This practice exam will do you very little good unless (1) you have mastered the homework and workshop problems, and tried to understand everything discussed in the online solutions for all these problems; (2) you have already composed your one-page Cheat Sheet and want to see if it really has everything you need on it, and nothing you don’t. If you’ve done (1) and (2), feel free to proceed.

If this were a real exam, you would be advised here of the exam rules:

“You may consult only one page of formulas and constants and a calculator while taking this test. You may not consult any books, nor each other. All of your work must be written on the attached pages, using the reverse sides if necessary. The final answers, and any formulas you use or derive, must be indicated clearly. Exams are due an hour and fifteen minutes after we start, and will be returned to you during the next lecture. Good luck.

“Note:

• “Work first on the problems you find easiest, and come back to harder or less familiar material later. Don’t get stuck.

• “The amount of space left for each problem is not necessarily an indication of the amount of writing it takes to solve it.”

Name: ______________________
Problem 1. (40 points)

a. A $10^8 M_\odot$ black hole at the center of an elliptical galaxy accretes matter from a surrounding disk at a rate limited by radiation pressure. What is the luminosity thus produced (in $L_\odot$)?

b. We usually think of the radiated energy in black hole accretion as originally comprising a fraction $\varepsilon = 0.1$ of the rest energy of the infalling material. What is the rate (in $M_\odot$ year$^{-1}$) at which the black hole in part (a) must accrete mass in order to produce its luminosity?
Problem 1. (continued)

c. A very young $1M_\odot$ protostar, still completing its formation, accretes matter from a surrounding disk at a rate limited by radiation pressure. What is the luminosity thus produced (in $L_\odot$)?

d. In this case we account for the radiated energy in the gravitational potential energy of the infalling material. Gas originating hundreds of AU away from the star is passed through the disk and deposited on the surface of the star, considered here to lie $1R_\odot$ from the star’s center. According to the virial theorem only half of the potential energy drop shows up as an increase in thermal kinetic energy; the rest is available for radiation. What is the rate (in $M_\odot \text{ year}^{-1}$) at which the star in part (c) must accrete mass in order to produce its luminosity?
Problem 2. (30 points)

a. Consider a universe with curvature $K \neq 0$, in which the cosmological constant $\Lambda$ has the same sign as $K$, and in which $\Omega_M \ll \Omega_\Lambda$. Solve the Friedmann equation to obtain a relation between time and scale factor for this universe.

(Hint: As in the case of the de Sitter universe considered in recitation, start your integration at $t_0$ instead of zero.)
Problem 2. (continued)

b. At what time \( t(0) \) does \( R = 0 \)? Is this time in the future or the past, relative to \( t_0 \)?

c. The most distant quasars we know have redshifts of about \( z = 6.4 \). In this model of the Universe, how long ago did those quasars emit the photons we receive from them now?
Problem 3. (40 points)

A giant elliptical galaxy lies 20 Mpc away. Observations are made of the total flux from starlight, and stellar velocity dispersion along the line of sight, within circles on the sky that correspond to radii of 100 and 1000 pc. Within these two areas, the results are as follows:

<table>
<thead>
<tr>
<th>Radius</th>
<th>Total Flux (erg s(^{-1}) cm(^{-2}))</th>
<th>Radial Velocity Dispersion (km s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 pc radius</td>
<td>2.1×10(^{-11})</td>
<td>290</td>
</tr>
<tr>
<td>1000 pc radius</td>
<td>1.5×10(^{-9})</td>
<td>250</td>
</tr>
</tbody>
</table>

Use these data to answer the following questions about the galaxy.

a. Calculate the mass, luminosity, and mass-to-light ratio, all in solar units, for the part of the galaxy contained within the large (1 kpc-radius) area.
Problem 3. (continued)

b. Repeat for the small (100 pc-radius) area.

c. On the basis your answers to parts a-b, argue for or against the presence of a supermassive black hole at the center of this galaxy. If a black hole is likely to be present, estimate its mass.
Problem 3. (continued)

d. Estimate the stellar relaxation time for the 100-pc area, and compare to the likely age of the galaxy. Comment on the validity of the virial theorem in this case. (Hint: recall the similar issue of galaxy clusters.)
Problem 4. (20 points)

Short answers. Answer any four of these eight questions.

a. The galaxies in a cluster have an average radial velocity of 15000 km/sec. How far away is the cluster?

b. A galaxy with redshift 6.4 contains a particular A0 V star, which like all others of its kind has surface temperature $T = 10^4$ K when viewed from within that galaxy. What does its temperature appear to be from our viewpoint?
Problem 4. (continued)

c. List two strong pieces of evidence for the existence of dark matter.

d. Give one strong piece of evidence for the existence of dark energy.
Problem 4. (continued)

e. Over a certain range of radius the rotational velocities in a certain galaxy increase linearly with radius. What does this day about the mass density of that galaxy in this radius range?

f. It is claimed that Sirius, Arcturus and the Orion Nebula are visible from Dublin, late at night on Bloomsday. True or false? Explain.
Problem 4. (continued)

g. The classical Cepheids in the Small Magellanic Cloud typically have period 10 days and average apparent magnitude $m_V = 14.6$. What is the distance modulus, $m-M$, to the SMC?

h. Give the Hubble types of the two galaxies in the accompanying images.

A

B