Today in Astronomy 142: the Milky Way, continued

- ❑ More on stars as a gas: stellar relaxation time, equilibrium
- ❑ Differential rotation of the stars in the disk
- ❑ The local standard of rest
- ❑ Rotation curves and the distribution of mass
- ❑ The rotation curve of the Galaxy

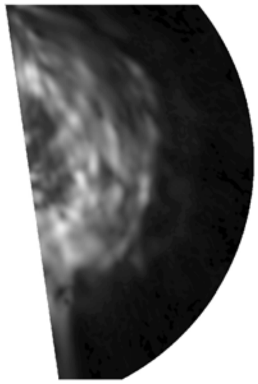


Figure: spiral structure in the first Galactic quadrant, deduced from CO observations (Clemens, Sanders, Scoville 1988)

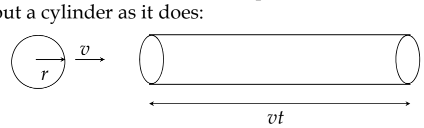
Stellar encounters: relaxation time of a stellar cluster

In order to behave like a gas, as we assumed last time, stars have to collide elastically enough times for their random kinetic energy to be shared in a thermal fashion.

- ❑ But stellar encounters, even distant ones, are rare. How long does it take a cluster of stars to "thermalize?"
- ❑ One characteristic time: the time between stellar elastic encounters, called the **relaxation time**. If a gravitationally bound cluster is a lot older than its relaxation time, then the stars will be describable as a gas (the star system has temperature, pressure, *etc*.).

Stellar encounters: relaxation time of a stellar cluster (continued)

Suppose a star has a gravitational "sphere of influence" with radius r ($>>R$, the radius of the star), and moves at speed v between encounters, with its sphere of influence sweeping out a cylinder as it does:



$$V = \pi r^2 vt$$

If the number density of stars (stars per unit volume) is n, then there will be exactly one star in the cylinder if

$$nV = n\pi r^2 v t_c = 1 \quad \Rightarrow \quad t_c = \frac{1}{n\pi r^2 v}$$ **Relaxation time**

Stellar encounters: relaxation time of a stellar cluster (continued)

What is the appropriate radius, r? Choose that for which the gravitational potential energy is equal in magnitude to the average stellar kinetic energy.

$$\frac{Gm^2}{r} = \frac{1}{2}mv^2 \Rightarrow \quad t_c = \frac{v^3}{4\pi G^2 m^2 n}$$

Done in more detail (Astronomy 232-level): for a spherical cluster with a **"core" radius** R, it can be shown that

$$t_c = \frac{v^3}{4\pi G^2 m^2 n} \frac{1}{\ln(2R/r)} \quad .$$

Not that far from our rough estimate, as the logarithm is a very slow function.

Stellar encounters: relaxation time of a stellar cluster (continued)

In this week's homework, you will show among other things that for such a cluster of N stars, with core radius R and typical stellar mass m,

$$v^2 = \frac{G(N-1)m}{2R} \quad \text{and} \quad \frac{Gm^2}{r} = \frac{G(N-1)m^2}{4R}$$

Assume $N >> 1$ and substitute these into the expression for relaxation time:

$$t_c \cong \left(\frac{2R}{v}\right) \frac{N}{24\ln\left(\frac{N}{2}\right)}$$

The time $t_x = 2R/v$ is called the **crossing time**; it's the time it takes a star moving at the mean speed v to traverse the core of the cluster (diameter $2R$) if it doesn't collide.

Thermal equilibrium

The virial theorem,

In an isolated system of particles that exert forces on each other describable by scalar potentials, the system's moment of inertia I, total kinetic energy K, total potential energy U and total mechanical energy E are related by

$$d^2I/dt^2 = 2K + U = K + E \quad .$$

applies only to thermal equilibrium or steady-state motion.

- Thus, before every use, one should check to see whether the system on which one's using it has been around long enough to be in thermal equilibrium.
- Long enough means the system has been together for a time **much longer than the relaxation time.**

Thermal equilibrium (continued)

Recall that usually, and especially in the situations you've encountered so far (e.g. spherically-symmetric collapse, as in Homework #2),

$$d^2I/dt^2 = 0$$

so in this case system's average total kinetic energy $\langle K \rangle$ and average total potential energy $\langle U \rangle$ are related by

$$\boxed{\langle K \rangle = -\frac{1}{2}\langle U \rangle = -E} \quad .$$

We will resist the temptation to prove these things here; though the proof isn't difficult, it's long and complicated. It will be proven for you in PHY 235 and AST 232. (If you can't wait, see the proof in *APP*, pp. 225-227.)

Rotation of the stellar population

Averaging over the random motions, one can detect **differential** rotation in the disk of the galaxy, from the radial velocities of nearby stars.

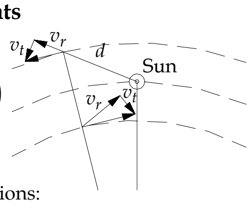
- The rotation is differential in the sense that different radii have different angular velocities. The angular velocity decreases monotonically as radius from the Galactic center increases.
- Measurement of average stellar motions along the line of sight and perpendicular to the line of sight can be used to determine the local angular velocity.
- Unfortunately, motions perpendicular to the line of sight are very hard to measure for enough stars, and stars that are far away.

Rotation of the stellar population, and Oort's constants

Oort's constants, defined:

$$A = -\frac{r}{2}\frac{d\Omega}{dr} \qquad B = -\frac{1}{2r}\frac{d}{dr}\left(r^2\Omega\right)$$

whence $\Omega = A - B$



In terms of the average radial velocities and average proper motions:

$$v_r = Ad\sin 2\ell \qquad v_t = Ad\cos 2\ell + Bd$$

Proper motions are difficult to measure for very distant objects, so B is usually obtained less directly from the statistics of random motions, with the result $\sqrt{1 - A/B} = 1.6$.

The local standard of rest

From A and B we get the average rotational motion of the Sun's orbit, called the **local standard of rest (LSR):**

$$\Omega = (9.8 \pm 0.9) \times 10^{-16} \text{ radians s}^{-1}$$

$$v_\phi = 254 \pm 16 \text{ km s}^{-1} \quad r_\odot = 8.4 \pm 0.6 \text{ kpc}$$

$$P = (200 \pm 20) \times 10^6 \text{ years}$$

The solar system actually moves slightly with respect to the LSR, at about 7 km/s.

From the motion of the LSR, the Galaxy within r = 8.4 kpc can be weighed:

$$M = \frac{v_\phi^2 r_\odot}{G} = (1.3 \pm 0.2) \times 10^{11} M_\odot .$$

(New experimental values from Reid et al. 2009.)

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Rotation curves

The average orbits in the disk of the Galaxy seem to be circular, centered on the Galactic center.

- A measurement of average angular velocity at any radius allows a determination of the mass within that radius of the Galactic center.
- Done as a function of radius: **rotation curve**
  - Enables determination of enclosed mass, and in turn the density, as a function of r.

Interstellar gas has far smaller random motions than stars, is widespread, and detectable throughout the galaxy; atomic (e.g. H I 21 cm) and molecular (e.g. CO 2.6 mm) lines are the best to use for determination of the Galactic rotation curve.

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Example rotation curves

Point mass, M:

$$F = \frac{GMm}{r^2} = \frac{mv^2}{r} \quad \Rightarrow \quad v(r) = \sqrt{\frac{GM}{r}}$$

Keplerian motion
v decreases with increasing r

Constant density, spherically symmetric:

$$M(r) = \frac{4\pi}{3} \rho_0 r^3$$

$$\frac{Gm}{r^2} M(r) = \frac{Gm}{r^2} \frac{4\pi}{3} \rho_0 r^3 = \frac{mv^2}{r}$$

$$\Rightarrow \quad v(r) = r\sqrt{\frac{4\pi G \rho_0}{3}}$$

Solid-body rotation
v increases linearly with increasing r

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Example rotation curves (continued)

Spherical symmetry, $1/r^2$ density distribution:

$$\rho = \rho_0 \frac{r_0^2}{r^2} \qquad (r_0: \text{ core radius of galaxy})$$

$$M(r) = \int_0^r \rho(r') 4\pi r'^2 dr' = 4\pi\rho_0 r_0^2 \int_0^r dr'$$

$$= 4\pi\rho_0 r_0^2 r \propto r$$

$$\frac{GmM(r)}{r^2} = \frac{Gm}{r^2} 4\pi\rho_0 r_0^2 r = \frac{mv^2}{r}$$

$$v(r) = \sqrt{4\pi G\rho_0 r_0^2} = \text{constant}$$ **Flat rotation curve**

As we will see, many rotation curves of disk galaxies, including ours, look like this one.

Measurement of Galaxy's rotation curve from H I and CO line profiles

Wavelength or frequency shift and radial velocity: the Doppler effect.

$$\frac{\lambda - \lambda_0}{\lambda_0} = \frac{\Delta\lambda}{\lambda_0} = -\frac{\Delta\nu}{\nu_0} = \frac{v_r}{c}$$

Maximum radial velocity must come from orbit tangent to line of sight: distance and rotational motion of **tangent points** very well determined.

Distance ambiguity: for lines of sight toward the inner galaxy (first and fourth quadrant), there are two locations with the same radial velocity.

Measurement of Galaxy's rotation curve from H I and CO line profiles (continued)

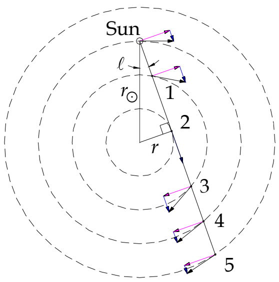
Resolution of the ambiguity usually involves information other than velocities:

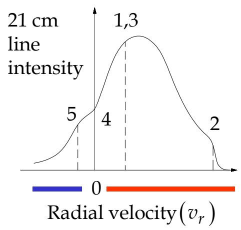
- association or lack thereof with visible-wavelength nebulosity (less extinguished = nearer).
- cloud angular size (bigger ones tend to be nearer by).
- height above Galactic plane (clouds that appear higher would be nearer by).

In the outer galaxy it is much harder to determine the distance to clouds, so the uncertainties are larger.

- Best method so far: association of clouds with H II regions or star clusters; cluster distances determined by main-sequence fitting.

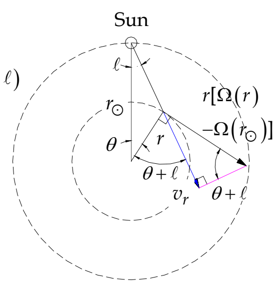
Interpretation of H I line profiles



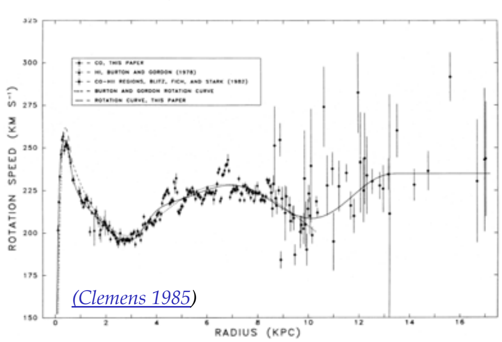


Measurement of the (inner) Galaxy's rotation curve

$$v_r = r\left[\Omega(r)-\Omega(r_\odot)\right]\sin(\theta+\ell)$$
$$= r_\odot\left[\Omega(r)-\Omega(r_\odot)\right]\sin\ell$$
$$v(r) = r\Omega(r)$$
$$= v_{r,\max} + r_\odot\Omega(r_\odot)\sin\ell$$



Results, from CO observations



Notable features of the Galaxy's rotation curve

- Central region has v increasing linearly with increasing r, as in solid body rotation. (Constant density if spherical.)
- Most of the disk has a rather "flat" rotation curve (i.e. differential rotation), meaning that the enclosed mass increases linearly with increasing radius - as if the mass were dominated by a spherical, $1/r^2$ density.
- This is the case in spite of the fact that the observed stellar density decreases more sharply.
- Keplerian rotation is expected eventually, at large enough distances, but is not seen.
    - Dark matter again? (Yes, as we will see next time.)

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