Today in Astronomy 142

Normal stars: the main sequence

- Relationships among luminosity, mass and effective temperature

Stellar evolution

- Changes on the main sequence
- Shell hydrogen fusion and subgiants

The central star in this planetary nebula, NGC 2440, is well on its way to becoming a white dwarf. *(Hubble Space Telescope/ NASA and STScI)*

Back to Live Stars

We will now learn how to scale our results on stellar structure and luminosity to normal stars of all masses.

Note: it saves writing and thus keeps things a little clearer if we use some common astro-jargon: leave multiplicative *constants* out of equations, and speak of proportionality, e.g.

$$P_c \propto \frac{M^2}{R^4} \quad \text{instead of} \quad P_c = 19\frac{GM^2}{R^4} \quad ,$$

because in the end we will express everything in terms of ratios to the results on the Sun, and all the constants will cancel out, e.g.

$$\frac{P_c(M)}{P_{c\odot}} = \frac{19GM^2/R^4}{19GM_\odot^2/R_\odot^4} = \left(\frac{M}{M_\odot}\right)^2\left(\frac{R_\odot}{R}\right)^4 \quad \Rightarrow \quad P_c = P_{c\odot}\left(\frac{M}{M_\odot}\right)^2\left(\frac{R_\odot}{R}\right)^4$$

The theoretical luminosity-mass relation

Escape of photon produced at star's center: as we have seen (27 January), in terms of the mean free path ℓ, the number of steps N and time t are given by

$$N = \frac{3R^2}{\ell^2} \qquad t = \frac{N\ell}{c} = \frac{3R^2}{\ell c}$$

The average photon mean free path is ℓ = 0.5 cm in the Sun. How does this scale with average temperature and density?

- Very complicated (~AST 453) to show; skip to answer:

$$\ell \propto T^{3.5}/\bar{\rho}^2 \qquad \text{if } M < 1M_\odot \text{ or so;}$$
$$\propto 1/\bar{\rho} \qquad \text{if } M > 1M_\odot \text{ or so.}$$
$$(\bar{\rho} = 3M/4\pi R^3)$$

The theoretical luminosity-mass relation (cont'd)

This tells us how luminosity scales (lecture on 27 January):

$$L = \frac{u_r V}{t} \qquad (u_r = \text{radiation energy density})$$

$$\approx \left(\frac{\ell c}{3R^2}\right)\left(\frac{4}{c}\sigma T^4\right)\left(\frac{4\pi R^3}{3}\right) \propto \ell R T^4$$

Hydrostatic equilibrium and ideal-gas pressure support against the star's weight (also on 27 January) also imply

$$P \propto \frac{GM^2}{R^4} \propto \bar{\rho} T \quad ,$$

$$\text{so} \quad T \propto \frac{P}{\bar{\rho}} \propto \frac{GM^2}{R^4}\frac{R^3}{M} \propto \frac{M}{R}$$

The theoretical luminosity-mass relation (cont'd)

Thus, for low-mass stars $\left(M \le 1 M_\odot\right)$,

$$L \propto \ell R T^4 \propto \frac{T^{3.5}}{\bar{\rho}^2} R T^4 \propto \left(\frac{M}{R}\right)^{3.5}\left(\frac{R^3}{M}\right)^2 R \left(\frac{M}{R}\right)^4$$

$$\propto \frac{M^{5.5}}{R^{0.5}}$$

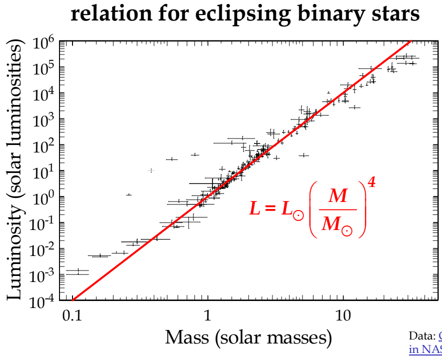
and for higher-mass stars $\left(M \ge 1 M_\odot\right)$,

$$L \propto \ell R T^4 \propto \frac{1}{\bar{\rho}} R T^4 \propto \left(\frac{R^3}{M}\right) R \left(\frac{M}{R}\right)^4$$

$$\propto M^3$$

Compromise: $L \propto M^4 \quad \Rightarrow \quad \frac{L}{L_\odot} = \frac{M^4}{M_\odot^4} \quad \Rightarrow \quad \boxed{L = L_\odot \left(\frac{M}{M_\odot}\right)^4}$

Comparison to experiment: the luminosity-mass relation for eclipsing binary stars



Data: O. Malkov 1993, in NASA NSSDC.

The lifetimes of stars

Note how fast luminosity increases with increasing mass. Because of this, the more massive a star, the shorter its life.

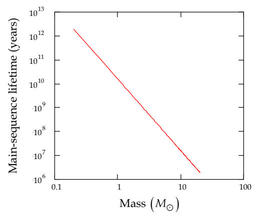
pp-chain fusion energy supply: $\Delta E \cong 0.03Mc^2 \propto M$

$L \propto M^4$

Thus $\tau = \dfrac{\Delta E}{L} \propto \dfrac{1}{M^3}$, or

$$\frac{\tau}{\tau_\odot} = \frac{M_\odot^3}{M^3}$$

$$\tau \cong 1.5\times10^{10} \text{ years } \times\left(\frac{M_\odot}{M}\right)^3$$



17 February 2009 Astronomy 142, Spring 2009 7

The theoretical radius-mass and temperature-mass relations

Fusion reactions comprise a sort of thermostat: temperature in the interior of a main sequence star is only slowly dependent upon M and R, as we saw before (again in class on 27 January). Take T therefore to be approximately constant within a given star; then since $T \propto M/R$,

$$R \propto M \quad \Rightarrow \quad \boxed{R = R_\odot\left(M/M_\odot\right)}$$

On one hand, $L = 4\pi R^2\sigma T_e^4$, and on the other (due to our "compromise"), $L \propto M^4$, so

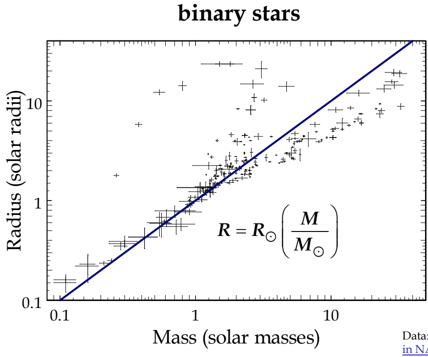
$$4\pi R^2\sigma T_e^4 \propto M^4$$

$$M^2 T_e^4 \propto M^4$$

$$T_e \propto M^{1/2} \quad \Rightarrow \quad \boxed{T_e = T_{e\odot}\sqrt{M/M_\odot}}$$

17 February 2009 Astronomy 142, Spring 2009 8

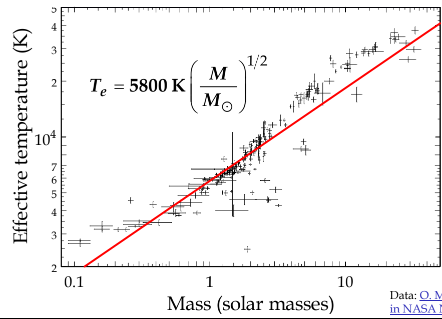
Comparison to experiment: radii of eclipsing binary stars



Data: O. Malkov 1993, in NASA NSSDC.

17 February 2009 Astronomy 142, Spring 2009 9

Comparison to experiment: effective temperatures of eclipsing binary stars



Data: O. Malkov 1993, in NASA NSSDC.

The theoretical main sequence in the luminosity-effective temperature relation (H-R diagram)

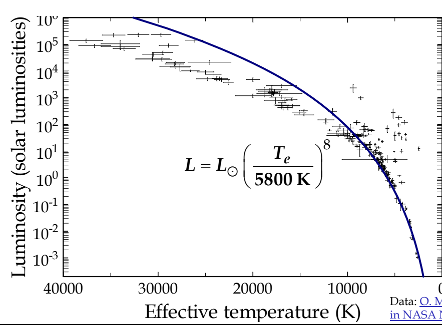
Combine this last result again with $L \propto M^4$:

$$L \propto M^4 \propto T_e^8$$

$$\boxed{L = L_\odot \left(\frac{T_e}{T_{e\odot}}\right)^8} \quad \text{The main sequence}$$

All of these results are in reasonable agreement with the data, which indicates that we have included most of the important physics in our discussions. In fact, one needs to build quite detailed models to do better – AST 453 style, not even AST 241 style.

Comparison to experiment: the L-T_e relation (H-R diagram) for eclipsing binary stars



Data: O. Malkov 1993, in NASA NSSDC.

Stellar evolution

First, a break for some pedantry:

- Individuals *develop*; populations and species *evolve*.
  - That is, evolutionary changes show up in the average, not in the individual.
  - The difference is important for scientists to keep in mind, as the non-scientific public often confuses one for the other, and can thus be misled in their ideas about the origin of species.
- Astronomers, unfortunately, use the term "evolution" loosely, to describe the changes of properties of individual stars through their lives (i.e. development) as well as the changes of stellar properties through time on the cluster or Galactic scale (evolution proper).

Mean molecular weight

For pure ionized hydrogen: $\mu m_p = \overline{m} = \frac{m_p + m_e}{2} \cong 0.5 m_p$

For pure ionized helium: $\mu m_p = \frac{3.97 m_p + 2 m_e}{3} \cong 1.32 m_p$

In general, and in terms of mass fractions: X, Y, Z = fraction of total mass in hydrogen, helium, total of all others, respectively, for fully ionized gas:

$$\frac{1}{\mu} \cong 2X + \frac{3}{4}Y + \frac{1}{2}Z$$

Mean molecular weight for ionized gas with X = 0.70, Y = 0.28, Z = 0.02 (abundances found on the Solar surface):

$$\mu m_p = \overline{m} = 0.62 m_p$$

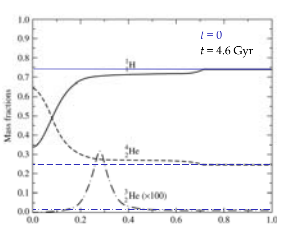
Stellar evolution on the main sequence

As hydrogen burns in the core, fusing into heavier elements, the mean molecular weight slowly increases. At a given temperature the ideal gas law says this would result in a lower gas pressure, and less support for the star's weight:

$$P = \frac{\rho k T}{\overline{m}} \qquad \overline{m} = \mu m_p \quad .$$

In the center of the Sun today,

$$\overline{m} = 1.17 m_p \quad .$$

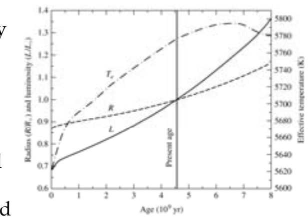


From Carroll and Ostlie, Modern Astrophysics, 2e.

Stellar evolution on the main sequence (continued)

Therefore, as time goes on,

- the core of the star slowly contracts and heats up.
- the radius and effective temperature of the star slowly increase, in response to the new internal temperature and density distribution.
- the luminosity slowly and slightly increases, in response to the increase in radius and effective temperature.

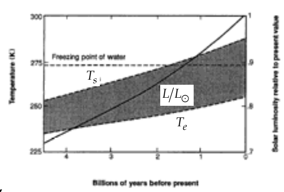


From Carroll and Ostlie, Modern Astrophysics, 2e.

Aside: the Faint Young Sun paradox

- So: before about 2 Gyr ago, the Sun should have been faint enough that Earth's surface would have been completely frozen.
- Yet there is abundant geological evidence for liquid water on the surface, **continuously** for the past 3.8 Gyr.

This was first pointed out by Carl Sagan and George Mullen in 1972.



Kasting and Catling 2003

Here,

$$T_e = \left(\frac{1-A}{\varepsilon} \frac{L}{16\pi\sigma r^2} \right)^{1/4}$$

Aside: the Faint Young Sun paradox (continued)

The resolution to this paradox is not yet clear:

- Majority opinion: large atmospheric CO_2 concentration at earlier times led to a much larger greenhouse effect than today; later decreased with more abundant plant life making CO_2 into O_2 (e.g. Kasting and Catling 2003).
- But this might not be enough; others invoke further the correlation (of unknown origin!) between cosmic-ray flux and surface temperature. The stronger solar wind at earlier times would have kept the cosmic-ray flux lower than it is today (e.g. Shaviv 2003).
- Also suggested but improbable: solar wind unusually large in the past, and the young Sun more massive. 10% larger mass would do.

Shell hydrogen burning and the subgiant phase

Eventually hydrogen is exhausted in the very center, and the temperature is insufficient to ignite helium burning, but is high enough just outside the center for a shell of hydrogen fusion to provide support for the star. Thus

- T is nearly constant in the core (**isothermal helium core**), which keeps increasing in mass owing to hydrogen depletion.
- there is increased luminosity and further expansion of the envelope of the star.
- there is a decrease in effective temperature.

This is called the **subgiant phase**: the star moves off the main sequence, upwards and to the right on the H-R diagram.

17 February 2009 Astronomy 142, Spring 2009 19

Degeneracy in the isothermal core

The subgiant phase ends when the mass of the isothermal core becomes too great for support of the star.

- Reason for a maximum in the weight that can be supported by pressure in the core: **electron degeneracy pressure**. The core is a lot like a white dwarf.
- Maximum fraction of mass in core (Schoenberg and Chandrasekhar, 1942):

$$\frac{M_{\text{isoth. core}}}{M_{\text{total}}} \cong 0.37\left(\frac{\mu_{\text{envelope}}}{\mu_{\text{isoth. core}}}\right)^2$$

$$\cong 0.37\left(\frac{0.62}{1.32}\right)^2 = 0.08 \quad \text{for the Sun.}$$

17 February 2009 Astronomy 142, Spring 2009 20