

Today in Astronomy 142: fusion

- ❑ Nuclear fusion reactions in stars
- ❑ Temperature dependence of fusion rate in stars
- ❑ Nucleosynthesis and the cosmic abundances of the elements




Figure: Ray Davis' 100,000 gallon perchloroethylene neutrino detector, almost a mile underground in the Homestake Gold Mine, Lead, SD. That's Davis on the catwalk above the tank. (Brookhaven National laboratory photo.)

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The four forces of nature

...are gravity, electromagnetism, the strong nuclear force, and the weak nuclear force.

- ❑ **Gravity**
 - Always attractive, weak, and can't be shielded, so is very long ranged.
 - Holds stars, solar systems, galaxies, and clusters together.
- ❑ **Electromagnetism**
 - Much stronger than gravity.
 - Long ranged but is generally shielded over very large distances because it can be either attractive or repulsive (charge can have either algebraic sign).
 - Holds atoms and molecules together. Light is a form of electromagnetic energy.

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The four forces of nature (continued)

These two forces are insufficient to explain nature; there must be other attractive forces.

- ❑ Example: protons in the helium nucleus repel each other electrostatically, and gravity is insufficient to overcome the repulsion (as you'll see this week).

Strong nuclear interaction

- ❑ Short ranged: only acts over nuclear dimensions; that's why we don't notice it in everyday life.
- ❑ Involves a different sort of charge than electromagnetism, that characterizes the strength of interparticle forces. The term for this sort of charge is color.
- ❑ Always attractive, much stronger than EM, gravity.

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The four forces of nature (continued)

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The four forces of nature (continued)

None of these three forces explains the slow transmutation of some nuclei: the phenomenon of radioactivity.

So for completeness we mention also the **Weak nuclear interaction**

- Even shorter range than strong interaction.
- Much stronger than electromagnetism and gravity but weaker than the strong interaction by quite a bit.
- Causes radioactivity.

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Elementary particles

Matter is made of **leptons** and **quarks**.

- Quarks participate in all four interactions. Leptons abstain from the strong interaction.
- Each family includes six **flavors** of particles:

Leptons	$\begin{bmatrix} e^- \\ \nu_e \end{bmatrix} \quad \begin{bmatrix} \mu^- \\ \nu_\mu \end{bmatrix} \quad \begin{bmatrix} \tau^- \\ \nu_\tau \end{bmatrix}$	Normal atoms and nuclei only contain members of the first generation of leptons and quarks.
Quarks	$\begin{bmatrix} u \\ d \end{bmatrix} \quad \begin{bmatrix} c \\ s \end{bmatrix} \quad \begin{bmatrix} t \\ b \end{bmatrix}$	

- All of these particles have spin one half (that is, $\hbar/2$); they all obey the Pauli exclusion principle.

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Elementary particles (continued)

- ❑ Quarks come in three different kinds of the strong-interaction analogue of charge, or **colors** (usually called red, white and blue).
- ❑ Individual quarks are never observed (“confinement”).
- ❑ Nuclear particles are made up of two or three quarks.
 - Quark-antiquark pairs: mesons
 - Three quarks: **baryons** (e.g. protons, neutrons).

It has been shown experimentally and theoretically that the electromagnetic, weak and strong interactions can be thought of as different aspects of the same interaction, that manifest themselves in different energy regimes; this notion is called the **standard model** of elementary particles and interactions.

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Quantities conserved in the four interactions

- ❑ Energy, momentum, angular momentum, etc.
- ❑ Electric charge
- ❑ Lepton number, separately for each generation of leptons; = +1 for particle and corresponding neutrino.
- ❑ Baryon number

Particle	Spin (\hbar)	Electric charge (e)	Baryon number
u	1/2	2/3	1/3
d	1/2	-1/3	1/3
proton (uud)	1/2	1	1
neutron (udd)	1/2	0	1

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Particles and antiparticles

To each of the particles mentioned above corresponds an **antiparticle**.

- ❑ Antiparticles have exactly the same mass and spin as the corresponding particle.
- ❑ They have electric charge, lepton number, and baryon number **opposite** that of the corresponding particle.

Notation: $\overset{\text{baryon number}}{\text{charge}} \text{Name} \leftarrow$ if antiparticle

Examples:

proton = 1_1p electron = ${}^{-1}_1e$ antiproton = ${}^{-1}_{-1}\bar{p}$
 positron = ${}^0_1\bar{e}$ antineutrino = ${}^0_0\bar{\nu}_e$ photon = ${}^0_0\gamma$

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Energy from nuclear fusion

Binding of nuclear particles into nuclei represents a very large negative potential energy due to the strong interaction.

□ Therefore if free nuclear particles become bound through the strong interaction, energy/momentum conservation demands that the mass of the bound object is less than the sum of the masses of the free objects, and the deficit appears in other forms of energy, such as the kinetic energies of the products of the reaction:

protons → heavier nuclei *Large negative potential energy*
 + other particles *Conservation of lepton number, baryon number, momentum, etc.*
 + heat *Kinetic energy of products*

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Fusion of two protons

The diagram illustrates the fusion of two protons. Two red spheres labeled 'Proton' with a '+' sign are shown on the left, moving towards a central point where they meet. From this point, a large yellow starburst represents the release of energy. Three particles are shown moving away from the center: a red sphere labeled 'Proton' with a '+' sign, a blue sphere labeled 'Positron' with a '-' sign, and a blue sphere labeled 'Neutrino' with a '0' sign. Below the starburst, a red and blue sphere labeled 'Deuteron' with a '+' sign is shown.

Figure: Chaisson and McMillan, *Astronomy Today*.
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Fusion and $E = mc^2$

Energy is a form of mass; mass is a form of energy. Even when it's at rest and far from attracting or repelling bodies, a body has a total **rest energy**

$$E = m_0c^2 \quad (m_0 \text{ is the rest mass})$$

Two protons fuse to make a deuteron and two lightweight particles. The deuteron has a proton and a neutron (close to the same mass as the proton), but it also has the large *negative* potential energy from the strong interaction, so its rest energy is less than the sum of the proton rest energies, or equivalently its rest mass is less than that of the protons.

This suggests a convenient accounting method for energy released in fusion processes.

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The proton-proton chains

❑ Several different sequences of reactions that start with the fusion of two protons are the most important such reactions in main sequence stars; collectively they are called the proton-proton (pp) chains.

❑ Here is pp chain I for example (70% of pp chain reactions):

$$2\text{}^1_1\text{H} \rightarrow \text{}^2_1\text{H} + \text{}^0_{-1}\text{e} + \text{}^0_0\nu_e \quad (\times 2)$$

$$\text{}^2_1\text{H} + \text{}^1_1\text{H} \rightarrow \text{}^3_2\text{He} + \text{}^0_0\gamma \quad (\times 2)$$

$$2\text{}^3_2\text{He} \rightarrow \text{}^4_2\text{He} + 2\text{}^1_1\text{H}$$

Total: $4\text{}^1_1\text{H} \rightarrow \text{}^4_2\text{He} + 2\text{}^0_{-1}\text{e} + 2\text{}^0_0\nu_e + 2\text{}^0_0\gamma$

❑ Rest mass of products less than reactants, so the products have more kinetic energy than the reactants (heat!).

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Proton-proton chain I (PPI)

Chaisson and McMillan, *Astronomy Today*.

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The proton-proton chains (continued)

Application of all the conservation laws:

$2\text{}^1_1\text{H} \rightarrow \text{}^2_1\text{H} + \text{}^0_{-1}\text{e} + \text{}^0_0\nu_e$
Mass and baryon number in deuteron; extra charge (+) must be carried off by a non-baryon (i.e. an anti-lepton); neutrino added to conserve lepton number.

$\text{}^2_1\text{H} + \text{}^1_1\text{H} \rightarrow \text{}^3_2\text{He} + \text{}^0_0\gamma$
Energy and momentum cannot both be conserved unless there is more than one particle in the final state. A neutrino-antineutrino pair would also work here, but would happen much less frequently.

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The proton-proton chains (continued)

How much kinetic energy do the products have?
 The difference in rest mass between products and reactants gives the difference in binding (potential) energy:

$$\Delta V = m_0(\text{He})c^2 + 2m_0(e)c^2 - 4m_0(\text{H})c^2$$

$$\cong m_0(\text{He})c^2 - 4m_0(\text{H})c^2 \quad (m_p/m_e = 1836)$$

$$= 3.97m_p c^2 - 4m_p c^2 = -0.03m_p c^2$$

$$= -4.5 \times 10^{-5} \text{ erg} = -\Delta KE$$

Measured mass of the helium nucleus

Compare to the **average kinetic energy of particles in an ideal gas** at 15.7×10^6 K:

$$\overline{KE} = \frac{3}{2}kT = 3 \times 10^{-9} \text{ erg}$$

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Nuclear catalysis: the CNO bi-cycle

One branch, the CN cycle:

$${}^{12}_6\text{C} + {}^1_1\text{H} \rightarrow {}^{13}_7\text{N} + \gamma$$

$${}^{13}_7\text{N} \rightarrow {}^{13}_6\text{C} + {}^0_{-1}e + {}^0_0\nu_e$$

$${}^{13}_6\text{C} + {}^1_1\text{H} \rightarrow {}^{14}_7\text{N} + \gamma$$

$${}^{14}_7\text{N} + {}^1_1\text{H} \rightarrow {}^{15}_8\text{O} + \gamma$$

$${}^{15}_8\text{O} \rightarrow {}^{15}_7\text{N} + {}^0_{-1}e + {}^0_0\nu_e$$

$${}^{15}_7\text{N} + {}^1_1\text{H} \rightarrow {}^{12}_6\text{C} + {}^4_2\text{He}$$

${}^{12}_6\text{C}$ is a catalyst: it's not used up in the reactions.

Total: $4 {}^1_1\text{H} \rightarrow {}^4_2\text{He} + 2 {}^0_{-1}e + 2 {}^0_0\nu_e + 3 \gamma$

-- same rest mass difference, and therefore kinetic energy of products, as pp chains. (Requires higher T , though.)

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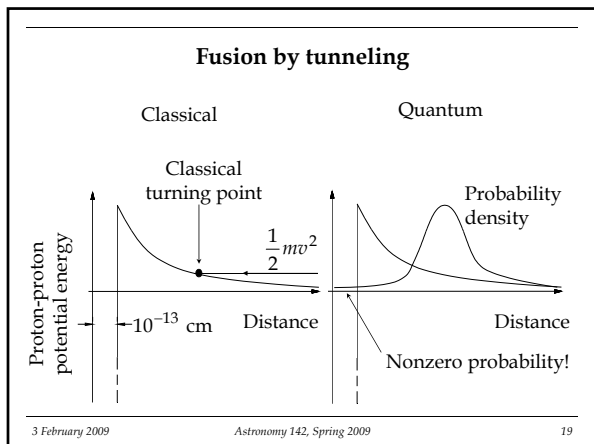
Hot fusion

Why don't pp chain reactions take place in the ocean, where there's plenty of hydrogen?

- ❑ The **Coulomb barrier**: average kinetic energy of oceanic H is far less than the height of the potential energy barrier due to electrostatic repulsion. Repulsion keeps the protons from getting close enough for the strong interaction to take over.
- ❑ In fact, even at tens of millions of degrees, as in the centers of main sequence stars, the average kinetic energy is too small for classical collisions to result in protons penetrating the Coulomb barrier.

Quantum-mechanical **tunneling** is still required for fusion to occur, even at stellar core temperatures.

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Temperature dependence of proton fusion rate

We'll do a simplified version of a calculation first done by George Gamow in the late 1930s. (See PU, page 115.) Consider in one dimension the fusion of two particles with masses m_1 and m_2 , charges q_1 and q_2 , speeds v_1 and v_2 , and separation r . Their reduced mass is $m = m_1 m_2 / (m_1 + m_2)$, and their relative speed is $v = v_1 - v_2$. Classically they can't get any closer together than r_{\min} where

$$W = \frac{1}{2} m v^2 = \frac{q_1 q_2}{r_{\min}} \Rightarrow r_{\min} = \frac{2 q_1 q_2}{m v^2} .$$

At this point the tunneling probability is equal to $A e^{-r_{\min}/\lambda}$, where A is a constant and λ is the de Broglie wavelength:

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Temperature dependence of proton fusion rate (continued)

$$\lambda = \frac{h}{m v} \quad (h = \text{Planck's constant})$$

This probability is usually written as

$$P(\text{tunneling}) = B \exp\left(-\frac{2\pi^2 r_{\min}}{\lambda}\right) = B \exp\left(-\frac{4\pi^2 q_1 q_2}{h v}\right) ,$$

where B is a constant. (Don't worry yet about the values of these constants.) The probability that the two particles have relative speed v is given by the Maxwell-Boltzmann distribution:

$$P(v) = C \exp\left(-\frac{m v^2}{2 k T}\right) ,$$

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Temperature dependence of proton fusion rate (continued)

so the probability of having tunneling *and* speed v is the product of these two probabilities:

$$P = D \exp\left(-\frac{4\pi^2 q_1 q_2}{h v} - \frac{m v^2}{2kT}\right).$$

The fusion rate is proportional to this probability P . For what v is the rate largest? Find by setting derivative equal to zero:

$$\begin{aligned} \frac{dP}{dv} &= D \exp\left(-\frac{4\pi^2 q_1 q_2}{h v} - \frac{m v^2}{2kT}\right) \frac{d}{dv} \left(-\frac{4\pi^2 q_1 q_2}{h v} - \frac{m v^2}{2kT}\right) \\ &= D \exp\left(-\frac{4\pi^2 q_1 q_2}{h v} - \frac{m v^2}{2kT}\right) \left(\frac{4\pi^2 q_1 q_2}{h v^2} - \frac{m v}{kT}\right) = 0, \end{aligned}$$

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Temperature dependence of proton fusion rate (continued)

or

$$\frac{4\pi^2 q_1 q_2}{h v^2} = \frac{m v}{kT} \Rightarrow v = \left(\frac{4\pi^2 q_1 q_2 kT}{h m}\right)^{1/3}.$$

for fastest rate. Take parameters for proton-proton fusion:

$$m = m_p / 2 = 0.84 \times 10^{-24} \text{ gm}, \quad q_1 = q_2 = 4.803 \times 10^{-10} \text{ esu}$$

and we get

$$v = \left(\frac{4\pi^2 q_1 q_2 kT}{h m}\right)^{1/3} = 1.5 \times 10^8 \text{ cm s}^{-1}$$

$$r_{\min} = \frac{2q_1 q_2}{m v^2} = 2.4 \times 10^{-11} \text{ cm} \gg 10^{-13} \text{ cm}.$$

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Temperature dependence of proton fusion rate (continued)

Thus

$$\begin{aligned} P_{\max} &= D \exp\left(-\frac{4\pi^2 q_1 q_2}{h v} - \frac{m v^2}{2kT}\right) \\ &= D \exp\left(-\frac{4\pi^2 q_1 q_2}{h} \left(\frac{4\pi^2 q_1 q_2 kT}{h m}\right)^{-1/3} - \frac{m}{2kT} \left(\frac{4\pi^2 q_1 q_2 kT}{h m}\right)^{2/3}\right) \\ &= D \exp\left(-\left(\frac{4\pi^2 q_1 q_2}{h}\right)^{2/3} \left(\frac{m}{kT}\right)^{1/3} - \frac{1}{2} \left(\frac{4\pi^2 q_1 q_2}{h m}\right)^{2/3} \left(\frac{m}{kT}\right)^{1/3}\right) \\ &= D \exp\left(-\frac{3}{2} \left(\frac{4\pi^2 q_1 q_2}{h}\right)^{2/3} \left(\frac{m}{kT}\right)^{1/3}\right) = D \exp\left(-\left(\frac{T_0}{T}\right)^{1/3}\right), \end{aligned}$$

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Temperature dependence of proton fusion rate (continued)

where

$$T_0 = \left(\frac{3}{2}\right)^3 \left(\frac{4\pi^2 q_1 q_2}{h}\right)^2 \frac{m}{k} = 3.85 \times 10^{10} \text{ K}$$

Thus the fusion rate is very sensitive to temperature. Some numbers:

Where	Temperature	P/D
Earth	300 K	7×10^{-220}
Sun's center	15.7 MK	1.4×10^{-6}
Hot star at center	100 MK	7×10^{-4}

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Implications

- Fusion is still fairly slow in the centers of stars. (Good!)
- Fusion rates are very sensitive to temperature; they are much higher at higher temperatures.
- Fusion rates are much lower for larger values of nuclear charge. This is why the CNO cycles require higher temperatures than the pp chains in order to be significant energy sources.
- Nucleosynthesis:** fusion in stellar cores produces heavier elements out of hydrogen, in amounts that should tend to decrease with increasing atomic number and nuclear weight.

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Results of nucleosynthesis: the cosmic abundances of the elements (not all due to stellar processes)

Figure: Shu, *The Physical Universe*

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Hotter fusion and heavier elements

Could stars in principle live forever simply by contracting gravitationally and increasing their temperature to ignite the next heavier source of nuclear fuel whenever they run out?

- No. The strong interaction's range is smaller than the diameters of all but the smaller nuclei, but the range of the Coulomb interaction still covers the whole nucleus.
- If nuclei get large enough the increase in electrostatic repulsion of protons becomes greater than the increase in binding energy from the strong interaction.
- Thus there is a peak in the binding-energy-per-baryon *vs.* atomic mass number relationship, which turns out to lie at iron (Fe).

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Hotter fusion and heavier elements (continued)

Figure: Shu, *The Physical Universe*

Implication:
once a star's core is composed completely of iron, it can no longer replenish its energy losses (from luminosity) by fusion. Stars therefore must die, eventually.

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