Astronomy 142 Recitation #4

8 February 2013

Formulas to remember

\[ E = mc^2 \]

Maximum fusion tunneling probability and tunneling rate

\[ P(T) = D \exp \left( -\left[ \frac{T_0}{T} \right]^{1/3} \right) , \]

where \( T_0 = \left( \frac{3}{2} \right)^3 \left( 8 \pi q_1 q_2 / h \right)^2 m / k = 1.565 \times 10^{10} \) K for proton-proton fusion, where \( q_1 \) and \( q_2 \) are the electric charges (in cgs units) of the two fusing nuclei, where \( m = m_1 m_2 / (m_1 + m_2) \) is the reduced mass of the two fusing nuclei, and where \( D \) is a constant.

Pulsating stars

Adiabatic speed of sound: \( v_s = \sqrt{\gamma \rho / \mu} = \sqrt{\gamma kT / \mu} ; \gamma = 5/3 \) for monatomic gases, 7/5 for diatomic gases at room temperature.

Pulsation period, fundamental mode, uniform-density star:

\[ \Pi = 4 \int_0^R dr / v_s = \frac{6 \pi}{\gamma G \rho} \]

Local sidereal time (LST)

LST = right ascension of celestial objects on the meridian (the meridian = arc through the poles and the zenith); thus same units as right ascension.

Earth rotates by \( 360^\circ = 2\pi \) radians in 24 h of sidereal time; Earth rotates by \( 2\pi \left( 1 + \text{day/TY} \right) \) in one day. 1 day = 24 hours, or 86400 seconds; 1 tropical year (TY) = 365.242189 days. The latter means that the Earth’s rotational angular speed is \( \Omega_{\oplus} = 7.292115855 \times 10^{-5} \) radians per second, and that a sidereal day – 24h of sidereal time, and the actual rotation period of the Earth – is 23.9344696 hours.

For a given time of day, the corresponding sidereal time advances by 24h as the date advances one year.

Workshop problems

Warning! The workshop problems you will do in groups in Recitation are a crucial part of the process of building up your command of the concepts important in AST 142 and subsequent courses. Do not, therefore, do your work on scratch paper and discard it. Better for each of you to keep your own account of each problem, in some sort of bound notebook.

1. Consider two protons (\( q = 4.8 \times 10^{-10} \) esu, \( m = 1.67 \times 10^{-24} \) gm), separated by 10^{-13} cm.
   a. Calculate the magnitude of the proton's electric repulsive force and gravitational attractive force. Can gravity hold the protons together?
b. Calculate the electrostatic and gravitational potential energies. Compare these to the binding energy of the deuteron, 2.2 MeV (3.55×10⁻⁶ erg). This binding energy is the opposite of the strong-interaction potential energy, and its relatively large value indicates the predominance of the strong interaction on nuclear size scales.

2. Assume that the Sun was initially composed 70% by mass of hydrogen. How many hydrogen nuclei, \( N \), were there? What is the total nuclear energy supply, \( NE/4 \), where \( E = 0.03m_ec^2 \)? If all of the hydrogen could be fused into helium? It turns out that only about 13% of the hydrogen in a solar-type star lies within the parts of the star hot enough to participate in fusion during the star’s stay on the main sequence. Under these assumptions, what will the Sun’s main sequence lifetime be?

3. The flux of pp-chain electron neutrinos from the Sun has been measured to be about half the flux that is expected at the temperature inferred for the Sun’s center, \( T_{\odot} = 1.57 \times 10^6 \) K. (This is the deficit to which we used to refer as the “Solar neutrino problem”.)

   a. What would the Sun’s central temperature have to be, in order to produce pp-chain electron neutrinos at half the rate that applies to \( T_{\odot} = 1.57 \times 10^6 \) K?

   b. Review briefly the means by which we inferred the Sun’s central temperature (in class on 29 January). By how much would the accepted values of other measurable properties of the Sun need to be in error for the temperature to be off by this much? Is it likely that such differences lie within present observational uncertainties?

4. Newton worked on sound, too, but he turned out to be slightly off about how it propagated. He thought it was an isothermal pressure disturbance (constant temperature as pressure changes) instead of an adiabatic one (no heat flows from the pressure fluctuations). In modern terms Newton’s formula for the (isothermal) speed of sound would be

\[
v_{s,isoT} = \sqrt{\frac{kT}{\mu}},
\]

where \( \mu \) is the average mass of particles (atoms or molecules) in the gas. (Newton, of course, didn’t know about Boltzmann’s constant or molecular masses, so he was unaware of what the proportionality constant should be in the relation he proposed for sound speed.)

If Newton had been right in these details, what would be the formula for pulsation period of a uniform-density star, and by what factor do the periods for adiabatic and isothermal pulsations differ?

Learn your way around the sky, lesson 4. (An exclusive feature of AST 142 recitations.) You may find the lab’s celestial globes, and TheSky running on the lab computers, useful in answering these questions about the celestial sphere and the constellations. Today’s lesson is on

5. A mystery, to be solved next week. The vernal equinox is when the Sun reaches the intersection of the celestial equator and the ecliptic, going north; this particular intersection defines the zero of the right ascension scale. This year the vernal equinox takes place on 20 March, at 7:02 AM EDT: 11:02 coordinated universal time (UTC; standard time at zero longitude).

   a. At what longitude is the Sun on the meridian at the moment of the vernal equinox? What is the local sidereal time at this location and moment?
b. What is the local sidereal time at zero longitude, and in Rochester (longitude 77.60° West), at the moment of the vernal equinox?

c. Now look at the result of an accurate calculation of local sidereal time, courtesy of Western Washington University's planetarium:


Calculate the local sidereal time the moment of the vernal equinox, at the longitude calculated in part a, at zero longitude, and at Rochester's longitude. If all has gone well you will find that your answers differ from the results in parts a and b by the same amount. What do you suppose is the cause of this offset? (That is, what extra effect does the calculator include, that we are currently leaving out?)

6. Match the celestial object to the constellation to which it belongs.

<table>
<thead>
<tr>
<th>Objects</th>
<th>Constellations</th>
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<tbody>
<tr>
<td>Brightest star in the sky</td>
<td>Andromeda</td>
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<td>Most massive, luminous star visible to the naked eye</td>
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Solutions

1. The magnitude of the repulsive force exerted by the protons on each other is

\[ F_E = \frac{q^2}{r^2} = 2.3 \times 10^7 \text{ dynes} \]

and that of the attractive gravitational force is quite a bit less:

\[ F_G = \frac{Gm^2}{r^2} = 1.9 \times 10^{-29} \text{ dynes} \]

The potential energies are

\[ U_E = \frac{q^2}{r} = 2.3 \times 10^{-6} \text{ erg} \]

and

\[ U_G = -\frac{Gm^2}{r} = -1.8 \times 10^{-42} \text{ erg} \]

both much smaller in absolute magnitude than the binding energy of the deuteron, which must therefore be contributed for the most part by a third interaction (the strong interaction).

2. The number of hydrogen nuclei in the sun is \( N_H = 0.7 M_\odot / m_p = 8.3 \times 10^{56} \), so the total energy supply is

\[ E = \frac{N_H}{4} \times 0.03 m_p c^2 = 9.4 \times 10^{51} \text{ erg} \]

The sun can't use up its entire hydrogen supply, however; it will only use an amount \( E_{\text{eff}} = 0.13E = 1.22 \times 10^{51} \text{ erg} \) before it starts to have trouble supporting itself. If it burns up the energy \( E_{\text{eff}} \) at a rate \( L_\odot \), this takes

\[ t = \frac{E_{\text{eff}}}{L_\odot} = 3.2 \times 10^{17} \text{ s} = 1.0 \times 10^{10} \text{ years} \]

So far, it's only lived \( 0.4567 \times 10^{10} \text{ years} \), so we have quite a ways to go.

3. a. As we learned in lecture on Thursday, the rate of neutrino production is proportional to the fusion probability \( P(T) = D \exp \left( -\left[ T_0 / T \right]^{1/3} \right) \), where \( T_0 = (3/2)^3 \left( \frac{8 \pi q^2}{\hbar} \right)^2 m/k = 1.565 \times 10^{10} \text{ K} \)

A reduction by a factor of two in the neutrino flux goes with a reduction by a factor of two in this probability:
\[
\frac{P(T)}{P(T_{\odot})} = e^{-\left(\frac{T_0}{T}\right)^{\frac{2}{3}}} = \frac{1}{2} .
\]

Solve this for \(T\):

\[
-\left(\frac{T_0}{T}\right)^{\frac{1}{3}} + \left(\frac{T_0}{T_{\odot}}\right)^{\frac{1}{3}} = -\ln 2
\]

\[
T = \frac{T_0}{\left[\left(\frac{T_0}{T_{\odot}}\right)^{\frac{1}{3}} + \ln 2\right]^3} = 1.28 \times 10^7 \text{ K} .
\]

This is smaller than the accepted value by \(\left(\frac{T_{\odot} - T}{T_{\odot}}\right)/T_{\odot} = 18\%\).

b. Here is the formula we got for the temperature, and expressions we got for the main ingredients of this formula:

\[
T_c = \frac{P_c \mu_c}{\rho_c k}, \quad P_c = 19 \frac{GM_{\odot}^2}{R_{\odot}^4}, \quad \rho_c = 25 \frac{M_{\odot}}{R_{\odot}^3} .
\]

So to change the inferred temperature we would need to infer a mistake in our assessment of the pressure or the typical particle mass smaller, or the density larger, or some combination of those things. We don’t have much choice about the particle mass, as this is fixed by the solar core’s element abundances. To have made a mistake in the pressure or density we’d need to have made an error in the factors of 19 and 25 in the expressions for \(P_c\) and \(\rho_c\). These factors come from (correct) solutions to the equations of stellar structure. Needless to say there’s very little uncertainty in the values of the mass and radius of the Sun. So the most likely place to look for an error is in the basic physics that give the equations of stellar structure: hydrostatic equilibrium, ideal-gas behaviour, etc. But this physics does a good job of explaining all other aspects of the Sun and other stars. Thus it is very unlikely that the temperature at the center of the Sun is 18\% smaller than we always thought.

(This is why the solar-neutrino deficit was taken seriously as an indication of neutrino flavor oscillation, even before the oscillations were observed in reactor neutrinos at Kamiokande and the missing electron neutrinos were detected as muon neutrinos at SNO.)

4. The “isothermal sound speed” in an ideal monatomic gas is

\[
\nu_{s,isoT} = \sqrt{\frac{kT}{\bar{m}}} = \sqrt{\frac{P}{\rho}} = \nu_s \sqrt{\frac{T}{Y}} = \nu_s \sqrt{1.3}
\]

\[\Pi_{isoT} = \sqrt{\frac{6\pi}{G\rho}}\]

so the pulsation periods would all get longer by a factor of \(\sqrt{Y} \approx 1.3\).
5. a. At the moment of the vernal equinox, it’s 11:02 AM at zero longitude (Royal Greenwich Observatory, London), and thus it’s true noon -- Sun on the meridian -- 58 time-minutes of longitude east of zero longitude:

\[ L_0 = \frac{360^\circ}{24 \times 60^m} = 14.50^\circ \text{ East} \]

And at that location, the local sidereal time is zero, practically by definition.

b. This is \( \Delta L = L_0 - L_R = 92.10^\circ \) east of Rochester, so at the moment of the vernal equinox the local sidereal time -- earlier than it is where the Sun is on the meridian -- is

\[
LST = 24^h - L_0 \left( \frac{24^h}{360^\circ} \right) = 23.03^h = 23^h 02^m \text{ in London},
\]

\[
= 24^h - \Delta L \left( \frac{24^h}{360^\circ} \right) = 17.86^h = 17^h 52^m \text{ in Rochester}.
\]

c. So, much to our surprise, we find on the oracular online LST calculator that

\[
LST = 23^h 52^m 34^s \text{ at 11:02 UTC and longitude } 14.50^\circ \text{ E}
\]

\[
= 22^h 54^m 34^s \text{ at 11:02 UTC and zero longitude}
\]

\[
= 17^h 44^m 10^s \text{ at 11:02 UTC and longitude } 77.60^\circ \text{ W}
\]

All of the sidereal times from the calculator are about \( 7^m 26^s \) earlier than our simple calculations in parts a and b. So our calculations are close to correct, but there’s an offset. This offset will be discussed next week, and has to do with the details of the Earth’s orbit around the Sun. (If you can’t wait to find out, google “equation of time” and read on. Also note that an offset in this amount is even given, by the name Equation of Time, in the LST calculator...)

6.

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</tr>
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