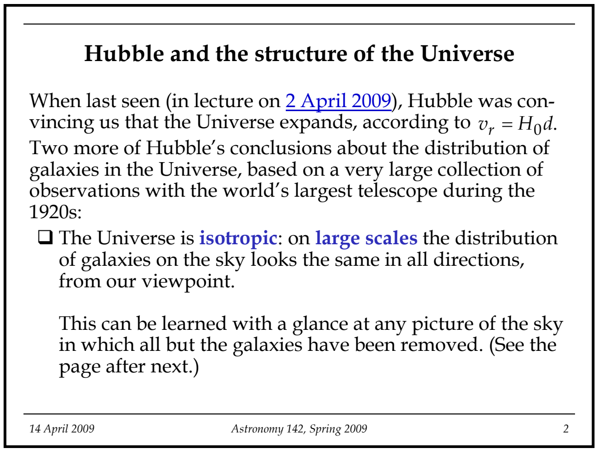
Today in Astronomy 142: cosmology

- ❑ The Universe expands, is homogeneous, and is isotropic.
- ❑ What Newton and Einstein would have to say about such a Universe.

The HST/ACS Ultra Deep Field (Beckwith et al. 2004).

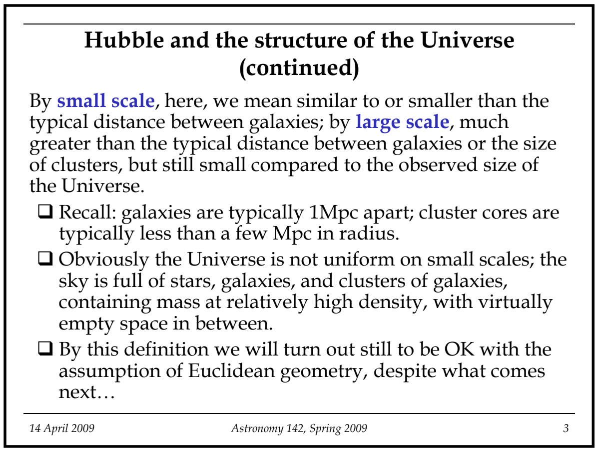


Hubble and the structure of the Universe

When last seen (in lecture on 2 April 2009), Hubble was convincing us that the Universe expands, according to $v_r = H_0 d$. Two more of Hubble’s conclusions about the distribution of galaxies in the Universe, based on a very large collection of observations with the world’s largest telescope during the 1920s:

- ❑ The Universe is **isotropic**: on **large scales** the distribution of galaxies on the sky looks the same in all directions, from our viewpoint.

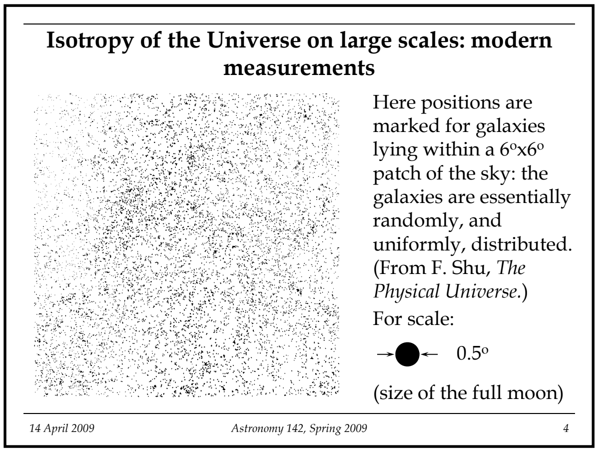
  This can be learned with a glance at any picture of the sky in which all but the galaxies have been removed. (See the page after next.)



Hubble and the structure of the Universe (continued)

By **small scale**, here, we mean similar to or smaller than the typical distance between galaxies; by **large scale**, much greater than the typical distance between galaxies or the size of clusters, but still small compared to the observed size of the Universe.

- ❑ Recall: galaxies are typically 1Mpc apart; cluster cores are typically less than a few Mpc in radius.
- ❑ Obviously the Universe is not uniform on small scales; the sky is full of stars, galaxies, and clusters of galaxies, containing mass at relatively high density, with virtually empty space in between.
- ❑ By this definition we will turn out still to be OK with the assumption of Euclidean geometry, despite what comes next…



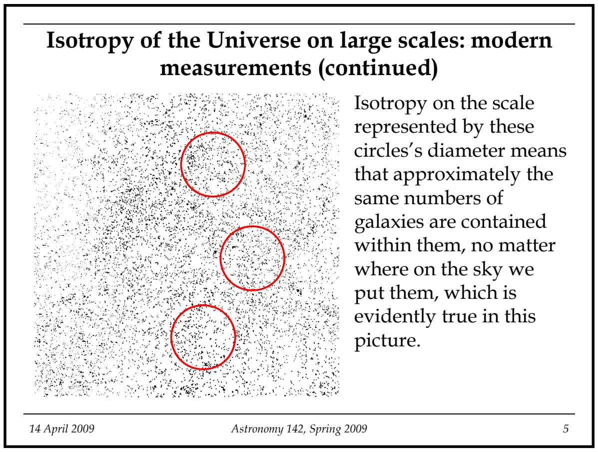
Isotropy of the Universe on large scales: modern measurements

Here positions are marked for galaxies lying within a 6°x6° patch of the sky: the galaxies are essentially randomly, and uniformly, distributed. (From F. Shu, *The Physical Universe*.)

For scale:

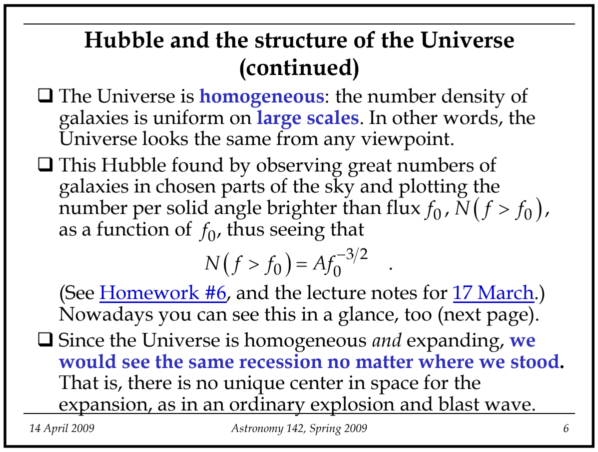
→●← 0.5°

(size of the full moon)



Isotropy of the Universe on large scales: modern measurements (continued)

Isotropy on the scale represented by these circles's diameter means that approximately the same numbers of galaxies are contained within them, no matter where on the sky we put them, which is evidently true in this picture.



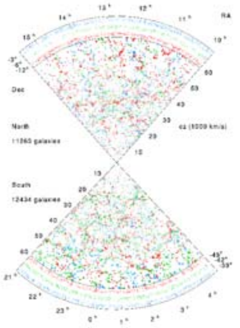
Hubble and the structure of the Universe (continued)

- ❑ The Universe is **homogeneous**: the number density of galaxies is uniform on **large scales**. In other words, the Universe looks the same from any viewpoint.
- ❑ This Hubble found by observing great numbers of galaxies in chosen parts of the sky and plotting the number per solid angle brighter than flux f_0, $N(f > f_0)$, as a function of f_0, thus seeing that

$$N(f > f_0) = Af_0^{-3/2} \quad .$$

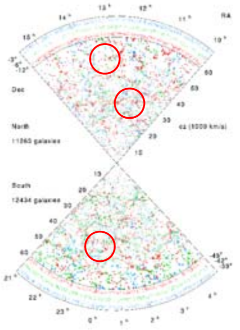
  (See Homework #6, and the lecture notes for 17 March.) Nowadays you can see this in a glance, too (next page).
- ❑ Since the Universe is homogeneous *and* expanding, **we would see the same recession no matter where we stood.** That is, there is no unique center in space for the expansion, as in an ordinary explosion and blast wave.

Homogeneity of the Universe on large scales: modern measurements



The Las Campanas Redshift Survey, showing the positions, out to distances of about 3x10^9 light years along the line of sight, of almost 24,000 galaxies in six different thin stripes on the sky. Data from stripes in the northern and southern sky are overlaid. Again, the galaxies and their larger groupings tend to be randomly distributed through volume on large scales. From Huan Lin, U. Toronto.

Homogeneity of the Universe on large scales: modern measurements (concluded)

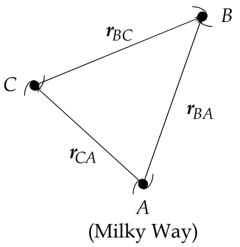


Homogeneity on the scale of these circles's diameter means that approximately the same numbers of galaxies are contained within them, no matter where we put them within the Universe's volume, which is evidently true in this picture.

Why the expansion looks the same from all viewpoints

In Euclidean (flat) space, $\boldsymbol{v} = d\boldsymbol{r}/dt$ is along $\boldsymbol{r}$, so

$$\begin{aligned} \boldsymbol{v}_{BC} &= \boldsymbol{v}_{BA} - \boldsymbol{v}_{CA} \\ &= H_0\left(\boldsymbol{r}_{BA} - \boldsymbol{r}_{CA}\right) = H_0 \boldsymbol{r}_{BC} \quad . \end{aligned}$$



Thus observers in galaxies *B* and *C* see the same Hubble law that we do: the expansion looks the same from *A*, *B* and *C*.

A Newtonian Universe?

In an infinite, homogeneous and isotropic universe, the gravitational acceleration *g* **should** be zero everywhere (averaged over large enough scales), since a mass would be equally attracted to all directions in such a universe.

The only way to do that if space is Euclidean and gravity is Newtonian is if **space is completely empty**. Reason:

- Through any point one can draw a large sphere.
- The acceleration *g* at this point due to matter outside this sphere is zero (that matter is uniform, infinite,…).
- The acceleration due to matter within this sphere is

$$g = GM(r)/r^2 \quad .$$

- Thus $g = 0$ only if the sphere is empty ($M(r) = 0$), and if all other such spheres are empty (by homogeneity!).

A general-relativistic Universe

It is patently false that the Universe is empty, so it cannot be Newtonian. Can it be general-relativistic?

A general-relativistic model of the expanding universe may teach us about the age, structure, and origin of universe and expansion.

More about general relativity related to our discussion of black holes (in lecture on 12 February 2009):

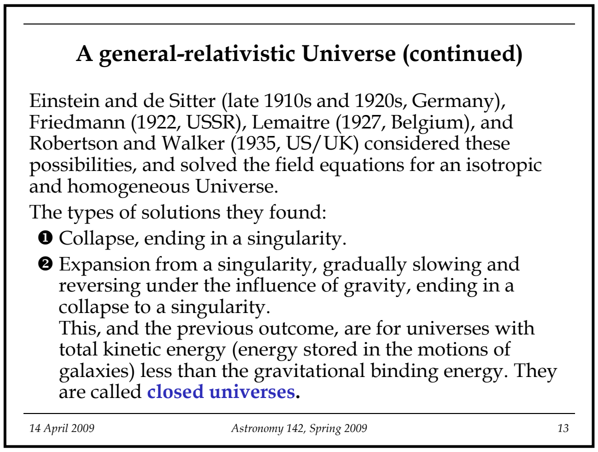
- The Schwarzschild (black-hole) solution to the Einstein field equations was a static solution, under the assumption that some pressure supported the star.
- It can be shown (as Oppenheimer did in 1939) that if the maximum neutron degeneracy pressure is exceeded, the system must collapse: that is, no static solution to the equations of GR is possible any more.

A general-relativistic Universe (continued)

- This collapse proceeds until the formation of a **singularity** (formally, infinite density: all the mass at one point, somewhere within the event horizon).
- Collapse to a singularity is the inevitable outcome of a system in which gravity is the only significant force and all of its matter is bound together by gravity.

So: how would a system behave if

- it were expanding initially, but still bound together by gravity? Or alternatively…
- if in fact the total energy of the objects in the system were positive (i.e. $KE > -PE$); that is, if the system were not gravitationally bound?



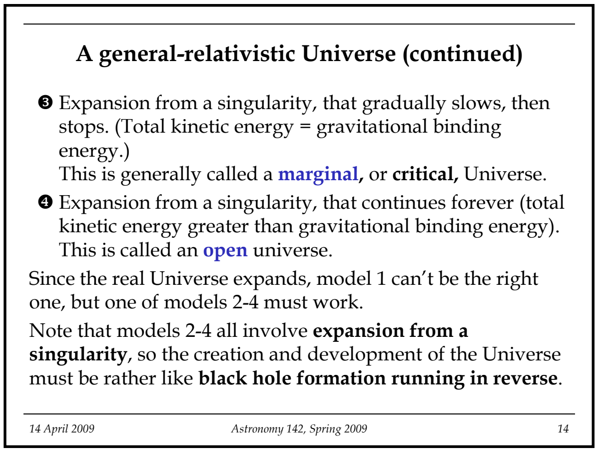
A general-relativistic Universe (continued)

Einstein and de Sitter (late 1910s and 1920s, Germany), Friedmann (1922, USSR), Lemaitre (1927, Belgium), and Robertson and Walker (1935, US/UK) considered these possibilities, and solved the field equations for an isotropic and homogeneous Universe.

The types of solutions they found:

❶ Collapse, ending in a singularity.

❷ Expansion from a singularity, gradually slowing and reversing under the influence of gravity, ending in a collapse to a singularity.
This, and the previous outcome, are for universes with total kinetic energy (energy stored in the motions of galaxies) less than the gravitational binding energy. They are called **closed universes.**



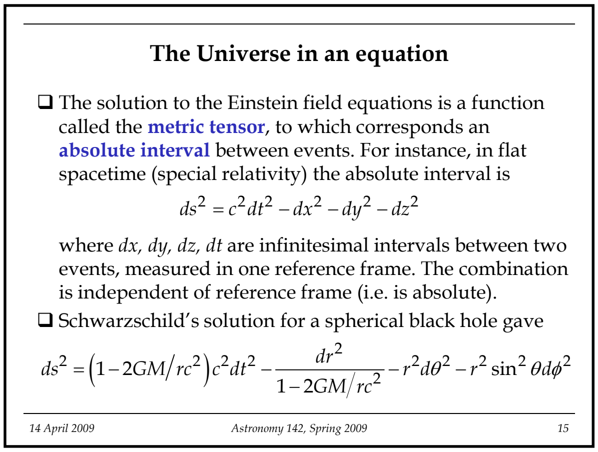
A general-relativistic Universe (continued)

❸ Expansion from a singularity, that gradually slows, then stops. (Total kinetic energy = gravitational binding energy.)
This is generally called a **marginal,** or **critical,** Universe.

❹ Expansion from a singularity, that continues forever (total kinetic energy greater than gravitational binding energy). This is called an **open** universe.

Since the real Universe expands, model 1 can’t be the right one, but one of models 2-4 must work.

Note that models 2-4 all involve **expansion from a singularity**, so the creation and development of the Universe must be rather like **black hole formation running in reverse**.



The Universe in an equation

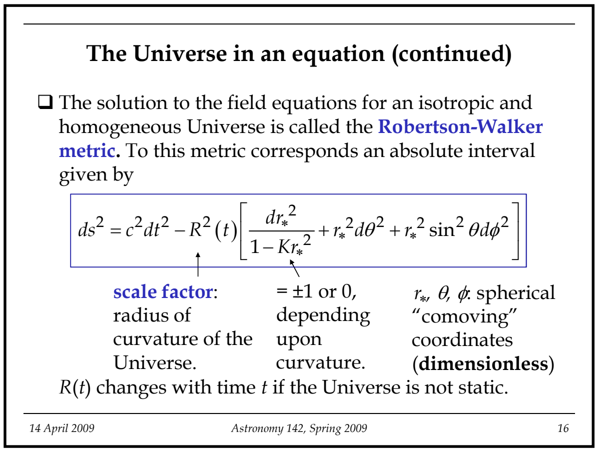
- ❑ The solution to the Einstein field equations is a function called the **metric tensor**, to which corresponds an **absolute interval** between events. For instance, in flat spacetime (special relativity) the absolute interval is

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$$

  where dx, dy, dz, dt are infinitesimal intervals between two events, measured in one reference frame. The combination is independent of reference frame (i.e. is absolute).

- ❑ Schwarzschild’s solution for a spherical black hole gave

$$ds^2 = \left(1 - 2GM/rc^2\right)c^2dt^2 - \frac{dr^2}{1 - 2GM/rc^2} - r^2d\theta^2 - r^2\sin^2\theta d\phi^2$$



The Universe in an equation (continued)

- The solution to the field equations for an isotropic and homogeneous Universe is called the **Robertson-Walker metric.** To this metric corresponds an absolute interval given by

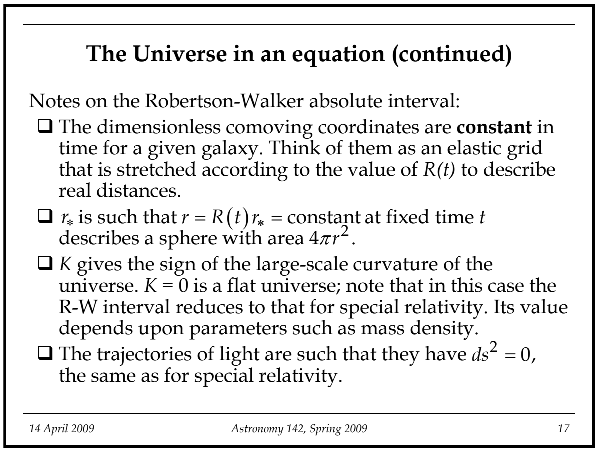
$$\boxed{ds^2 = c^2dt^2 - R^2(t)\left[\frac{dr_*^2}{1-Kr_*^2} + r_*^2 d\theta^2 + r_*^2\sin^2\theta\, d\phi^2\right]}$$

scale factor: radius of curvature of the Universe.

= ±1 or 0, depending upon curvature.

r_*, θ, ϕ: spherical "comoving" coordinates (**dimensionless**)

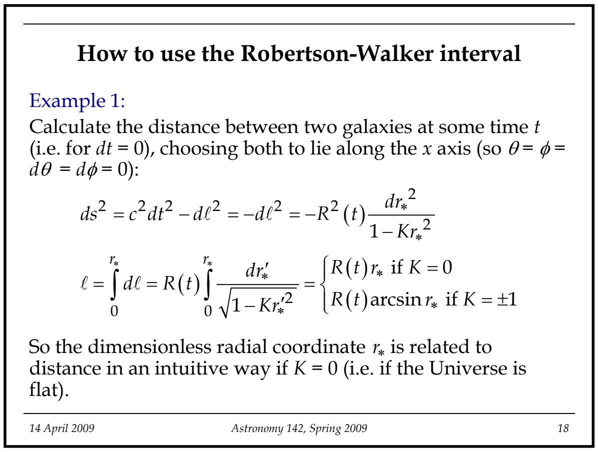
$R(t)$ changes with time t if the Universe is not static.



The Universe in an equation (continued)

Notes on the Robertson-Walker absolute interval:

- The dimensionless comoving coordinates are **constant** in time for a given galaxy. Think of them as an elastic grid that is stretched according to the value of $R(t)$ to describe real distances.
- r_* is such that $r = R(t)r_* =$ constant at fixed time t describes a sphere with area $4\pi r^2$.
- K gives the sign of the large-scale curvature of the universe. $K = 0$ is a flat universe; note that in this case the R-W interval reduces to that for special relativity. Its value depends upon parameters such as mass density.
- The trajectories of light are such that they have $ds^2 = 0$, the same as for special relativity.



How to use the Robertson-Walker interval

Example 1:

Calculate the distance between two galaxies at some time t (i.e. for $dt = 0$), choosing both to lie along the x axis (so $\theta = \phi = d\theta = d\phi = 0$):

$$ds^2 = c^2dt^2 - d\ell^2 = -d\ell^2 = -R^2(t)\frac{dr_*^2}{1-Kr_*^2}$$

$$\ell = \int_0^{r_*} d\ell = R(t)\int_0^{r_*}\frac{dr_*'}{\sqrt{1-Kr_*'^2}} = \begin{cases} R(t)r_* & \text{if } K = 0 \\ R(t)\arcsin r_* & \text{if } K = \pm 1 \end{cases}$$

So the dimensionless radial coordinate r_* is related to distance in an intuitive way if $K = 0$ (i.e. if the Universe is flat).

How to use the Robertson-Walker interval (continued)

Example 2

Calculate the expansion speed, if $r_* \ll 1$ (that is, if one views a nearby galaxy):

$$\arcsin r_* = r_* + \frac{1}{6}r_*^3 + \frac{3}{40}r_*^5 + \ldots \cong r_*, \quad \text{so}$$

$$\ell = R(t)r_* \quad \text{(all curvatures)}$$

$$v_r = v = \frac{d\ell}{dt} = \frac{dR}{dt}r_* \equiv \dot{R}(t)r_* = \frac{\dot{R}(t)}{R(t)}\ell \equiv H(t)\ell$$

The last result is of course just Hubble's Law. Our usual value, of the Hubble "constant", $H_0 = 65 \text{ km sec}^{-1} \text{ Mpc}^{-1}$, is the present value of $\dot{R}(t)/R(t)$.

How to use the Robertson-Walker interval (continued)

Example 3

Small distances, $r_* \ll 1$, as functions of time:

$$\ell = R(t)r_* \quad \Rightarrow \quad \frac{\ell_1}{\ell_0} = \frac{R(t_1)}{R(t_0)}$$

But if this works for small intergalactic distances, it should work even better for *wavelengths*, which are usually quite small distances. Suppose light is emitted at time t_1 and detected at time t_0; then its wavelengths at those two epochs are related by $\lambda_0/\lambda_1 = R(t_0)/R(t_1)$. But that ratio of wavelengths is related to redshift by $(\lambda_0 - \lambda_1)/\lambda_1 = z$, so

$$\boxed{1 + z = \frac{R(t_0)}{R(t_1)}} \quad .$$

Birkhoff's Rule

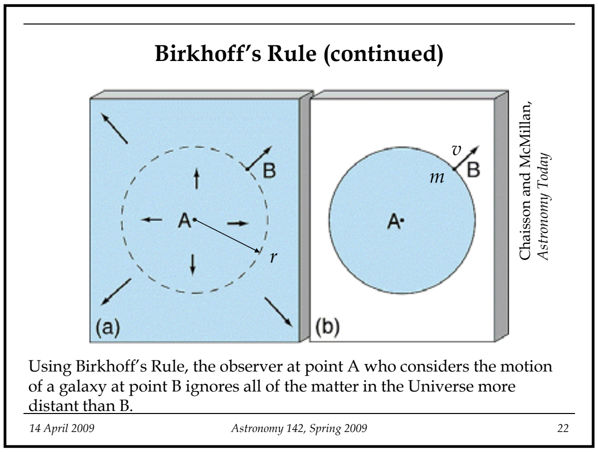
The following can also be derived from general relativity:

A galaxy a distance r away from the observer has motion that is only affected by the mass within the sphere of radius r centered on the observer, as long as the sphere isn't larger than the scale on which the Universe can be considered Euclidean $(r_* \ll 1)$. In other words, its total energy is

$$E = \frac{1}{2}mv^2 - \frac{GM(r)m}{r} \quad ,$$

where $M(r)$ is the mass contained within the sphere, m is the galaxy's mass, and v its speed. (See next page.)

We will use this rule next time to answer some basic questions about the large-scale structure of the Universe.



Birkhoff's Rule (continued)

Chaisson and McMillan, *Astronomy Today*

Using Birkhoff's Rule, the observer at point A who considers the motion of a galaxy at point B ignores all of the matter in the Universe more distant than B.