Today in Astronomy 142: stellar-mass black holes

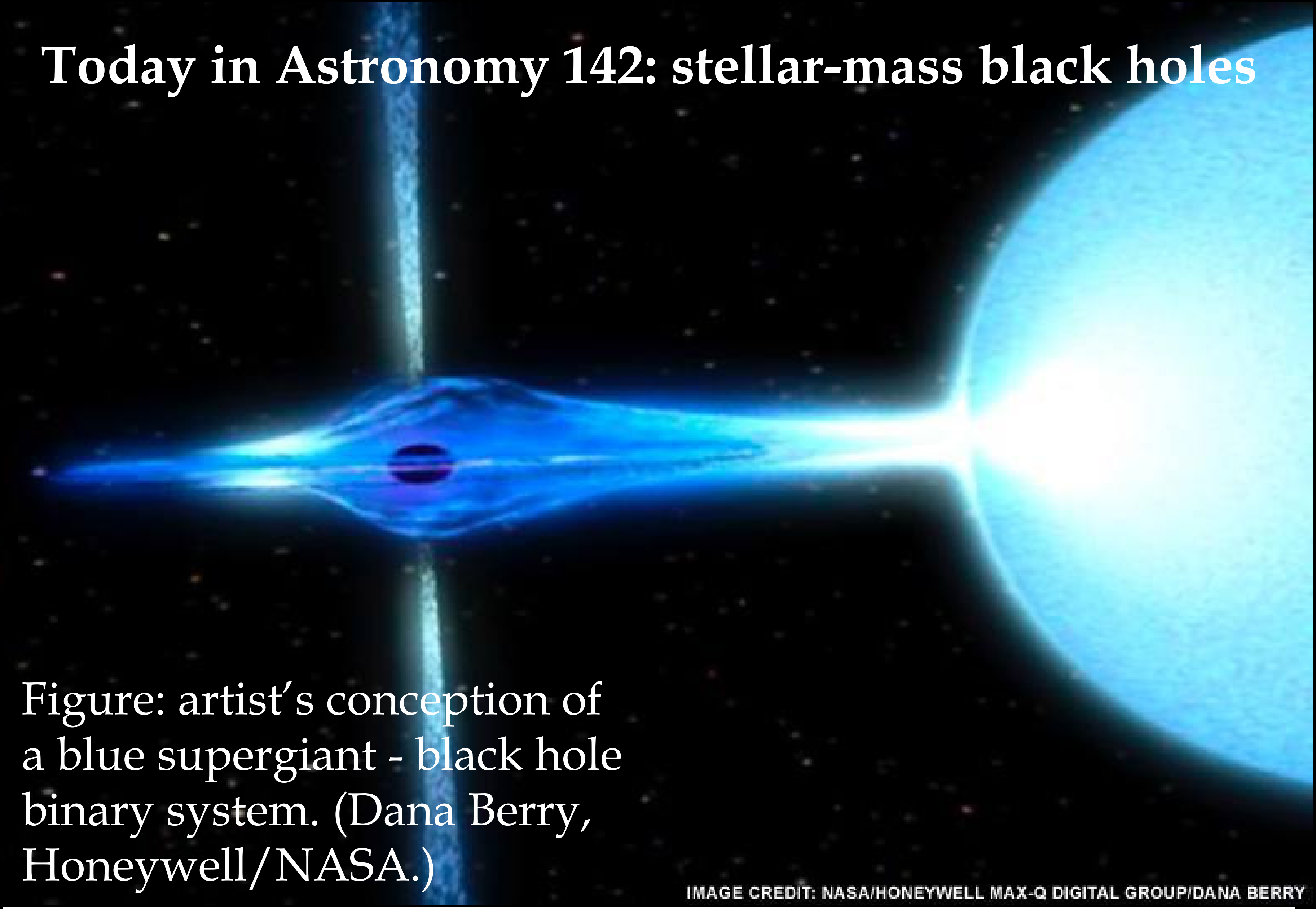


Figure: artist's conception of a blue supergiant - black hole binary system. (Dana Berry, Honeywell/NASA.)

Escape speeds from stars, white dwarfs and neutron stars: relativistic stars

Neglecting relativity,

$$E = \frac{1}{2} m v_{esc}^2 - \frac{GMm}{R} = 0$$

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

$= 619$ km s^{-1} from the Sun

$= 6970$ km s^{-1} from a $1 M_\odot$ white dwarf

$= 154000$ km s^{-1} from a $1 M_\odot$ neutron star

$v_{esc} = c = 299792$ km s^{-1} when $\boxed{R = 2GM/c^2}$ Schwarzshild radius

Beyond the neutron-star maximum mass: black holes

- The maximum mass of a neutron star is about 2 $M_{\odot}$. There is **no** physical process that can support a heavier object without internal energy generation.
- A heavier object will collapse past neutron-star dimensions, and soon thereafter becomes a **black hole**: an object from which even light cannot escape if it lies within

$$R_{\mathrm{Sch}} = \frac{2GM}{c^2}$$

  of the object, as measured by a distant observer; this spherical surface is called the **event horizon**.
- Obviously, with light speeds and strong gravity, one **cannot** ignore relativity (in particular, general relativity) in the description of neutron stars and black holes, but this result turns out accidentally to be the same as that obtained with relativistic machinery.

Interesting facts about black holes

- Time intervals on clocks near black holes appear to distant observers to be slow compared to their (identical) clocks, an effect known as **gravitational time dilation** or the **gravitational redshift**:

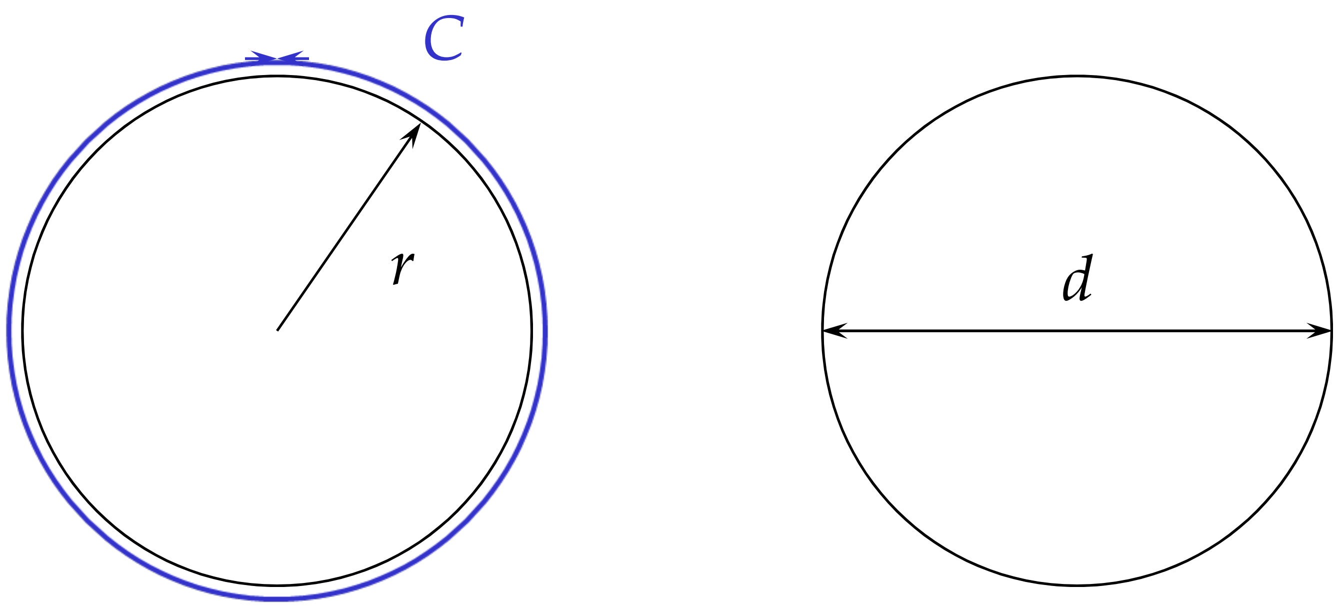
$$\Delta t = \Delta\tau \Big/ \sqrt{1 - 2GM/rc^2} > \Delta\tau$$

- Thus time appears (to a distant observer) to stop at the event horizon: $\Delta t \rightarrow \infty$ as $r \rightarrow R_{\mathrm{Sch}}$
- Near a black hole, a (small) length $\Delta\mathcal{L}$ measured instantaneously between two points on a **radial** line is greater than the **coordinate distance** Δr between the points (measured by a distant observer):

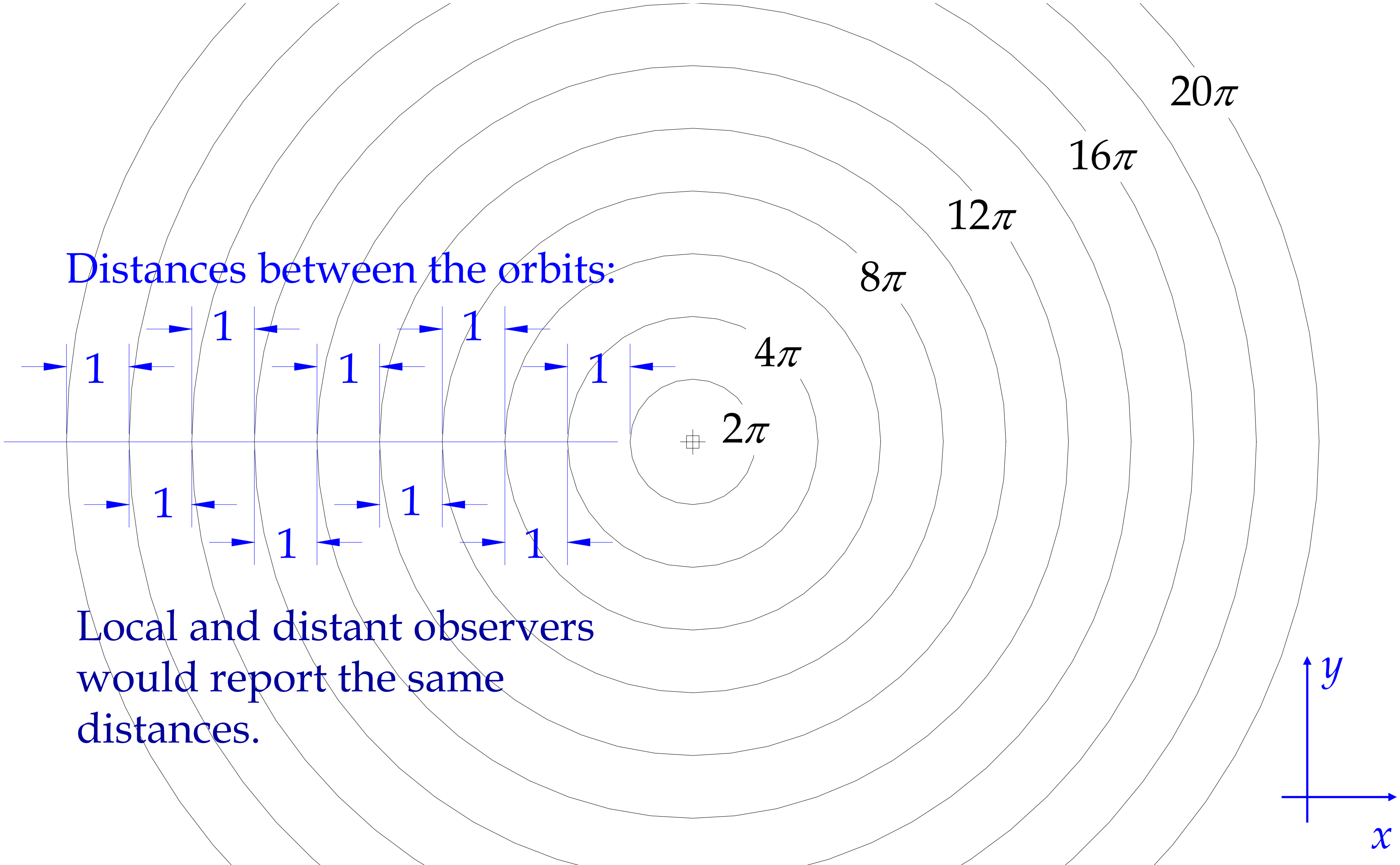
$$\Delta\mathcal{L} = \Delta r \Big/ \sqrt{1 - 2GM/rc^2} > \Delta r$$

Embedding diagrams, and why they're useful

Circles in flat spacetime: $C = 2\pi r = \pi d$. That, of course, is the very definition of π.

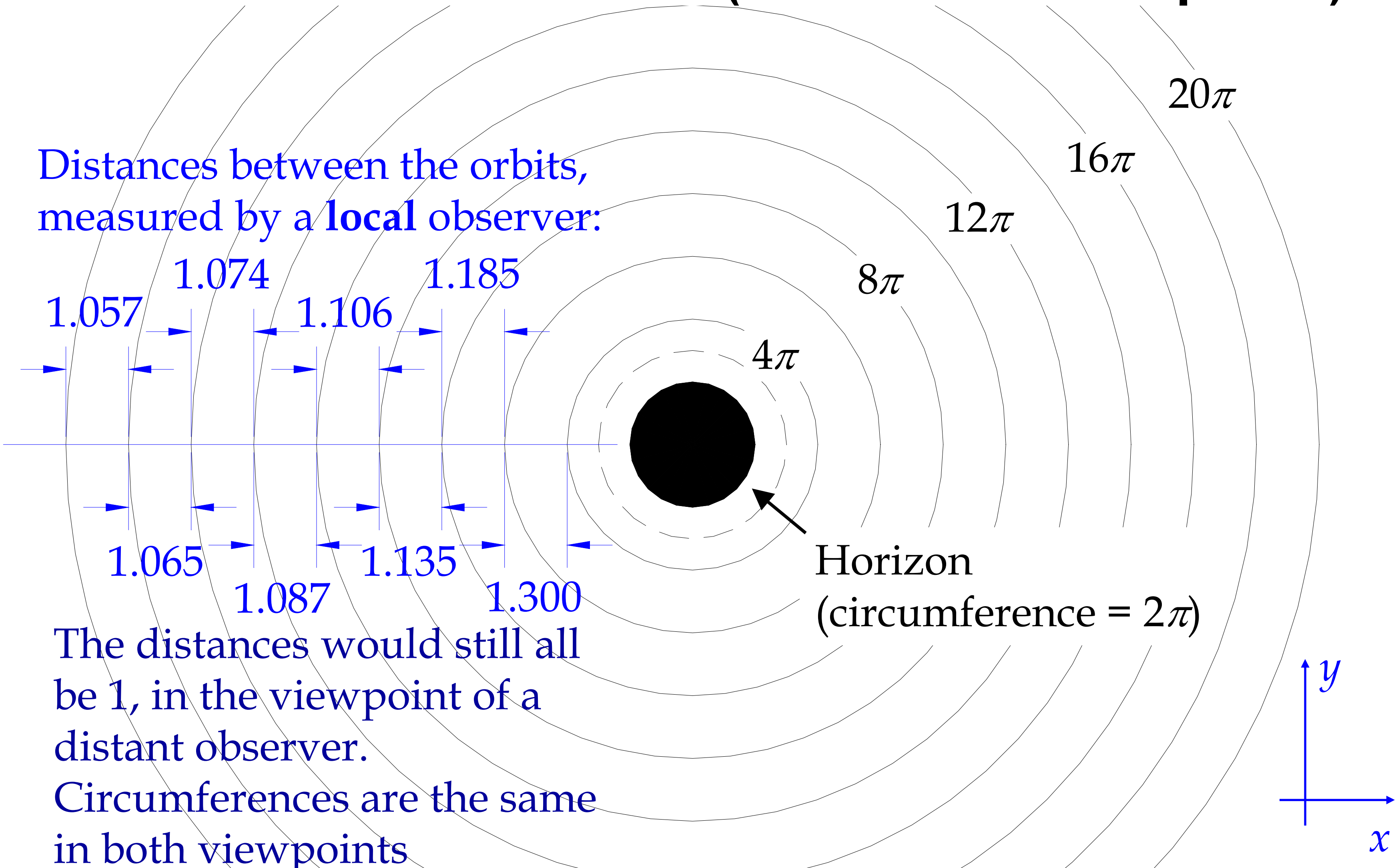


Circular orbits in flat space (all in the same plane)



Local and distant observers would report the same distances.

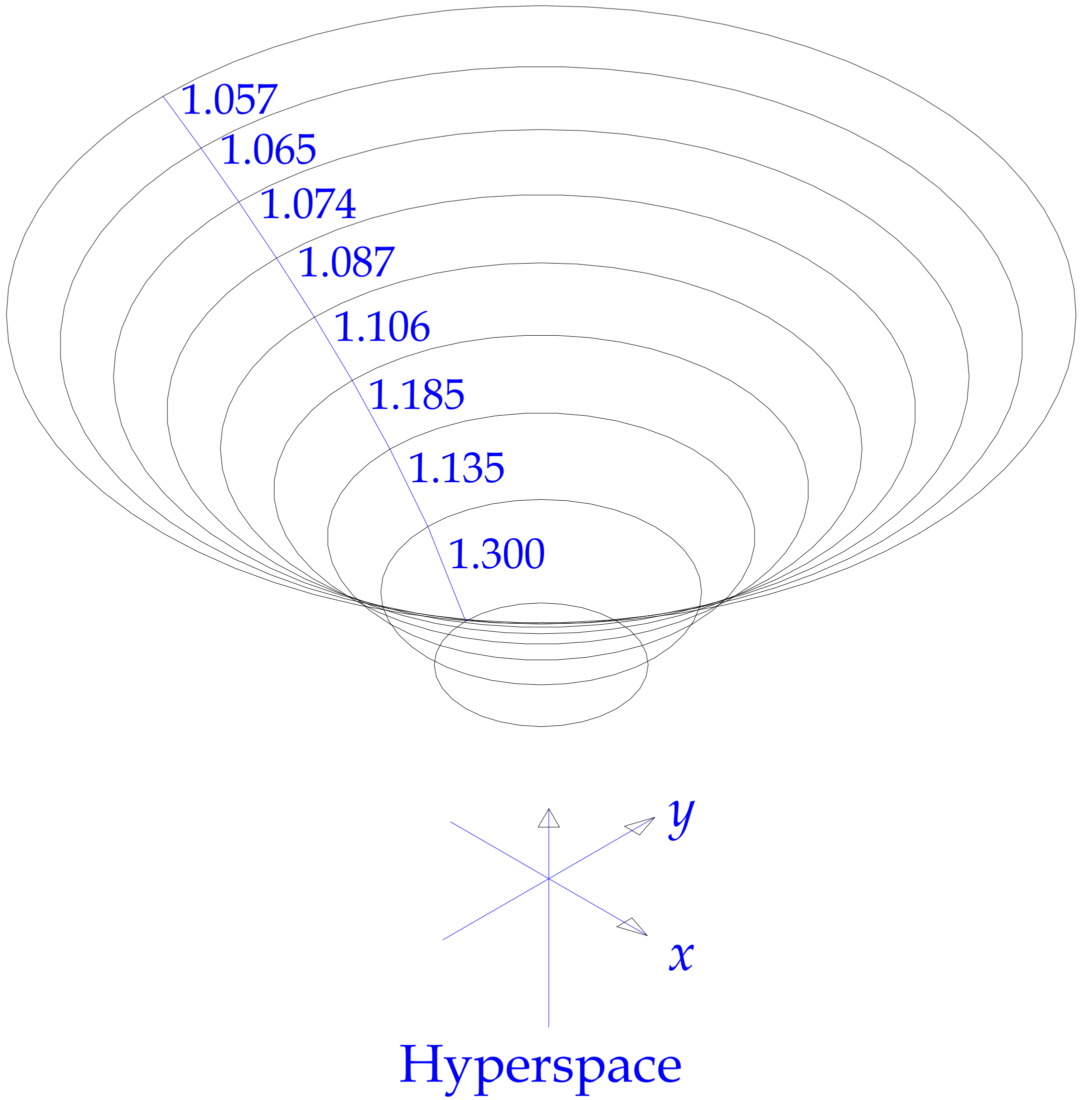
Circular orbits in space warped by a black hole, same circumferences as before (still all in same plane)



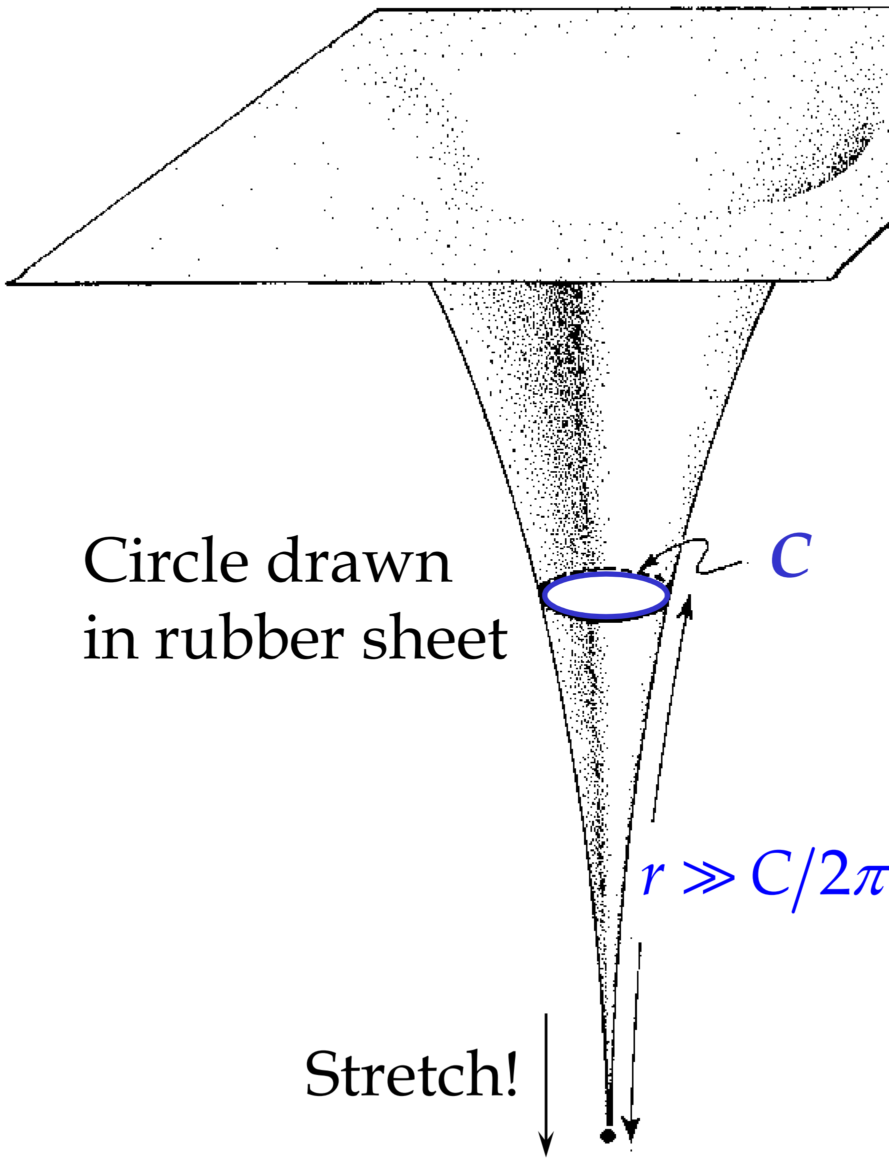
The distances would still all be 1, in the viewpoint of a distant observer.
Circumferences are the same in both viewpoints

One way to visualize warped space: "hyperspace"

To connect these circles with segments of these "too long" lengths, one can consider them to be offset from one another along some **imaginary** dimension that is perpendicular to x and y but is not z. (If it were z, the circles wouldn't appear to lie in a plane!). Such additional dimensions comprise **hyperspace**.



Embedding diagrams



This is why you often see the equatorial plane of a black hole represented as a funnel-shaped surface, as if made from a stretched rubber sheet. It's important to note that the direction of the stretch is in hyperspace, though, so the scene would not look like a funnel to your eyes, which see just the three usual spatial dimensions.

Interesting facts about black holes (continued)

- Orbits outside the BH, further away than 1.5 R_{Sch} (in the coordinate system of a distant observer) still turn out to be ellipses.
- The resulting **coordinate speed** in orbit (for the coordinate system of a distant observer) is the same as is obtained for Newtonian gravity:

$$v \equiv r\frac{d\phi}{dt} = \sqrt{\frac{GM}{r}}$$

- At the horizon, the radial component of the coordinate speed of light is zero: **light cannot escape**. Thus no information can reach a distant observer from, or within, the horizon.

Interesting facts about black holes (continued)

- Orbits with **coordinate radius** < 3 R_{Sch} are unstable to small perturbations, if the black hole doesn't spin.
- There are no orbits with coordinate radius < 1.5 R_{Sch} for a non-spinning black hole; at this radius the local orbital speed is the speed of light, and smaller orbits would require (impossibly) higher speeds.
  - Thus *you* can't orbit there, because your rest mass isn't zero, but if you could, you could train your binoculars straight ahead (in the ϕ direction) and see the back of your head.
  - To get closer to the horizon, one would have to descend vertically and balance gravity with thrust, as in a rocket launch.
  - If the black hole spins, the innermost stable orbit and the photon orbit are smaller than 3 and 1.5 R_{Sch} if the particle orbits in the same direction as the spin, and larger if it orbits in the opposite direction.

Interesting facts about black holes (continued)

Within r = 1.5 R_{Sch}, all geodesics (paths of light) terminate at the horizon.

- Thus: from near the horizon, the sky appears to be compressed into a small range of angles directly overhead; the range of angles is smaller the closer one is to the horizon, and vanishes at the horizon. (The objects in the sky appear bluer than their natural colors as well, because of the gravitational Doppler shift).
- Space itself is stuck to the horizon, since one end of all the geodesics are there. If the horizon began to rotate, the ends of the geodesics would rotate with it. (This harmonizes with time stopping there.)

Interesting facts about black holes (continued)

- Gravitational acceleration turns out to be

$$a = \frac{GM}{r^2}\frac{1}{\sqrt{1-2GM/rc^2}}$$

which has its familiar Newtonian form at large distances but blows up at $r = R_{Sch}$. Thus, in a vertical descent to a hovering position just above the horizon, very large gravitational accelerations would be encountered.

Interesting facts about black holes (continued)

- Tidal forces turn out the same near a black hole as you'd expect from Newtonian gravity, and are finite at the horizon. For an object of length Δr in the radial direction and Δx in the crosswise directions,

$$\Delta a_r = \frac{2GM}{r^3}\Delta r \qquad \Delta a_\phi = -\frac{GM}{r^3}\Delta x$$

- For a 2 m person and a 10 $M_\odot$ BH, the radial tidal acceleration at the event horizon is 2×10^{10} cm sec^{-2} ($2 \times 10^7 g$).
- It is 1g for a 4.6×10^4 $M_\odot$ BH. Thus if you want to fall freely past the horizon of a BH to see what happens, choose a large one, so as not to be torn apart before you get there.

Yet black holes emit light!

Details of the process, called Hawking radiation:

- Virtual particle-antiparticle pairs, produced briefly by vacuum fluctuations, can be split up by the strong gravity near a horizon. Both of the particles can fall in, but it is possible for one to fall in with the other escaping.
- The escaping particle is seen by a distant observer as emission by the black hole horizon: black holes emit light (and other particles)!
- The energy conservation debt involved in the un-recombined vacuum fluctuation is paid by the black hole itself: the black hole's mass decreases by the energy of the escaping particle, divided by c^2. The emission of light (or any other particle) costs the black hole mass and energy.

Black hole evaporation

- Hawking radiation is emitted more efficiently if the tides at the horizon are stronger. You will show in recitation this week that tides at the horizon are larger for smaller-mass black holes.
- Emission is the same as that of a blackbody at temperature

$$T = \frac{hc^3}{16\pi^2 kGM} \quad \left(\propto \sqrt{\Delta a_r} \right)$$

- Thus an isolated black hole will eventually evaporate (as you'll show in Homework #4).
  - Evaporation takes this long –
    $10^9\ M_\odot$ black hole: 10^{94} years.
    $2\ M_\odot$ black hole: 10^{67} years.
    10^8 gram black hole: 1 second (!)

"Black holes have no hair"

Meaning: after collapse is over with, the black hole horizon is smooth: nothing protrudes from it; and that almost everything about the star that gave rise to it has lost its identity during the black hole's formation. No "hair" is left to "stick out."

- Any protrusion, prominence or other departure from spherical smoothness gets turned into gravitational radiation; it is radiated away during the collapse.
- Any magnetic field lines emanating from the star close up and get radiated away (in the form of light) during the collapse.
- The identity of the matter that made up the star is lost. Nothing about its previous configuration can be reconstructed.

"Black holes have no hair" (continued)

- Even the distinction between matter and antimatter is lost: two stars of the same mass, but one made of matter and one made of antimatter, would produce identical black holes.

The black hole has only three quantities in common with the star that collapsed to create it: **mass, spin and electric charge**.

- That is, in common with the star as it was immediately before the formation of the horizon…
- Only very tiny black holes can have much electric charge; stars are electrically neutral, with equal numbers of positively- and negatively-charged elementary particles.
- Spin makes the black hole depart from spherical shape, but it's still smooth.

Distinctive features that can indicate the presence of a black hole

Observe **two or more** of these features to find a black hole:

Gravitational deflection of light, by an amount requiring black hole masses and sizes.

X-ray and/or γ-ray emission from ionized gas falling into the black hole.

Orbital motion of nearby stars or gas clouds that can be used to infer the mass of (perhaps invisible) companions: a **mass too large to be a white dwarf or a neutron star** might correspond to a black hole.

Motion close to the speed of light, or apparently greater than the speed of light ("superluminal motion").

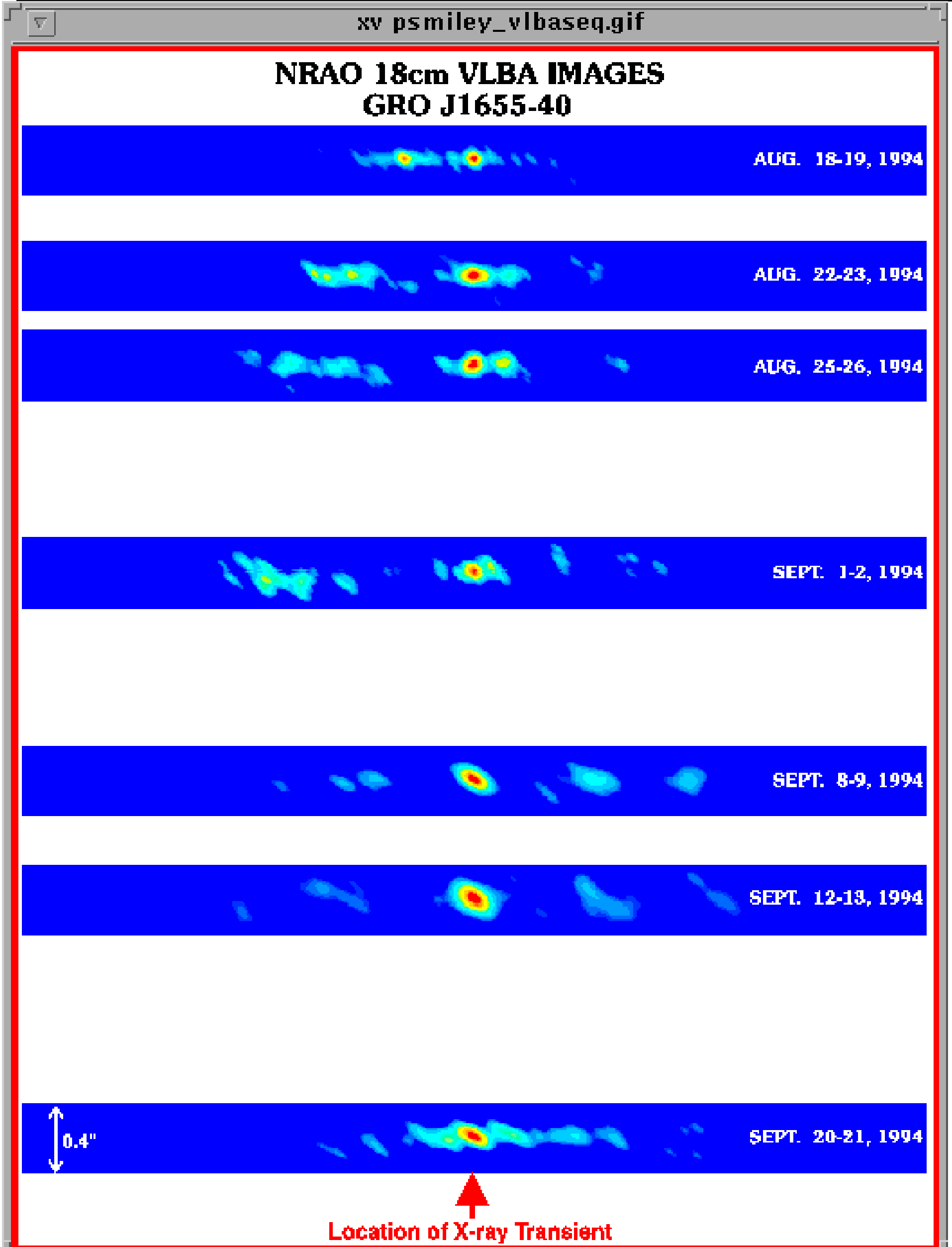
Extremely large luminosity that cannot be explained easily by normal stellar energy generation.

Direct observation of a large, massive **accretion disk**.

Paradigm stellar-mass black hole: GRO J1655-40

GRO J1655-40 is an X-ray transient source discovered by the *Compton* GRO in 1994. Appeared as a nova in visible light.

- Rapidly-variable X-ray emission : time scales of these variations show that the object is a few hundred km around.
- Has a stellar companion, a star rather similar to the Sun (about 1.1 $M_\odot$); the period is 2.92 days, and the velocity amplitude 227 km s^{-1}. Thus the **mass function** is 3.2 $M_\odot$.
- A stroke of luck: it is an **eclipsing** system, so the orbit is edge on to our line of sight.
- Thus we know the mass of the X-ray bright companion rather precisely: it must be between 5.5 and 7.9 $M_\odot$, with a most probable value of 7.0 $M_\odot$, way too much to be a neutron star (Shahbaz *et al.* 1999)
- Also has radio jets with motions close to the speed of light, tilted 85^o from the line of sight.



GRO J1655-40 (continued)

Jets: speed 0.92*c*, with some of the ejecta on the left moving at (projected) superluminal velocities. (We'll discuss superluminal motion later in the context of quasars.)

Thus: it's too small and massive to be a white dwarf or a neutron star, is X-ray bright, and is associated with relativistic ejection speeds: sounds like a black hole.

(radio maps from Hjellming and Rupen, NRAO)

GRO J1655-40 spins, too.

We expect it to spin (because stars do), but now we can demonstrate this:

A 7 $M_\odot$ nonspinning black hole has a horizon circumference 130 km, and an innermost stable orbit circumference of 390 km. Material in this orbit will circle the black hole **314 times per second.**

- However, one often sees the X-ray brightness of GRO J1655-40 modulate at **450 times per second** for long stretches of time (Tod Strohmayer 2001, *ApJL* 552, L49).
- Nothing besides very hot material in a stable orbit can do this so reproducibly at this frequency.
- Thus there are stable orbits closer to the black hole than they can be if it doesn't spin.

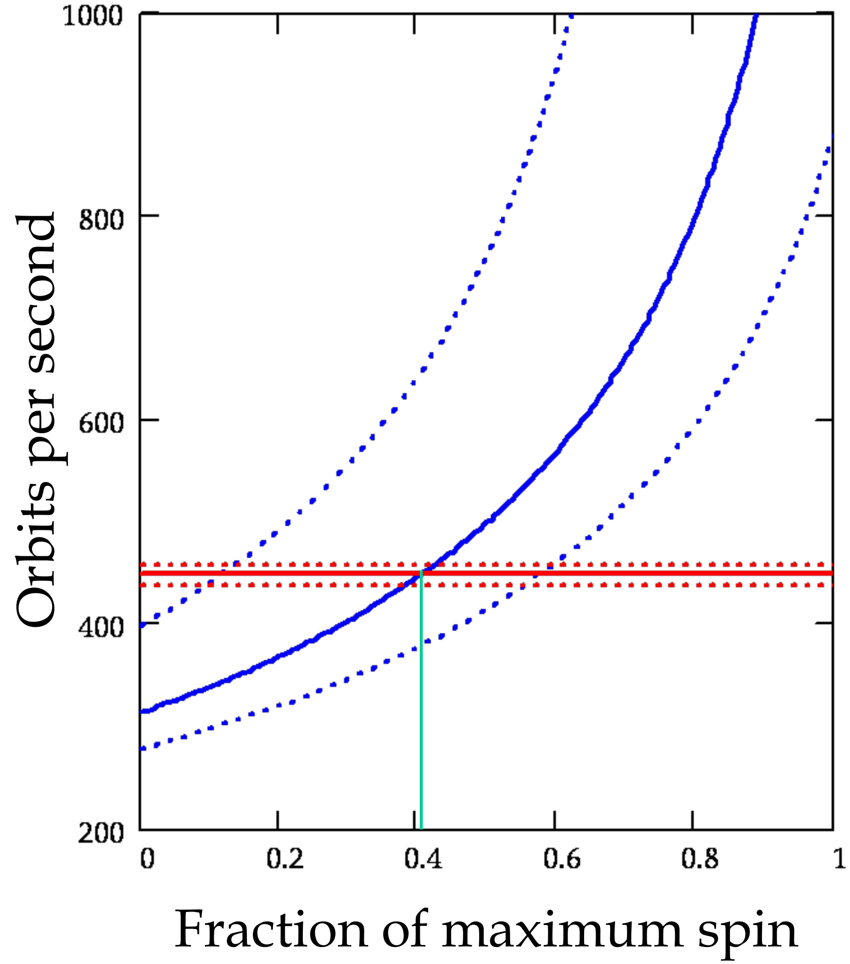
GRO J1655-40 spins, too (continued).

Most probably, the black hole in GRO J1655-40 is spinning at about 40% of its maximum rate. Within the uncertainties the spin rate lies in the range 12%-58% of maximum; zero spin is quite improbable.

In blue: innermost stable orbits per second for 7.0 $M_{\odot}$ black holes, with uncertainties.

In red: measured orbits per second, with uncertainties

(by Tod Strohmayer, with the *Rossi* X-ray Timing Explorer).



White dwarfs, neutron stars and black hole horizons