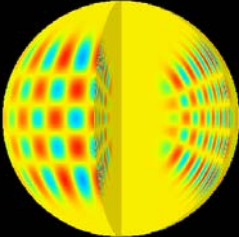


Today in Astronomy 142: seeing inside the stars

Light doesn't penetrate stars but neutrinos and sound do.

- Solar neutrinos and the former solar neutrino problem
- Radial pulsations in stars
- Pulsating stars
- The instability strip
- Nonradial pulsations in stars
- Helioseismology
- The standard solar model



Pulsation with $n, \ell, m = 14, 20, 16$; [Kosovichev et al. 1997](#)

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Solar neutrinos

The rate of emission of solar neutrinos should match the pp fusion rate in the center of the Sun. We know what the temperature is at the center, so we can predict accurately what the neutrino rate is. **The observed rate is lower than the electron-neutrino production rate, by factors of 2-4 depending on energy.**

Why?

- Are we wrong about the temperature? This seems unlikely (as you will show this week).
- Has the center cooled off suddenly, less than a photon-diffusion time ago? **No**, say solar-oscillation measurements.

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Solar neutrinos (continued)

- Can neutrinos change "flavor" from electron neutrinos to muon- or tauon-neutrinos in transit? **Yes**, say the particle physicists.
 - Several experiments on neutrinos produced in nuclear reactors (e.g. [Kamiokande](#)) have detected such flavor oscillations, even *via* deficit of electron neutrinos.
 - Confirmation comes from detection of solar muon neutrinos (e.g. [SNO](#)) at rates consistent with oscillation.
 - This is possible if neutrinos have nonzero mass!
 - For starting and inspiring this work, Ray Davis shared the 2002 Nobel Prize in physics.

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Sound waves in stars

Consider a star with constant density ρ . The pressure is given by

$$\frac{dP}{dr} = -\frac{GM(r)\rho}{r^2} = -\frac{G\rho}{r^2} \frac{4\pi}{3} \rho r^3 = -\frac{4\pi}{3} G\rho^2 r$$

$$P(R) - P(r) = \int_r^R \frac{dP}{dr} dr$$

$$P(r) = \frac{4\pi}{3} G\rho^2 \int_r^R r dr = \frac{2\pi}{3} G\rho^2 (R^2 - r^2)$$

Pressure waves travel at the speed of sound.

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Sound waves in stars (continued)

The (adiabatic) speed of sound is

$$v_s = \sqrt{\frac{\gamma P}{\rho}}$$

where γ is the adiabatic index - the ratio of specific heats - which is equal to 5/3 for monatomic gases, like the ionized gases that make up most stars. (See, e.g., [here](#) in AST 111.)

If sound waves can propagate in a medium of finite size, then it is possible for there to be **standing waves** of pressure, due to reflection of sound from the "ends" of the object.

- Sharp changes in sound speed can result in reflection of sound; any sharp changes in P/ρ can do that. Like the surface, the center, the edge of the convection zone...

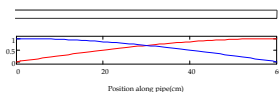
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Sound waves in clarinets

Notes played by wind instruments correspond to their acoustic standing waves.



- For instruments that consist of pipes open on one end, like clarinets and organs, the standing waves have a node of pressure on the open end and a peak at the closed end.

- Thus the lowest note has a wavelength four times the length of the pipe, and frequency and period given by

$$v = \frac{v_s}{\lambda} = \frac{v_s}{4L} = \frac{34000 \text{ cm sec}^{-1}}{4 \times 60 \text{ cm}} \cong 142 \text{ Hz} \approx \text{D below middle C}$$

$$\Pi = 1/v = 4L/v_s = 7 \text{ ms}$$

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Sound waves in stars (continued)

Now consider the “pipe” represented by the surface and the center of a star. This “pipe” is open on the surface and closed at the center, so the period of the longest-wavelength standing wave is

$$\begin{aligned} \Pi &= 4 \int_0^R \frac{dr}{v_s} = \frac{4}{\sqrt{\frac{2\pi}{3} \gamma G \rho}} \int_0^R \frac{dr}{\sqrt{R^2 - r^2}} = 4 \sqrt{\frac{3}{2\pi\gamma G \rho}} \int_0^1 \frac{dx}{\sqrt{1-x^2}} \\ &= 4 \sqrt{\frac{3}{2\pi\gamma G \rho}} \int_0^{\pi/2} \frac{\cos u du}{\sqrt{1-\sin^2 u}} = 4 \sqrt{\frac{3}{2\pi\gamma G \rho}} \int_0^{\pi/2} du \\ &= 4 \sqrt{\frac{3}{2\pi\gamma G \rho}} \frac{\pi}{2} = \sqrt{\frac{6\pi}{\gamma G \rho}} \end{aligned}$$

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Sound waves in stars (continued)

For a uniform-density star the mass and size of the Sun, we get

$$\Pi = \sqrt{\frac{6\pi}{\gamma G \rho}} = 11700 \text{ sec} = 183 \text{ min} = 3 \text{ hours}$$

and the period increases if the density decreases.

- Pressure waves manifest themselves as oscillations of surface temperature and radius of the star, which in turn cause oscillation of the magnitude of the star.
- The details of the periods and amplitudes of the oscillations are sensitive to the density, its variations, and the equation of state (P - ρ relation) **inside** the star.
- Good agreement with the loudest pulsating stars...

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Loud sound waves in stars: pulsators

Beginning with the discovery around 1600 of the pulsation of Mira (α Ceti), and in 1784 of that of δ Cephei, seven major types of pulsating stars have been found.

Type	Period	Amplitude, Δm
Long-period (Mira) variables	100-700 days	2-7 mag
Classical (Pop I) Cepheids	1-50 days	0.5-1.5 mag
W Virginis stars (Pop II Cepheids)	2-45 days	0.5-1.5 mag
RR Lyrae stars	1-48 hours	1-1.5 mag
δ Scuti stars	1-3 hours	0.1-0.6 mag
β Cephei stars	3-7 hours	0.05-0.2 mag
ZZ Ceti stars white dwarfs	100-1000 sec	0.1-0.3 mag

The rest are giant or supergiant stars. [\(General Catalogue of Variable Stars\)](#)

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Patterns of pulsators: the instability strip

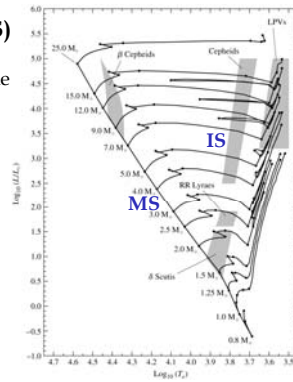
Suspiciously, the pulsating stars are not spread randomly all over the luminosity-temperature parameter space, nor are they all a certain composition.

- ❑ Mira variables are all supergiant stars, belonging to a part of the H-R diagram called the **asymptotic giant branch**.
- ❑ β Cep stars are all B stars, lying just above the main sequence.
- ❑ All the others lie along a long, narrow, nearly vertical patch of the H-R diagram, at effective temperatures just a bit less than 10^4 K: the **instability strip**.
 - δ Scv stars are close to the main sequence, but most IS inhabitants won't be there long: they develop rapidly after leaving the main sequence.

The instability strip (IS)

Extending the strip below the main sequence (MS) til it intersects the white dwarfs, one finds there the ZZ Ceti stars.

From Carroll and Ostlie, [Modern Astrophysics, 2e.](#)



Origin of the instability strip

- ❑ In stars without substantial convection zones, energy is transported predominantly by diffusion of photons, at a rate determined by the opacity of the material.
- ❑ Under almost all stellar-interior conditions, opacity increases with increasing density, but *decreases* even more sharply with increasing temperature. Thus opacity tends to decrease with increasing *compression*.
- ❑ If instead opacity were to increase with increasing compression a mechanical instability would result:
 - a random compression of some layer in the star would cause that layer to become more opaque, in turn damming up the energy that was flowing toward the surface.

Origin of the instability strip (continued)

- This would result in the compressed layer being pushed upwards, along with everything above it.
- As the compressed layer is pushed upwards it spreads out laterally and rarifies. As soon as it rarifies enough to drop its opacity, the dammed-up heat can flow through it again, and the layer can sink back toward its former level.
- But it compresses as it sinks, becomes more opaque as it compresses, and dams up the heat again. The original random fluctuation has ignited a self-sustaining, global, radial oscillation.

This is called the **kappa mechanism**, named after the symbol most astrophysicists use for opacity.

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Origin of the instability strip (continued)

Under what conditions would opacity increase with increasing compression?

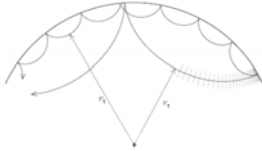
- ❑ Partial ionization zones: that is, in the domain of density and temperature in which a major opacity source is changing its ionization state...
 - ...because, then, compression can just increase the density and change the ionization state without changing the temperature.
- ❑ **Helium ionization** occurs deep enough in the star to drive oscillations for all stars with $T_c \leq 7 - 10 \times 10^4$ K.
- ❑ But at just slightly lower temperatures stars develop convection zones. Thus only a narrow strip would be **unstable - as indeed we observe.**

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Nonradial stellar oscillations

Some pressure waves may have transverse components as well as radial components.

- ❑ Such waves can be trapped between the surface and a certain density within the interior as they propagate.
 - Surface: reflect due to density drop-off
 - Interior: refract, eventually bending back up, due to increase of sound speed with depth.




Propagation of acoustic waves, corresponding to modes with $\ell = 30$, $\nu = 3$ mHz (deeply penetrating rays) and $\ell = 100$, $\nu = 3$ mHz (shallowly penetrating rays). From [Christiansen-Dalgaard 2003](#).

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Nonradial stellar oscillations (continued)

- ❑ The deeper the waves penetrate, the fewer surface reflections.
- ❑ Still can make standing waves, and thus stable oscillations, by having an integer number of wavelengths per circuit.
- ❑ On the surface the pressure variations of the standing waves are described by the spherical harmonic function Y_ℓ^m , where m is the number of pressure nodes, and $\sqrt{\ell(\ell+1)}$ the number of wavelengths, along the equator.

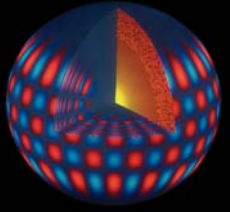


Again, modes with $\nu = 3$ mHz; in order of decreasing depth of penetration they have $\ell = 0, 2, 20, 25$ and 75 (Christiansen-Dalgaard 2003).

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Nonradial stellar oscillations (continued)

- ❑ For a given surface pattern indexed by ℓ and m , different radial components of the oscillation are also possible, indexed by the number n (> 0) of pressure nodes in the radial direction.
- ❑ In general each n - ℓ - m combination represents a different frequency or period of oscillation.
- ❑ Can be ignited or driven by energy from convection.



Snapshot of one particular mode ($n, \ell, m = 10, 22, 18$) of nonradial oscillation. Red/blue = motion inwards/ outwards. (GONG/NSO/AURA/NSF)

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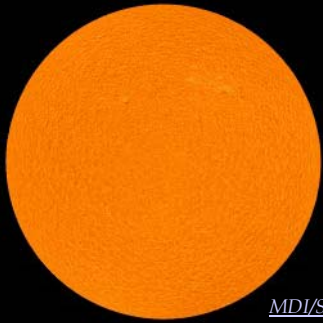
Solar oscillations and helioseismology

The Sun oscillates in many different modes, with periods around 5 minutes. This was discovered in 1962 by Bob Leighton and his students, and was explained later by Roger Ulrich as nonradial oscillations.

- ❑ Took a while to notice these oscillations because they are so small. The velocity amplitudes are typically 100 cm/sec; the displacements, tens of meters.
- ❑ Many **thousands** of modes (standing waves of given n, ℓ and m) have now been identified. The period of each mode gives the integral of $1/v_s = 1/\sqrt{\gamma P/\rho}$ along a different path through the Sun's interior.
- ❑ Modes with smaller ℓ penetrate more deeply into the interior. Good coverage for the outer 90% of the volume.

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The solar five-minute oscillations



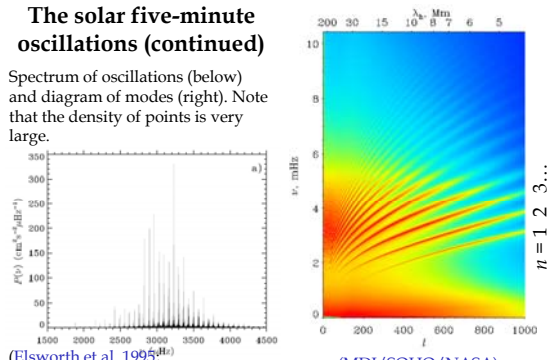
Sped up a bit, of course.

MDI/SOHO/NASA

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The solar five-minute oscillations (continued)

Spectrum of oscillations (below) and diagram of modes (right). Note that the density of points is very large.



[\(Elsworth et al. 1995²¹\)](#)
[\(Christiansen-Dalsgaard 2003\)](#) *(MDI/SOHO/NASA)*

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Precise knowledge of the solar interior

So we have

- ❑ a direct measure of the pp fusions per unit time at the very center, from pp-fusion neutrinos (corrected for oscillation).
- ❑ thousands of measurements of integrals of $1/v_s = 1/\sqrt{\gamma P/\rho}$ from which the density, pressure, temperature, abundances, etc. can be determined over most of the Solar volume.
- ❑ and thus confidence that we know precisely and accurately the thermodynamic parameters of all parts of the Solar interior, without being able to see the interior directly. The result is the Standard Solar Model (e.g. [Bahcall et al. 2005](#))

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