Today in Astronomy 142: dead and stillborn stars

- ❑ Degeneracy pressure of electrons and neutrons
- ❑ White dwarfs
- ❑ Neutron stars
- ❑ Brown dwarfs and giant planets

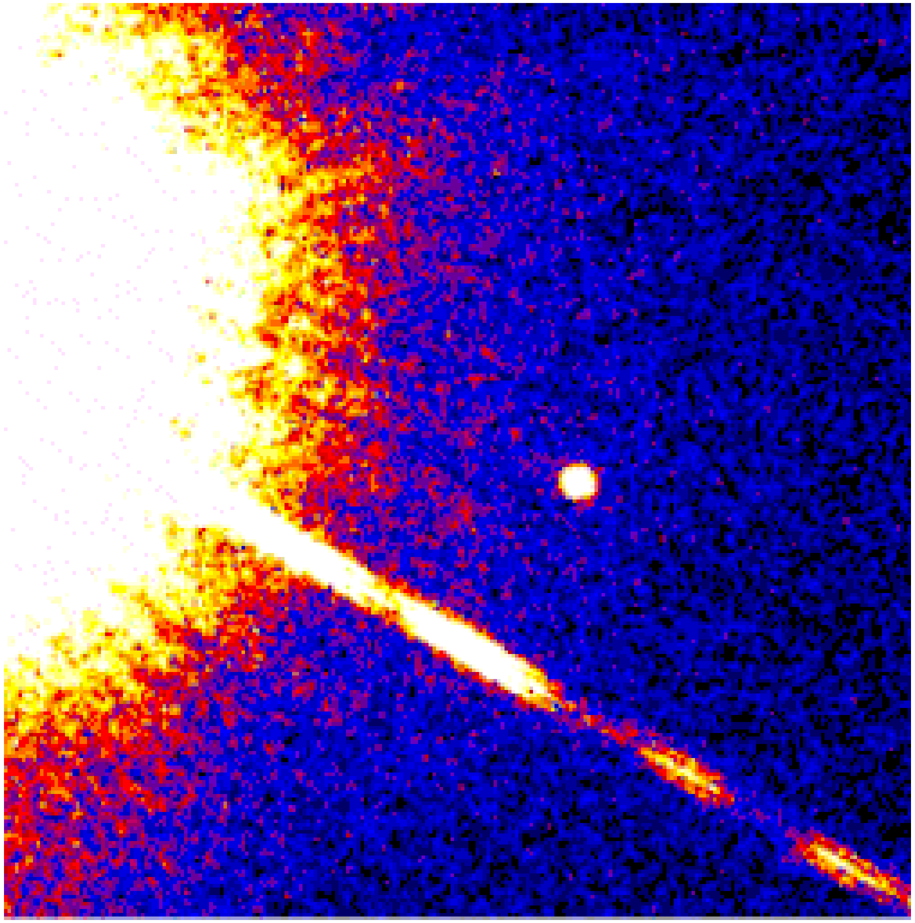


Figure: HST picture of the faint main sequence star Gliese 229A on the left -- heavily overexposed -- and its brown-dwarf companion Gliese 229B on the right (Nakajima *et al.* 1995/Caltech and NASA).

How stars can support themselves against gravity

- **Thermodynamics**: gas and radiation pressure….

$$P_{\text{gas}} = nkT \qquad P_{\text{radiation}} = \frac{4\sigma T^4}{3c}$$

….support stars in which thermonuclear energy generation occurs.

- **Quantum mechanics**: degeneracy pressure sets in under extreme states of compression and/or low temperatures.

  This is the means of support for objects with no fusion: dead stars (**white dwarfs, neutron stars**), stillborn stars (**brown dwarfs**), and the cores of **giant planets** (e.g. Jupiter, Saturn).

Degeneracy pressure

Degeneracy pressure is due to:

the **Pauli exclusion principle**: no two identical fermions (spin-1/2 particles) can be in the same state simultaneously.

- Other things equal, this means that a larger density of identical fermions (e.g. electrons) involves confinement of each particle to a smaller space by mutual exclusion.

the **Heisenberg uncertainty principle**: $\Delta x \Delta p_x \geq h$ (Δ=statistical uncertainty in..., p = momentum, x = position)

- Other things equal, this means that an electron confined to a smaller box (smaller Δx) could have a larger momentum component along the box's side (larger Δp_x).

Handwaving, 1-D derivation of the equation of state (P *vs.* n relationship) for degeneracy pressure follows….

Degeneracy pressure, continued

Consider identical fermions in a stack of 1-D boxes bounded by a pair of walls with area A. If particles have finite momentum they hit the walls and exert pressure:

$$P \cong \frac{F}{A} = \frac{1}{A}\frac{dp}{dt} \cong \frac{1}{A}\frac{nA\delta x / 2}{\delta t} p_x$$

$$= \frac{n v_x p_x}{2} \quad .$$

Number of particles that hit one wall

Typical momentum per particle

Time interval in which they hit

Length of boxes determined by Pauli principle from space not taken up by other particles past the walls.

n = density (particles per unit volume)

$1/n$ = volume per particle

$\ell = (1/n)^{1/3}$ = length of box

$\Delta x = \ell$ Each particle must be somewhere within its box.

Degeneracy pressure, continued

Typical momentum per particle:

$$p_x \approx \Delta p_x \qquad \text{(not exactly, but within a factor)}$$
$$\approx h/\Delta x \qquad \text{(by the uncertainty principle)}$$
$$= hn^{+1/3}$$

Nonrelativistic motion: $v_x = p_x/m$

Thus

$$P = \frac{nv_x p_x}{2} = \frac{1}{2}\frac{np_x^2}{m} = \frac{1}{2}\frac{h^2 n^{5/3}}{m}$$

Done with the Fermi-Dirac probability distribution for *p, v, etc.* (AST 241 or 453-style) and you get the leading factor right:

$$P = 0.0485\frac{h^2 n^{5/3}}{m}$$

Very different from an ideal gas!

Electron degeneracy pressure

Consider a gas of electrons, produced by ionization from atoms with nuclear charge Ze and baryon number A. Then the electron and nucleus densities are related by

$$n_e = Zn_+$$

and the mass density is

$$\rho = m_e n_e + A m_p n_+ \quad (m_p \cong m_n)$$
$$\cong A m_p n_+ \quad (m_p >> m_e)$$

so $n_e = Z\rho / A m_p$, and

$$P_e = 0.0485 \frac{h^2}{m_e} \left(\frac{Z}{A} \right)^{5/3} \frac{\rho^{5/3}}{m_p^{5/3}}$$

Electron degeneracy pressure

Degenerate electrons in stars

Let's use the new equation of state, instead of the ideal gas law, to balance gravity and hold up a star. (Recall that the last time we did this we got useful scaling relationships for *M, R, T,* P_C, *etc.*)

Our former results from the ideal gas law and gravity:

$$P_C \approx \frac{GM^2}{R^4} \qquad \rho_C \approx \frac{M}{R^3}$$

Precise calculations for $P \propto \rho^{5/3}$ and gravity turn out to give:

$$P_C = 0.77\frac{GM^2}{R^4} \qquad \rho_C = 1.43\frac{M}{R^3}$$

Suppose this pressure from weight is balanced by electron degeneracy pressure…..

Degenerate electrons in stars, continued

$$0.77\,\frac{GM^2}{R^4} = 0.0485\,\frac{h^2}{m_e}\left(\frac{Z}{A}\right)^{5/3}\,\frac{\rho^{5/3}}{m_p^{5/3}}$$

$$= 0.0485\,\frac{h^2}{m_e}\left(\frac{Z}{A}\right)^{5/3}\left(\frac{1.43M}{m_p}\right)^{5/3}\frac{1}{R^5}$$

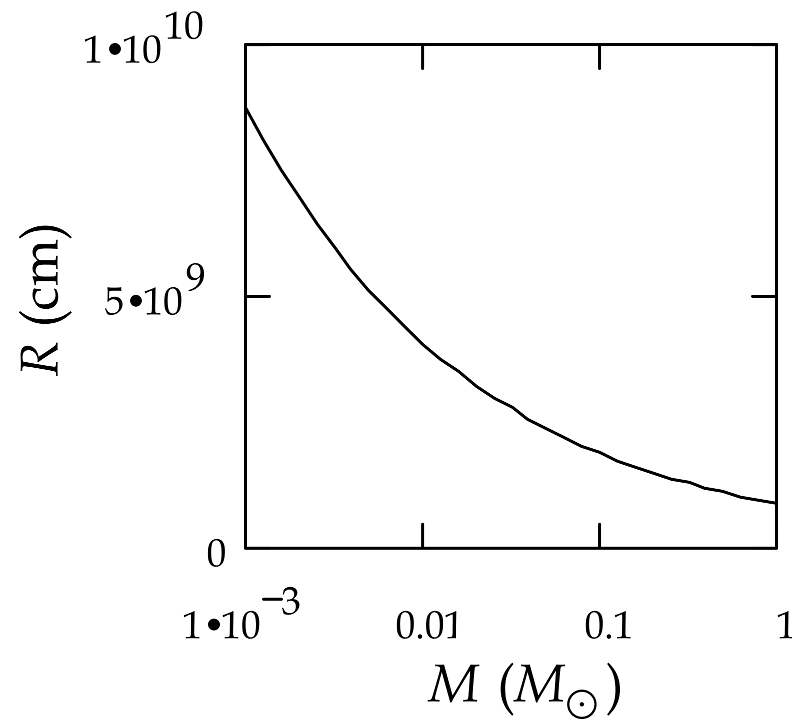
$$\Rightarrow R = 0.114\,\frac{h^2}{Gm_e m_p^{5/3}}\left(\frac{Z}{A}\right)^{5/3} M^{-1/3}$$

- No temperature dependence! Much simpler than a normal star.
- Much smaller than a normal star of the same mass.

White dwarf stars

Numerical example: $M = 1M_{\odot}$, $Z / A = 0.5 \Rightarrow R = 9\times 10^{8}$ cm.
(compare $R_{\odot} = 6.96\times 10^{10}$ cm, $R_{\oplus} = 6.4\times 10^{8}$ cm)
Mass of a star, size of a planet: **white dwarf**.

- Remarkable feature of *R*-*M* relation: ***R* decreases with increasing *M*.** Reason: larger mass requires larger supporting pressure, which in turn requires larger electron momenta, which in turn requires that each electron be confined to a smaller box.



Massive white dwarfs: relativity and Chandrasekhar's WD mass limit

- To support higher mass (smaller) white dwarfs, larger electron momenta (and speeds) are required.
- Speeds cannot exceed the speed of light! And when they get close to c, p isn't simply given by mv any more.
- Electron degeneracy pressure in extreme relativistic limit (v approaching c; see slide 5) is

$$P_e = 0.123 hc n_e^{4/3}$$

The relativistic and nonrelativistic expressions for electron degeneracy pressure are equal at $n_e = 10^{30}\ \mathrm{cm}^{-3}$, about that of the core of a 0.3 $M_\odot$ white dwarf.

Massive white dwarfs: relativity and Chandrasekhar's WD mass limit (continued)

For $P \propto \rho^{4/3}$ and gravity, the central pressure and density turn out to be

$$P_C = 11\frac{GM^2}{R^4} \qquad \rho_C = 12.9\frac{M}{R^3}$$

Balance that with relativistic degeneracy pressure, and even radius disappears from the equation (see Homework #3):

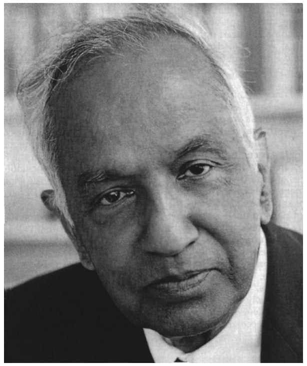
$$M_{\mathrm{Ch}} = 0.2\left(\frac{Z}{A}\right)^2\left(\frac{hc}{Gm_p^2}\right)^{3/2} m_p$$

$$= 1.44 M_\odot \quad \text{for } Z/A = 0.5.$$

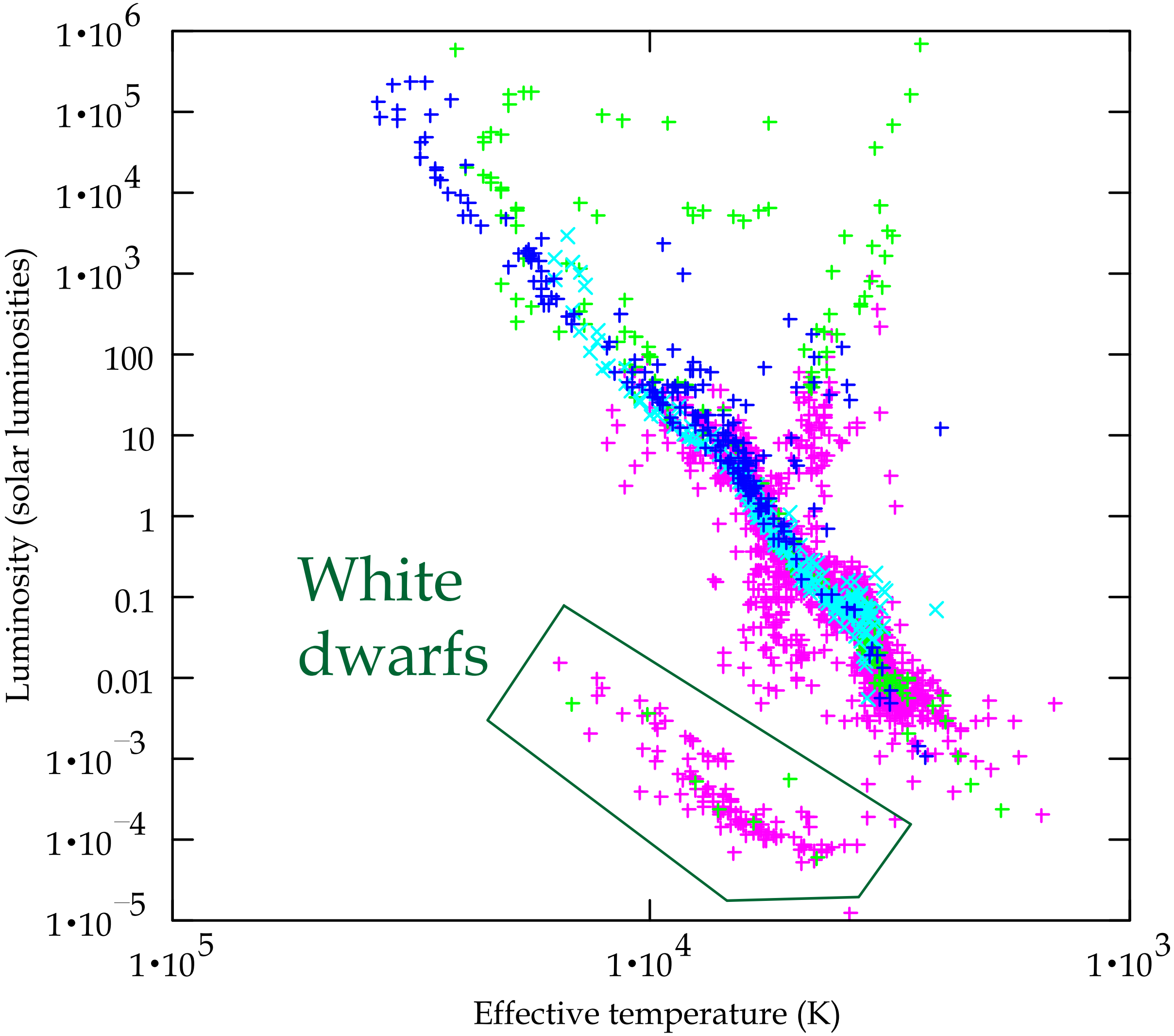
Maximum mass of a white dwarf (corresponds to zero radius)

How well does the theory work?

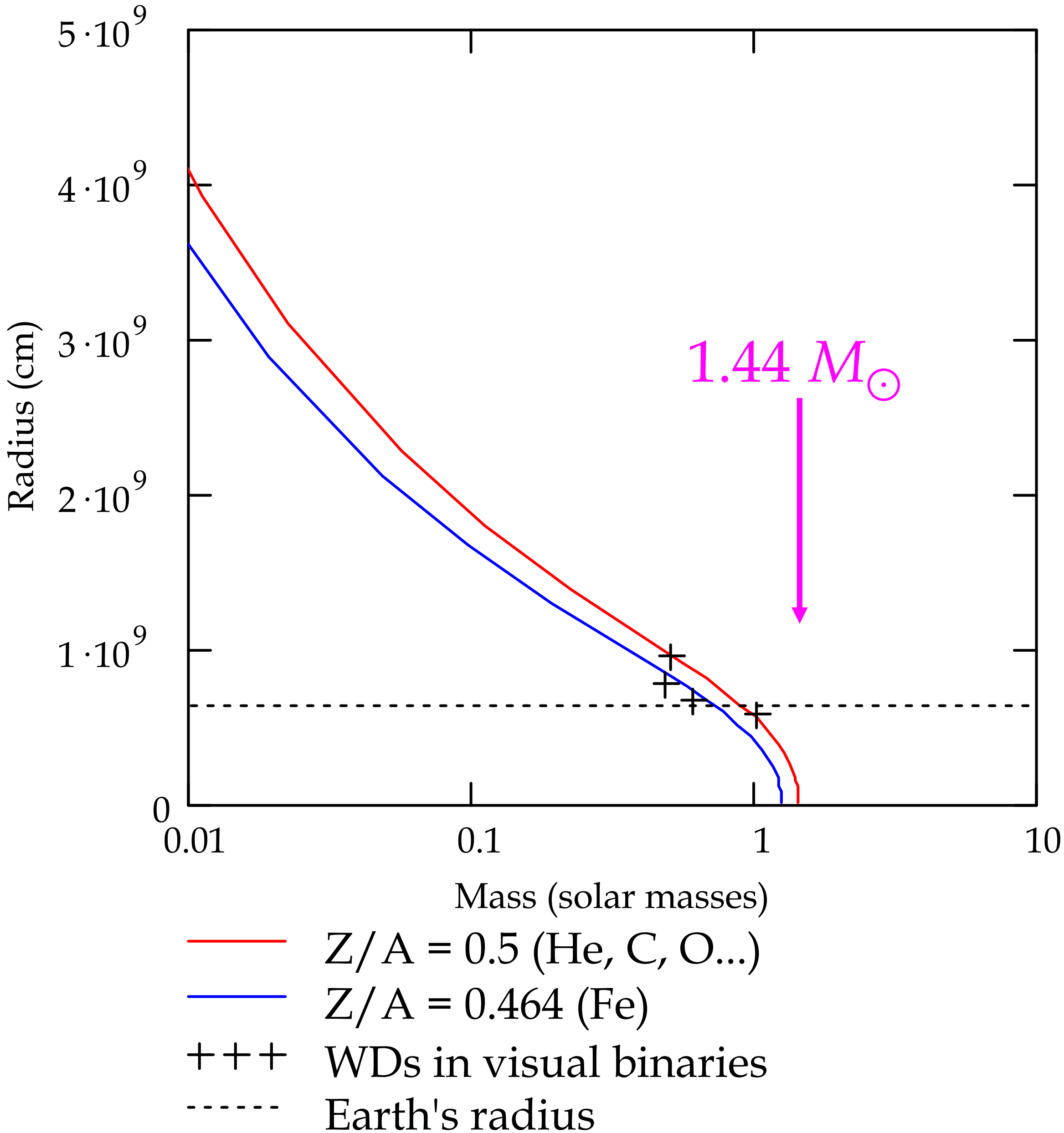
- White dwarfs are hard to detect in binaries because they are so much fainter than main-sequence stars. The handful detected in binaries such that masses could be worked out all weigh less than 1.44 $M_{\odot}$ (see slide 14).
- WD cooling is simple and well understood, though, and provides an indirect way to estimate the masses of observed, isolated white dwarfs. The masses of WDs determined in this way are tightly clustered about 0.5-0.6 $M_{\odot}$ (see slide 15). None weigh more than 1.44 $M_{\odot}$.
- For this work, involving one of the first correct combinations of relativity and quantum mechanics as well as an important astrophysical application, Chandrasekhar shared the 1983 Nobel Prize in Physics.



Nearby white dwarfs



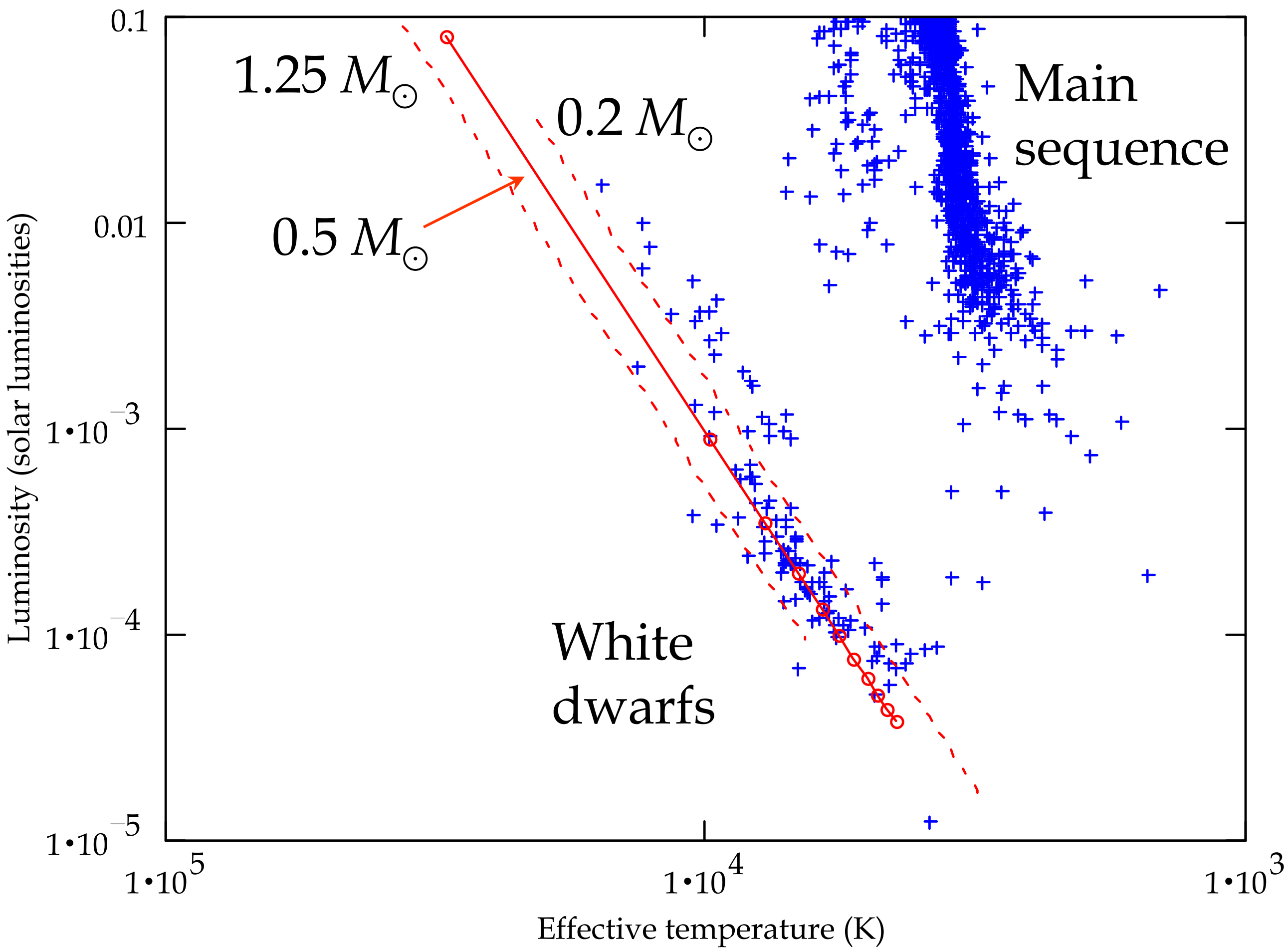
- Stars within 25 parsecs of the Sun (Gliese and Jahreiss 1991)
- Nearest and Brightest stars (Allen 1973)
- Pleiades X-ray sources (Stauffer *et al.* 1994)
- Binaries with measured temperature and luminosity (Malkov 1993)



Chandrasekhar's relativistic white dwarf theory (result of calculation à lá AST 241 or 462)

Data on white dwarfs in visual binary systems (all four of them) from the *Hipparcos* satellite, by Provencal *et al.* 1998.

White dwarf cooling, masses and ages (result of calculation à lá AST 241)



Cooling curve for a 0.5 $M_\odot$ carbon white dwarf, with time from zero to 10^{10} years marked in 10^{9} year intervals (circles) and compared to the white dwarfs in the third Gliese catalogue (crosses). The starting central temperature was 10^{8} K.

Beyond the Chandrasekhar mass: neutron stars

A dead star more massive than 1.4 $M_\odot$ simply cannot be supported by electron degeneracy pressure; add a little too much mass and it will collapse gravitationally or explode.

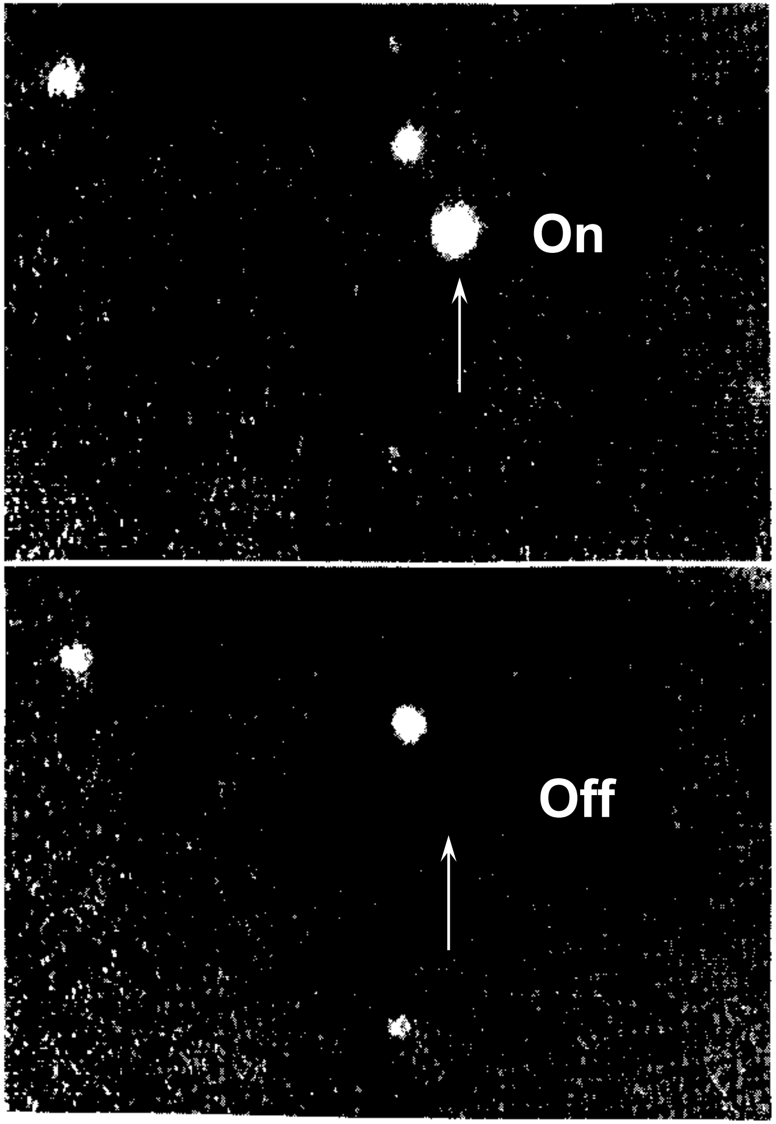
- During the collapse, the extra energy liberated from gravity, and the high density, can help drive some endothermic nuclear reactions, notably $\Delta E + e + p \rightarrow n + \nu_e$.
- But neutrons are fermions, and neutron degeneracy pressure can balance gravity: a **neutron star** is formed. Nonrelativistic formula turns out to be:

$$R = 0.114 \frac{h^2}{G m_p^{8/3}} M^{-1/3}$$

That's 1.5×10^6 cm (~10 miles) for 1.4 $M_\odot$.

Beyond the Chandrasekhar mass: neutron stars

- The maximum mass is not as easy to calculate as in the white dwarf case; it involves general relativity and an equation of state that includes the strong interaction.
- The maximum mass turns out to be about 2 $M_\odot$, according to the best models; it could not possibly be > 3 $M_\odot$.
- Neutron stars generally have very large magnetic fields (conservation of flux) and rotate rapidly (conservation of angular momentum), and are observed as **pulsars**: apparently pulsed radio (or visible/X-ray) emission from high energy electrons moving along poloidal field lines.
- A handful of pulsars are observed in binary systems and their masses have been measured. Curiously, they all come out to about 1.4 $M_\odot$.



A neutron star observed directly

The neutron star at the center of the Crab Nebula, the remnant of the supernova visible in the year 1054. It is seen as a **pulsar** in these images taken 0.03 second apart.

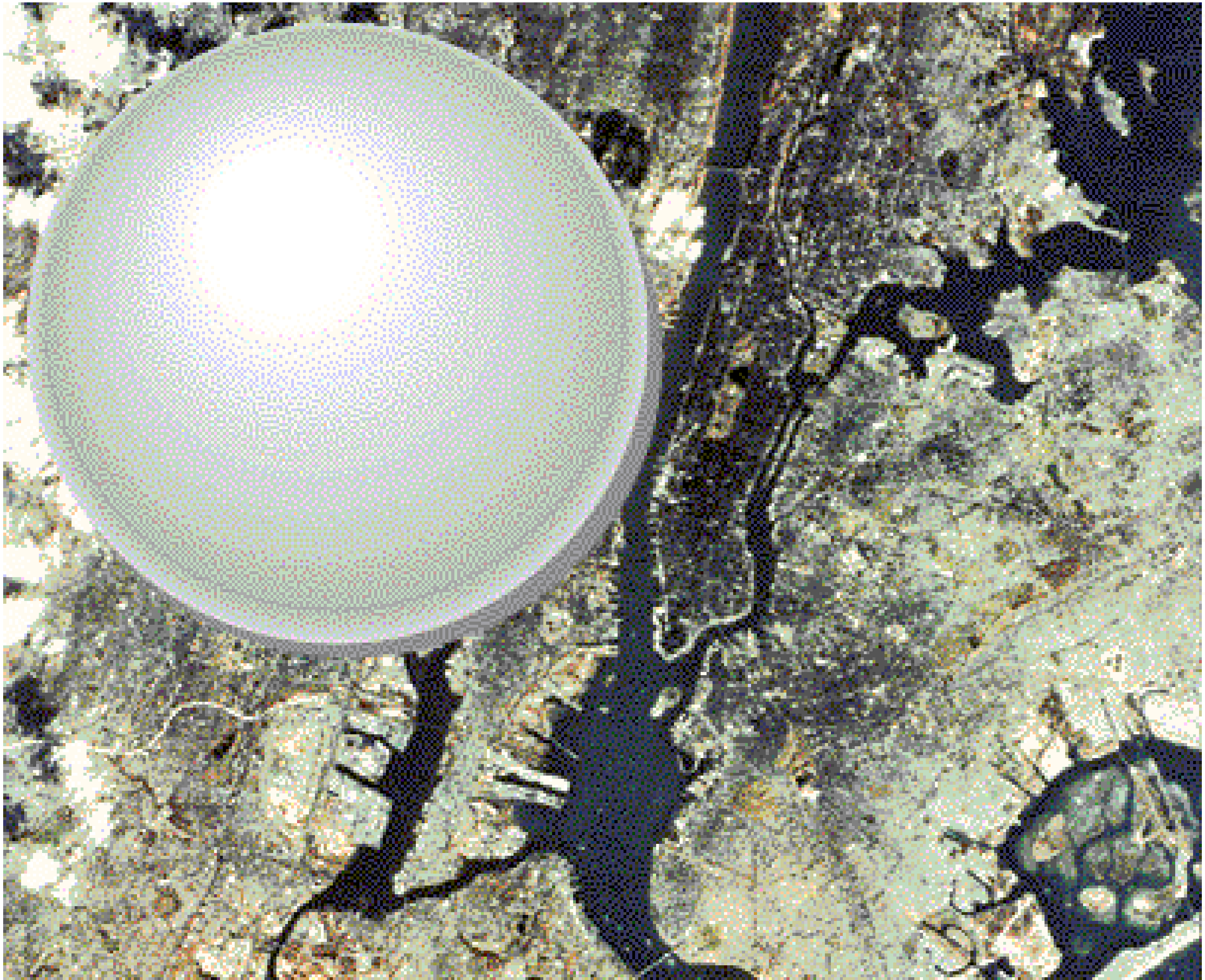


Figure: a 1.4 $M_\odot$ neutron star and New York City, shown at the same scale. From Chaisson and McMillan, *Astronomy Today*.

Way smaller than the Chandrasekhar mass: brown dwarfs and giant planets

When stars form, they **contract** until they are hot enough in the center (about 3×10^6 K) to ignite the pp-chain fusion reactions. Recall that

$$T_C = \frac{P_C \bar{m}}{\rho_C k} \cong \frac{\bar{m}}{k}\left(19\frac{GM^2}{R^4}\right)\left(\frac{1}{150}\frac{R^3}{M}\right) = 15.7\times10^6\left(\frac{M}{M_\odot}\right)\left(\frac{R_\odot}{R}\right)\text{ K} .$$

for solar-type stars, if gravity is supported by gas pressure.

- For **small** masses this involves gas pressures that become smaller than electron degeneracy pressure – so that degeneracy pressure can stop the contraction and prevent the object from reaching fusion temperatures. This imposes a lower mass limit on what can become a star. The limit turns out to be about 0.08 $M_\odot$ (HW #3).

Brown dwarfs and giant planets (continued)

Depending upon how they are formed and what their mass is, such objects are called **brown dwarfs** or **giant planets**.

- Because they cannot replace the energy that leaks away in the form of light, they simply remain at the size determined by degeneracy pressure, and cool off forever.
- Thus if they are very old, they are *very* faint. This prevented their detection until just a few years ago. Now thousands are known deep near-infrared surveys (e.g. 2MASS), and from Spitzer Space Telescope observations.
- But these objects could be very numerous in our galaxy and comprise a significant – and largely invisible – component of the galaxy. We will touch on this again, under the rubric of **dark matter** or **missing mass**.