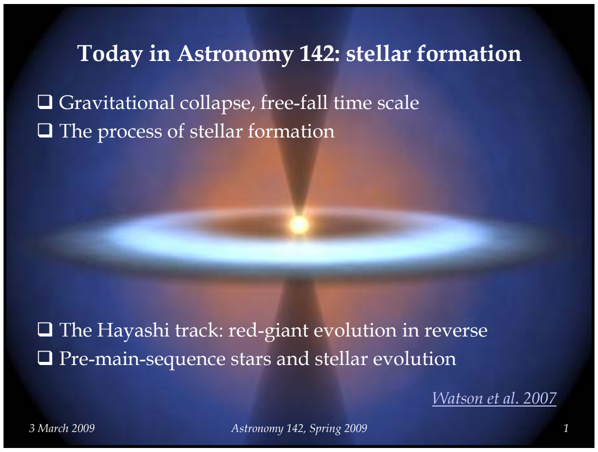



## Time scale of spherical gravitational collapse

For spherically-symmetric collapse, in free fall:

$$t_{ff} = \sqrt{\frac{3\pi}{32G\rho_0}} \qquad \rho_0 = \bar{m}n = \text{initial mass density}$$

$$= 10^{12}\ \text{sec} = 3\times10^{4}\ \text{years for}\ \rho_0 = 4.7\times10^{-19}\ \text{gm cm}^{-3}.$$

as we will derive in the following slides.

- ❑ This is very fast by astrophysical standards. We should therefore have to get very lucky to catch a star in the act of formation.
- ❑ Note that this is for spherical geometry and we know the collapse can't really be very spherical, so this is offered only as a crude estimate.



## Spherical free-fall time: derivation

Follow a test particle that starts from rest, a distance $r_0$ from cloud center, at $t$ = 0. How long until it reaches the center (i.e. until the clump collapses to a point)?

$$ma = F$$

$$m\frac{d^2r}{dt^2} = -\frac{GMm}{r^2} \qquad M\text{: mass interior to } r.$$

Multiply both sides by $dr/dt$, integrate over time (RHS first):

$$\int_0^t dt' \frac{dr}{dt'}\frac{d^2r}{dt'^2} = -GM\int_0^t \frac{1}{r^2}\frac{dr}{dt'}dt' = -GM\int_{r_0}^{r}\frac{dr'}{r'^2}$$

$$= \frac{GM}{r} - \frac{GM}{r_0}$$



## Spherical free-fall time: derivation (continued)

On LHS, substitute $v = dr/dt$, $dv = (d^2r/dt^2)dt$:

$$\int_0^v v'dv' = \frac{GM}{r} - \frac{GM}{r_0}$$

$$\frac{1}{2}v^2 =$$

Use $v = dr/dt$ again:

$$\frac{1}{2}\left(\frac{dr}{dt}\right)^2 = \frac{GM}{r} - \frac{GM}{r_0}$$

$$\frac{dr}{dt} = -\sqrt{\frac{2GM}{r} - \frac{2GM}{r_0}}$$

Choose negative root, so that it falls in.



## Spherical free-fall time: derivation (continued)

Insert $M = \frac{4\pi}{3} r_0^3 \rho_0$ :

$$\frac{dr}{dt} = -r_0\sqrt{\frac{8\pi G\rho_0}{3}\left(\frac{r_0}{r} - 1\right)}$$

Substitute $r = r_0 \cos^2 u$, $\frac{dr}{dt} = r_0 \frac{d}{dt}\cos^2 u = -2r_0 \cos u \sin u \frac{du}{dt}$ :

$$\frac{du}{dt} = \frac{1}{2\cos u \sin u}\sqrt{\frac{8\pi G\rho_0}{3}\left(\frac{1}{\cos^2 u} - 1\right)}$$

$$= \frac{1}{2\cos^2 u \sin u}\sqrt{\frac{8\pi G\rho_0}{3}\left(1 - \cos^2 u\right)}$$

$$= \frac{1}{2\cos^2 u}\sqrt{\frac{8\pi G\rho_0}{3}}$$



## Spherical free-fall time: derivation (continued)

Separate and integrate over time, RHS first:

$$\int_{u_0}^{u} \cos^2 u' du' = \frac{1}{2}\sqrt{\frac{8\pi G\rho_0}{3}}\int_0^{t_{ff}} dt = \frac{t_{ff}}{2}\sqrt{\frac{8\pi G\rho_0}{3}}$$

Now, what are $u$ and $u_0$?

$$t = 0: \quad r = r_0,\; r_0 = r_0\cos^2 u_0 \quad \Rightarrow u_0 = 0$$

$$t = t_{ff}: \quad r = 0,\; 0 = r_0\cos^2 u \quad \Rightarrow u = \frac{\pi}{2}$$

Thus $\frac{t_{ff}}{2}\sqrt{\frac{8\pi G\rho_0}{3}} = \int_0^{\pi/2}\cos^2 u\, du = \left[\frac{u}{2} + \frac{\sin 2u}{4}\right]_0^{\pi/2} = \frac{\pi}{4}$

$$t_{ff} = \sqrt{\frac{3\pi}{32G\rho_0}}$$

## Star formation synopsis

Molecular clump collapses gravitationally

- …to a disk shape at first.
- Collapse along the other two dimensions happens more slowly because of rotation (angular momentum) and magnetic forces.
- Smaller scales collapse faster ("inside-out collapse").

Central "core" gradually accretes some of the rest of the disk.

- To get rid of angular momentum and ***B***, this is accompanied by a bipolar outflow.

Core becomes star; remnants of surrounding disk become planetary system.

In more detail…



## Stellar formation

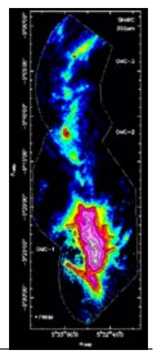
1. A fragment of an interstellar molecular cloud becomes gravitationally unstable and begins to collapse…

- …either because the material has cooled, or because it has been compressed (**triggered**) by pressure from outside.

2. Collapse proceeds from the **inside out**, and **anisotropically.**

- The central – denser – region collapses faster:

$$t_{ff} = \sqrt{\frac{3\pi}{32 G \rho_0}}$$



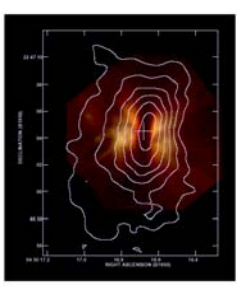
**At right**: map of the Orion molecular cloud at λ = 350 μm; Lis *et al.* 1999, CSO.



## Stellar formation (continued)

- Because of conservation of angular momentum, and to a lesser degree magnetic forces, collapse can always proceed much faster in one dimension than the other two: the fragment **flattens** as it collapses.

3. Soon a **disk** configuration is established, with a well-defined, very dense central condensation: a **protostar**.

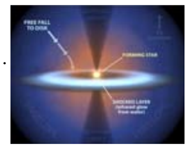


**At right:** contours of $^{13}CO$ emission (OVRO) and scattered near-infrared light (HST) in IRAS 04016. From D. Padgett *et al.* 1999.

## Stellar formation (continued)

- Henceforth the disk **accretes** further material from the surrounding molecular cloud, and the protostar accretes material from the disk.
- **Class 0 protostar:** central object still has much of its final mass to accrete; surrounding envelope still substantial; $dM/dt \sim 10^{-4}\ M_\odot\ \text{year}^{-1}$.
- **Class I protostar**: central object nearly complete, envelope settling onto disk; $dM/dt \sim 10^{-(5-7)}\ M_\odot\ \text{year}^{-1}$.

**At right:** artist's conception of accretion disk and jet, and cartoon of Class 0 object (Watson et al. 2007).



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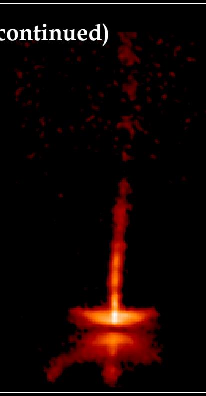
## Stellar formation (continued)

4. As the disk and central object accretes gas and dust, they drive a **bipolar outflow** into their surroundings, which is thought to carry off the accreted material's angular momentum.
   - In low-luminosity (low-mass) objects the outflow takes the form of highly collimated **jets.** In more massive objects the outflow has a wider opening angle.

HST images, HH30 (Alan Watson et al, UNAM/STScI/NASA)



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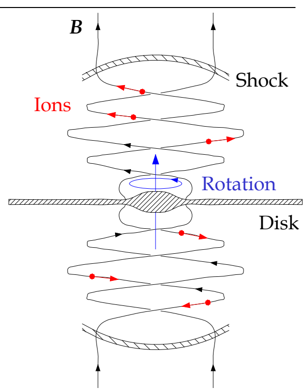
**Simplified view of magneto-centrifugal acceleration**. The lines of ***B*** are stuck to the (ionized) disk and the distant ambient medium. As the disk rotates, the nearer lines of ***B*** wind up as shown, and ions in the gas above and below the disk – which are stuck to ***B*** – are driven **away** from the disk, along lines of ***B***, like beads sliding on a wire. The ions can collide with neutral particles and drive them, too. The energy of the outflow comes from disk rotation, so the disk rotation must slow down as the outflow proceeds.

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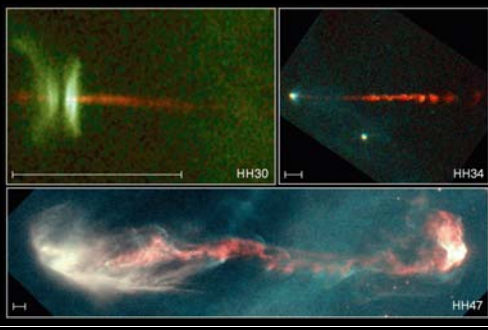
## Bipolar jets and disks associated with young stars (courtesy Arizona State U., STScI and NASA)






## Stellar formation (continued)

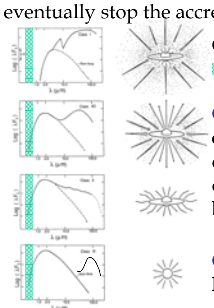
The outflows are supersonic, compared to the molecular surroundings, and thus **shocks** are driven into the ambient medium. The energy and momentum injected thereby contributes to the support of the molecular cloud, and can induce *or* hinder further star formation in the neighborhood.

*L1641 VLA1/HH1-2 (Bally et al. 2001)*



## Stellar formation (continued)

5. The central object becomes a star, and its wind and radiation eventually stop the accretion and dissipate the gas in the disk.



Class 0/I protostar
= visible wavelengths

**Class II protostar**: opaque disk, envelope almost gone, onto star from disk at rates that decrease with age, dropping below $10^{-10} M_{\odot}$ year$^{-1}$ by age 5 Myr.

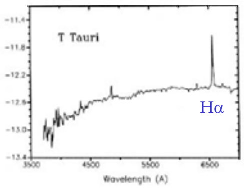
**Class III protostar**: little dust and very little gas left in disk.

after Wilking 1989, *PASP* **101**, 229

## Pre-main-sequence stars

The young stars themselves can often be seen at visible wavelengths, unless the disk is edge on, and have two distinctive features:

- their spectra show **emission lines**, notably hydrogen recombination lines, and UV excess emission, both produced in **accretion shocks** in material falling onto the star from the disk.
- Spectral type G-M = **T Tauri stars.** Earlier types: **Herbig Ae/Be stars.**




Visible spectrum of T Tau, by Bruce Weaver (MIRA).



## Pre-main-sequence stars (continued)

It takes young stars millions of years to settle down to their final sizes and get the fusion fires blazing.

- During this slow gravitational collapse, the conversion of gravitational potential energy dominates the luminosity of young stars. For it is written (Homework #2):

$$L = -\frac{3}{10}\frac{GM^2}{R^2}\frac{dR}{dt}$$

- The gravitational-collapse luminosity decreases with time, and is emitted at a fairly constant temperature. So the young star descends vertically through the H-R diagram. This path is called the **Hayashi track**.



## Pre-main-sequence stars (continued)

- The Hayashi track happens to be precisely the reverse of the path each star will follow as it becomes a red giant.
- For a given age the redder (lower mass, later spectral type) stars lie further above the main sequence than the bluer ones.

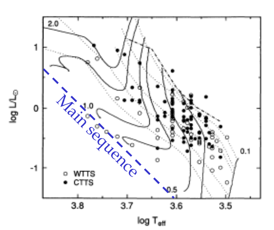



FIG. 15.—H-R diagram for Taurus-Auriga TTSs. Solid lines indicate CMA model tracks for $M_* = 0.1, 0.3, 0.5, 0.8, 1.0, 1.5$, and $2.0\ M_\odot$ (D'Antona & Mazzitelli 1994). Several tracks are labeled with their mass. Dotted lines denote isochrones for $10^5$, $3\times10^5$, $10^6$, $3\times10^6$, and $10^7$ yr. The heavy dot-dashed line indicates the stellar birth line (Fletcher & Stahler 1994a, Table 1).

Figure by Scott Kenyon and Lee Hartmann (1995, *Ap.J.S.* **101**, 117)