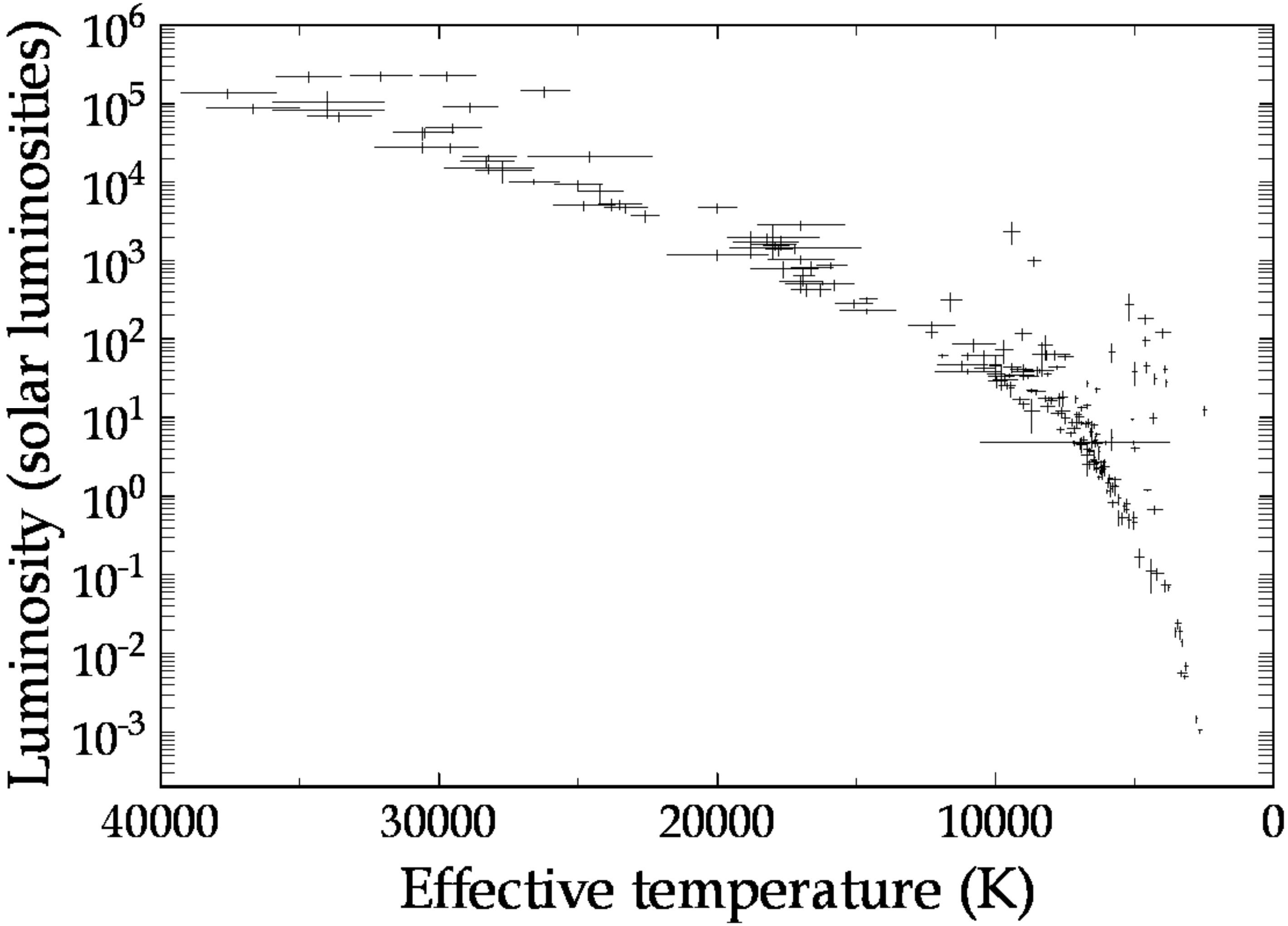
Radii of eclipsing binary stars



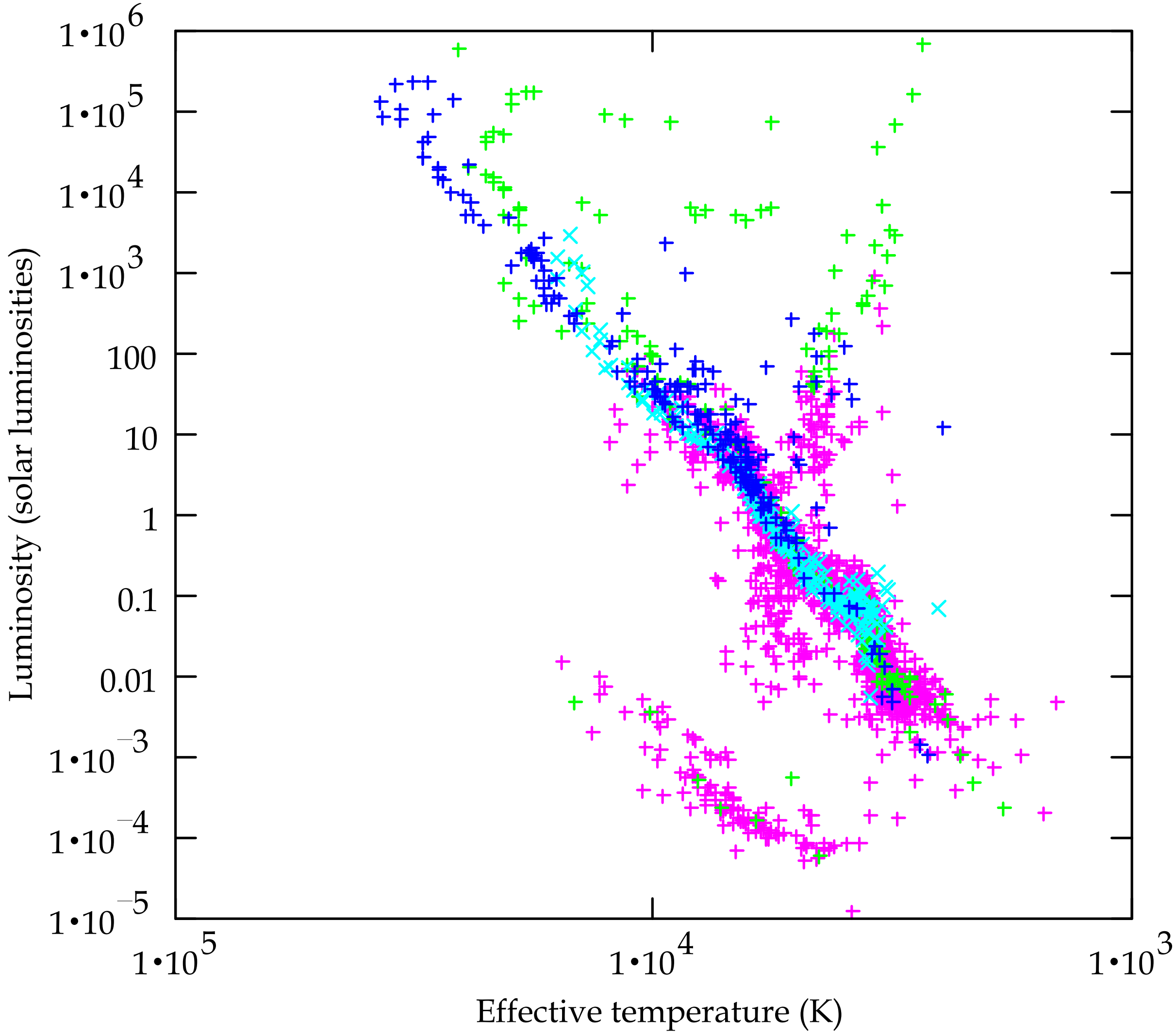
Effective temperatures of eclipsing binary stars



Hertzsprung-Russell (H-R) diagram for eclipsing binary stars



H-R diagram for binaries and other nearby stars



Stars within 25 parsecs of the Sun (Gliese and Jahreiss 1991)

Nearest and Brightest stars (Allen 1973)

Pleiades X-ray sources (Stauffer et al. 1994)

Binaries with measured temperature and luminosity (Malkov 1993)