Astronomy 142 Recitation #7

1 March 2013

Formulas to remember

Reddening and extinction

Magnitude reminder: \( U = m_U, B = m_B, V = m_V \) (apparent magnitudes); \( B - V = m_B - m_V \) (color index).

Visual extinction toward a star: \( A_V = RE(B - V) \),

where the star’s color excess \( E(B - V) \) is the difference between that star’s \( B - V \) color index and the \( B - V \) color index of an unextinguished main-sequence star of the same spectral type; and where \( R \), the ratio of total to selective extinction, is 3.06 for the extinction provided by the Milky Way’s diffuse interstellar clouds.

For that same extinction, \( E(U - B)/B(B - V) = 0.72 \), on the average.

The following works for any pair of wavelengths, but we will use the handy example of \( B \) and \( V \).

Extinction correction: if you measure magnitudes \( B \) and \( V \) for a star, and determine \( E(B - V) \) and \( A_V \), then the extinction corrected color and \( V \) magnitude are

\[
\begin{align*}
(B - V)_0 &= B - V - E(B - V) \\
V_0 &= V - A_V
\end{align*}
\]

The distance modulus can be converted reliably to distance if extinction-corrected apparent magnitudes are used:

\[
DM = m - M = V_0 - M_V = 5 \log (r/10 \text{ pc})
\]

Jeans mass

Weight and pressure in hydrostatic equilibrium for uniform, spherical gas cloud with mass density \( \rho \), temperature \( T \) and average molecular mass \( \mu \) if its mass is

\[
M = M_J = \left( \frac{kT}{\mu G} \right)^{3/2} \left( \frac{3}{4 \pi \rho} \right)^{1/2}
\]

Ideal gas reminder

\[
P = nkT = \frac{\rho kT}{\mu} \quad E = \frac{3}{2} NkT
\]

where \( N \) is the number of molecules in the system \( (= M/\mu, = nV) \).
Workshop problems

Warning! The workshop problems you will do in groups in Recitation are a crucial part of the process of building up your command of the concepts important in AST 142 and subsequent courses. Do not, therefore, do your work on scratch paper and discard it. Better for each of you to keep your own account of each problem, in some sort of bound notebook.

1. Extinction at some wavelengths besides those of the astronomical $B$ and $V$ filters are given in Table 1.

Table 1: extinction at the wavelengths of ultraviolet, visible and near-infrared astronomical filters, relative to the $J$ ($\lambda = 1.25 \, \mu m$) filter (Mathis 1990).

<table>
<thead>
<tr>
<th>Band</th>
<th>Wavelength, $\lambda$ (\mu m)</th>
<th>$A_\lambda / A_J$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U$</td>
<td>0.36</td>
<td>5.53</td>
</tr>
<tr>
<td>$B$</td>
<td>0.44</td>
<td>4.70</td>
</tr>
<tr>
<td>$V$</td>
<td>0.54</td>
<td>3.55</td>
</tr>
<tr>
<td>$R$</td>
<td>0.70</td>
<td>2.66</td>
</tr>
<tr>
<td>$I$</td>
<td>0.90</td>
<td>1.70</td>
</tr>
<tr>
<td>$J$</td>
<td>1.25</td>
<td>1.00</td>
</tr>
<tr>
<td>$H$</td>
<td>1.65</td>
<td>0.624</td>
</tr>
<tr>
<td>$K$</td>
<td>2.21</td>
<td>0.382</td>
</tr>
<tr>
<td>$L$</td>
<td>3.40</td>
<td>0.182</td>
</tr>
<tr>
<td>$M$</td>
<td>4.80</td>
<td>0.095</td>
</tr>
</tbody>
</table>

a. An A0 V star (absolute $V$ magnitude $M_V = 0.6$, visible color index $B - V = 0$, all visible and near-infrared color indices similarly zero) is placed at the distance of the Galactic center $(r = 8.0 \times 10^3 \, \text{pc})$. What is its apparent $V$ magnitude and visible color index?

b. The same star is placed in the real Galactic center, toward which the visual extinction is $A_V = 28$. Now what its apparent $V$ magnitude and visible color index?

c. What is its magnitude at $K$ ($\lambda = 2.2 \, \mu m$)?

d. With ground-based telescopes we can detect stars as faint as about 30th magnitude at $V$ and about 24th at $K$. At what wavelength should we search for stars in our Galactic center?

2. A uniform-density, pure-molecular-hydrogen, spherical cloud has mass $M = 2M_\odot$, temperature $T = 50$ K, and molecular-hydrogen number density $n = 10^5 \, \text{cm}^{-3}$.

a. Show that it is not unstable: that is, that it is not collapsing gravitationally.

b. At what temperature would it become gravitationally unstable, if its density remained at its original value?

c. Suppose it were possible to compress the cloud from the outside: by a passing supernova shock, for example. By what factor would the volume need to be compressed for the cloud to become gravitationally unstable and proceed to collapse on its own (without further squeezing), if the temperature remained at its original value?
Learn your way around the sky will be taking the week off. We will resume on the other side of Midterm #1 with the rest of the Equation of Time.

Learn your way around Microsoft Excel, lesson 2. (Another exclusive feature of AST 142 recitations.)

3. The spreadsheet ps_05.xls that you will use in Homework #5 contains data on 342 candidate members of the nearest large open cluster, the Hyades. It also contains data on the Zero-Age Main Sequence (ZAMS): the absolute magnitudes of main sequence stars of all spectral types, constructed from observations of very nearby (and therefore unextinguished) stars and accurate stellar models. Here we will use the ZAMS to measure and correct the small extinction toward the Hyades (which also turns out to be pretty nearby), and to measure the distance of the Hyades by main sequence fitting, the technique you will use in Homework #5 to measure the distances to a bunch of further-off open clusters. Unlike the clusters in Homework #5, though, we can check our main-sequence fitting distance to the Hyades by comparison to measured parallax for its members.

a. Choose two cells in columns different from the data, and define names for their content, using Formulas…Define name… Call them something like “EBV” and “EUV”. This is where you will put your adjustable color excess values.

b. Insert or identify two empty columns to the right of the data you’re given, and use them to calculate de-reddened values of $B-V$ and $U-B$. For example, if you have created a name called “EBV”, and your data for $B-V$ is in column C, then the formula “=C2 – EBV” placed in the second row of your first empty column will result in a calculation of the dereddened version, $(B-V)_0$. Selecting the rest of that column by dragging the cursor down from your formula, then using Fill Down (control-D), you can copy the formula, correctly updated, for all the rest of your data entries.

c. Create a chart in which your dereddened $(U-B)_0$ is plotted as a function of $(B-V)_0$, on top of those same quantities for the ZAMS. It will be most convenient if that chart is on the same worksheet page as the EBV and EUV names. The best way to do this is to create a chart with just one of the data sets, say $(U-B)_0$ vs. $(B-V)_0$, and then use Add Data (under Chart Tools...Design...Select Data…) to put in the other set. Plot $(U-B)_0$ vs. $(B-V)_0$ as data points unconnected by a curve, preferably with small symbols; plot the ZAMS points without symbols, as a continuous curve. In good astronomical style for plots of magnitudes and colors, make the vertical axis backwards: that is, horizontal axis increases from left to right, but vertical increases from top to bottom.

d. Note that the curves thus generated are slightly, but noticeably, displaced from one another. Adjust your “EBV” and “EUV” values until you get the best match between the data points and the ZAMS.

This should result in “EUV” being pretty close to 0.72 times “EBV”. How close did you come? (You may feel free to set “EUV” to be this value, by entering the formula =0.72*EBV in its cell.)

e. Define the name of another blank cell, near “EBV” and “EUV”, to be called something like “AV.” Set its value with the formula =3.06*EBV. This of course is the visual extinction corresponding to color excesses you measured by adjustment of “EBV” and “EUV”.

f. Define another name for a cell below this, called something like “DM.” This will be for the distance modulus, another thing you’ll need to measure by adjustment.
g. And, just below that, put in a formula for calculating the distance in parsecs from the distance modulus: $d = 10 \times 10^{(DM/5)}$.

h. Insert or identify a blank column to the right of your $(B-V)_0$ and $(U-B)_0$ points, and use this to calculate the extinction-corrected, absolute $V$ magnitude, $M_V = V_0 - DM = V - A_V - DM$.

i. Create another chart, near your DM cell, and use it to plot $M_V$ (as points, not connected by a curve) as a function of $(B-V)_0$, along with those same quantities for the ZAMS (as a connected curve without symbols). Adjust the value of “DM” until your points match the ZAMS curve, giving the highest weight to points with $(B-V)_0 \approx 0.5-1$. When you are done you will have completed the fitting of the Hyades main sequence to the zero-age main sequence. What values of $DM$ and $r$ (in parsecs) result?

j. You will note that some of your data points seem offset systematically from the rest. If they appear to correspond to a common distance, different from the rest, calculate the value of that distance, in parsecs. What do you suppose is the origin of those discrepant points?

k. And now the test. Included in your data on the Hyades are Hipparcos measurements of parallax for members of the Hyades. Compare the median value of the parallax distances to the cluster distance you determined by main-sequence fitting. (One handy way is to use Excel’s Median function; for instance $=\text{MEDIAN(F2:F342)}$ returns the median value of the entries in the second through 342nd rows of column F.) Note that the parallax is given in milliarcseconds (0.001 arcseconds).

l. Generate a new column with absolute magnitudes of the Hyades calculated from $M_V = V - A_V - 5\log(r/10 \text{ pc})$. Make a new chart with this new $M_V$ plotted as a function of $(B-V)_0$ (as points), along with those same quantities for the ZAMS (as a continuous curve). Why did I ask you, in part I, to give the greatest weight to the intermediate color excess values in your main sequence fitting? Why do the Hyades points depart systematically from the ZAMS in the way that they do, for both the bluest and reddest stars?
Solutions

1. a. \[ V_0 = M_V + 5 \log \left( \frac{r}{10 \text{ pc}} \right) = 0.6 + 14.5 = 15.1. \]

b. Since all the intrinsic color indices of an A0V star are zero, the absolute \( V \) and \( B \) magnitudes are the same:

\[ V = V_0 + A_V = 43.1 \]
\[ B = V_0 + A_B = V_0 + 28 \frac{4.7}{3.55} = 52.2 \]
\[ B - V = 9.1 \]
\[ \text{Or } B - V = (B - V)_0 + \frac{A_V}{R} = 0 + \frac{28}{3.06} = 9.1 \]

c. Ditto for the absolute \( K \) magnitude:

\[ K = V_0 + A_K = V_0 + 28 \frac{0.382}{3.55} = 18.1. \]

d. The star is undetectably faint at \( V \) but fairly easily detectable at \( K \). This makes the study of stars near the Galactic center an infrared-astronomy project.

2. a. \[ M_f = \left( \frac{kT}{\mu G} \right)^{3/2} \left( \frac{3}{4 \pi \rho} \right) \left( \frac{3}{4 \pi \rho} \right)^{1/2} = 2.3 M_\odot > M, \] so it’s currently not unstable.

b. It would be unstable if \( M_f = M \), i.e. if

\[ T = \frac{G \mu}{k} \left( \frac{4 \pi \rho}{3} \right)^{1/3} M^{2/3} = 45 \text{ K}. \]

c. Similarly, it would be unstable if compressed to

\[ \rho' = \frac{3}{4 \pi M^2} \left( \frac{kT}{\mu G} \right)^3, \text{ or } \]
\[ \frac{\rho'}{\rho} = \left( \frac{2.3}{2} \right)^2 = 1.4. \]

Equivalently, the volume would have to be reduced by this same factor, 1.4.

3. I will post my own spreadsheet for these solutions, on the Recitations page of the website. Here’s what I got:
The agreement is OK; greater care in constraining which points line up would improve the agreement. The plots are on the following pages.

The discrepant points form a locus parallel to the Hyades MS and below it; apparently they’re all stars in a cluster further away than the Hyades. Fitting these points to the ZAMS gives a distance modulus of about 5.3, and a distance of 115 parsecs, not far from the accepted distance to the Pleiades (see the lecture notes for 21 February). The Pleiades and Hyades overlap on the sky, so it’s easy to mistake their members for each other’s. Let that be a lesson to you: establishment of membership is important in main-sequence-fitting measurements of distance, and similarly-discrepant points will appear in the clusters you will analyze in Homework #5.

The Hyades with parallax distances lie slightly above the main sequence on the blue and red ends because of stellar evolution: the most massive stars are beginning to leave the main sequence (note the four red giant stars!), but the least massive haven’t yet had time to settle down onto the main sequence. It turns out all the clusters you will use for Homework #5 are a good deal younger than the Hyades, so they will match the ZAMS better for the bluest stars and worse for the reddest ones.
Hyades, DM = 3.16

ZAMS

Pleiades?