Astronomy 142 Recitation #10

5 April 2013

Formulas to remember

Leavitt's Law (classical Cepheid variables):

\[
\overline{M_V} = -2.77 \log \Pi - 1.69
\]

\[
\overline{m_V - M_V} = 5 \log \left( \frac{d}{10 \text{ pc}} \right)
\]

Hubble's Law (galaxies in the uniform Universal expansion):

\[
v_r = H_0 d
\]

\[H_0 = 74.2 \text{ km sec}^{-1} \text{ Mpc}^{-1} = 22.8 \text{ km sec}^{-1} \text{ Mly}^{-1}\]

Redshift

\[z = (\lambda - \lambda_0)/\lambda_0\]

SN Ia magnitude (dereddened)

\[m_V^0 = M_V^0 + 5 \log \frac{d}{\text{Mpc}} + 25\]

\[M_V^0 = -19.14\]

Black hole accretion

\[L = \frac{dE}{dt} = \varepsilon c^2 \frac{dm}{dt}, \varepsilon \approx 0.1.\]

Eddington luminosity

\[L < L_E = \frac{3GMmp^2c^5}{2e^4} \quad \text{and} \quad M > \frac{2e^4L}{3Gmp^2c^5}\]

Workshop problems

**Warning!** The workshop problems you will do in groups in Recitation are a crucial part of the process of building up your command of the concepts important in AST 142 and subsequent courses. Do not, therefore, do your work on scratch paper and discard it. Better for each of you to keep your own account of each problem, in some sort of bound notebook.

1. **(Team discussion)** A type Ia supernova happens when a the mass of a white dwarf, accreting material from a close-by normal or giant stellar companions, approaches the Stoner-Anderson-Chandrasekhar mass, \(M = 1.4M_\odot\). Review your previous experience with degenerate stars and answer the following questions, in order.

   a. If mass is added to a white dwarf, does its radius get larger, smaller, or stay the same? Is this different from what happens when mass is added to an ordinary, nondegenerate star?
b. If heat is added suddenly to a white dwarf – for example, along with the accreted mass – does its radius get larger, smaller, or stay the same? Is this different from what happens when heat is added suddenly to an ordinary, nondegenerate star?

c. In white dwarfs, electrons are degenerate but nuclei behave as an ideal gas. What happens to the temperature and density of the gas of nuclei, as a white dwarf accretes mass and (because of the energy conservation) heat?

d. Suppose that during this process, additional heat, besides that accreted, is generated within the star – for instance, by thermonuclear fusion of the nuclei. What happens to the temperature and density of the gas of nuclei? Is this different from what happens to temperature and density when heat is created within an ordinary, nondegenerate star?

e. As a result: why does mass accretion by white dwarfs near the Chandrasekhar limit result in runaway thermonuclear deflagration and explosion, rather than stable, slow fusion power generation as in ordinary stars?

2. (Requires even more teamwork). In Homework #5, you each measured the distance to two open clusters by main-sequence fitting. Now the reason may be told: these clusters were chosen, not because they are particularly good data sets or particularly illustrative clusters, but simply because each of them has at least one classical Cepheid variable as a member: there are ten in all.

<table>
<thead>
<tr>
<th>Name</th>
<th>Cluster</th>
<th>log ( \Pi ) (days)</th>
<th>( \overline{m}_{\text{V}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>DL Cas</td>
<td>NGC 129</td>
<td>0.9031</td>
<td>8.92</td>
</tr>
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<td>1.0526</td>
<td>8.20</td>
</tr>
<tr>
<td>S Nor</td>
<td>NGC 6087</td>
<td>0.9892</td>
<td>6.31</td>
</tr>
<tr>
<td>CEb Cas</td>
<td>NGC 7790</td>
<td>0.6512</td>
<td>10.39</td>
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<tr>
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<td>0.6880</td>
<td>10.62</td>
</tr>
<tr>
<td>CEa Cas</td>
<td>NGC 7790</td>
<td>0.7111</td>
<td>10.30</td>
</tr>
</tbody>
</table>

a. Consult with your classmates who measured the clusters you didn’t, collect all the distances and extinction values that you are missing, and compare your results to those by classmates who chose the same clusters you did. Then use these distances, visual extinctions, and the average apparent magnitudes given above to calculate the extinction-corrected, average, absolute \( V \) magnitudes of the ten classical Cepheids. (Since the calculations are repetitive, it’s easiest to do this in Excel.)

b. Using Excel, plot the average absolute \( V \) magnitude as a function of log\( \Pi \). Then use the Add Trendline feature of Excel to produce a line that is the best fit to the points. Have Excel print the equation for this line in the plot as well. Compare this result with the Leavitt Law (\( \overline{M}_{\text{V}} - \log \Pi \) relation) for classical Cepheids that was given in class on Tuesday,

\[
\overline{M}_{\text{V}} = -2.77 \log \Pi - 1.69
\]

and comment on the agreement, or lack thereof.
c. If all has gone well you will notice that three of the points – those from NGC 7790 – seem to differ somewhat from the trend of the rest. Look back at the main-sequence fits to this cluster, and speculate on the origin of this difference.

Learn your way around the sky, lesson 9. (An exclusive feature of AST 142 recitations.) Use the lab’s celestial globes, TheSky running on the lab computers, the SIMBAD database at http://simbad.harvard.edu/simbad/, and any other resources you would like to use, to answer these questions about the celestial sphere and the constellations.

3. Secant ZA. If one wants to measure stellar magnitudes accurately while the stars rise from low altitude to high, and then set back to low altitude again, one must correct for atmospheric extinction. One would have to make such a correction, for example, in the RR Lyrae option for this semester’s observing project. Fortunately this is easier than correcting for interstellar extinction. Here’s how.

Suppose the atmosphere is an infinite plane-parallel layer, which we can assume to be uniform in density. Suppose further that the flux we would measure from a star at the zenith, in the absence of the atmosphere, is \( f_0 \). Then the flux with the atmosphere in place is \( f = f_0 e^{-\tau_0} \), where \( \tau_0 \) is the extinction, expressed as an optical depth, of the atmosphere toward the zenith. (See the lecture notes for 22 February 2011 for a reminder of extinction in terms optical depth and in terms of magnitudes.)

a. Now suppose that the star has moved over to a zenith angle of \( ZA \). If the thickness of the atmosphere is \( d \), what is the length of the path through the atmosphere for the line of sight toward the star?

b. If the optical depth is proportional to path length through the atmosphere, what is the atmospheric optical depth toward the star with zenith angle \( ZA \)?

c. As you know, if the sky is clear and dry, the extinction must be pretty small, because one can see stars well as they rise and set, not just when they’re close to zenith. Make a first-order approximation and derive a simple formula for the flux \( f \) received from a star at zenith angle \( ZA \), in terms of the unextinguished flux \( f_0 \) and the minimum atmospheric optical depth \( \tau_0 \).

d. Describe how you could, thereby, measure \( f_0 \) and \( \tau_0 \), assuming these quantities to be constant through the night.

4. Literary astronomy. You are now upperclasspeople, or on the verge of being upperclasspeople. And thus professors can assume that you’re familiar with all of the great works of world literature. So of course you’ve read all of Joyce. Congratulations on your hard-won erudition.

Near the end of James Joyce’s Ulysses, the two main characters, Leopold Bloom and Stephen Dedalus, walk into the back yard of Bloom’s home in Dublin. It is very late at night:

What spectacle confronted them when they, first the host, then the guest, emerged silently, doubly dark, from obscurity by a passage from the rere of the house into the penumbra of the garden?
The heaventree of stars hung with humid nightblue fruit.

With what meditations did Bloom accompany his demonstration to his companion of various constellations?
Meditations of evolution increasingly vaster: of the moon invisible in incipient lunation, approaching perigee: of the infinite lattiginous scintillating uncondensed milky way, discernible by daylight by an observer placed at the lower end of a cylindrical vertical
shaft 5000 ft deep sunk from the surface towards the centre of the earth: of Sirius (alpha in Canis Major) 10 lightyears (57,000,000,000,000 miles) distant and in volume 900 times the dimensions of our planet: of Arcturus: of the precession of the equinoxes: of Orion, with belt and sextuple sun theta and nebula in which 100 of our solar systems could be contained: of moribund and of nascent new stars such as Nova in 1901: of our system plunging toward the constellation of Hercules; of the parallax or parallactic drift of so-called fixed stars, in reality evermoving from immeasurably remote eons to infinitely remote futures in comparison with which the years, threescore and ten, of allotted human life formed a parenthesis of infinitesimal brevity.

In this stiff, formal, question-and-answer style, Joyce casts the section as a parody of the Catechism. After four paragraphs on geology, math and comparative planetology he returns to astronomy:

Which various features of the constellations were in turn considered?
The various colours significant of various degrees of vitality (white, yellow, crimson, vermilion, cinnabar): their degrees of brilliancy: their magnitudes revealed up to and including the 7th: their positions: the waggoner's star: Walsingham way: the chariot of David: the annular cinctures of Saturn: the condensation of spiral nebulae into suns: the interdependent gyrations of double suns: the independent synchronous discoveries of Galileo, Simon Marius, Piazzi, Le Verrier, Herschel, Galle: the systematisations attempted by Bode and Kepler of cubes of distances and squares of times of revolution: the almost infinite compressibility of hirsute comets and their vast elliptical egressive and reentrant orbits from perihelion to aphelion: the sidereal origin of meteoric stones: the Libyan floods on Mars about the period of the birth of the younger astroscopist: the annual recurrence of meteoric showers about the period of the feast of St. Lawrence (martyr, 10 August): the monthly recurrence known as the new moon with the old moon in her arms: the posited influence of celestial on human bodies: the appearance of a star (1st magnitude) of exceeding brilliancy dominating by night and day (a new luminous sun generated by the collision and amalgamation in incandescence of two nonluminous exsuns) about the period of the birth of William Shakespeare over delta in the recumbent neversetting constellation of Cassiopeia and of a star (2nd magnitude) of similar origin but of lesser brilliancy which had appeared in and disappeared from the constellation of the Corona Septentrionalis about the period of the birth of Leopold Bloom and of other stars of (presumably) similar origin which had (effectively or presumably) appeared in and disappeared from the constellation of Andromeda about the period of the birth of Stephen Dedalus, and in and from the constellation of Auriga some years after the birth and death of Rudolph Bloom, junior, and in and from other constellations some years before or after the birth or death of other persons: the attendant phenomena of eclipses, solar and lunar, from immersion to emersion, abatement of wind, transit of shadow, taciturnity of winged creatures, emergence of nocturnal or crepuscular animals, persistence of infernal light, obscurity of terrestrial waters, pallor of human beings.

Let us set aside the obvious error ("equinoxes" for equinoctes – actually I am surprised Joyce got that wrong) and analyze the accuracy of Joyce’s astronomical statements.

a. What is the date on which the action takes place? (You don’t need to read the book to know; just ask Google when Bloomsday is.)

b. On this date, could Bloom and Stephen see the stars they are claimed to see?

c. How does Joyce do on the dimensions of interstellar space, the sizes of celestial objects, and the brightness of stars? Which properties are quoted correctly, which incorrectly?
d. Evaluate the theory of star formation Joyce includes in his discussion.

e. What is the significance of the star which "appeared in and disappeared from the constellation of Andromeda about the period of the birth of Stephen Dedalus," and how old is Stephen when his conversation with Bloom takes place? (No fair looking that last fact up in the book or on the net.)

f. Describe a modern astronomical example of “the appearance of a star (1st magnitude) of exceeding brilliancy dominating by night and day (a new luminous sun generated by the collision and amalgamation in incandescence of two nonluminous exsuns).”

g. Review from AST 111. Evaluate Joyce’s two statements about meteors, and identify the Keplerian law to which he refers.
Solutions

1. a. If mass is added to a white dwarf, its radius gets smaller; when this happens to a normal star it gets larger.

   b. If heat is added suddenly to a white dwarf – by which I should mean, more quickly than it could be radiated away – without adding significant extra mass, its radius does not change. This is different from an ordinary, nondegenerate star, which would swell up a bit due to the sudden increase in pressure. Indeed the heat increases the temperature (of the nondegenerate nuclei), but not the pressure (of the degenerate electrons): degeneracy pressure is independent of temperature.

   c. As a white dwarf accretes mass and heat, the temperature and density of the gas of nuclei both rise: the latter because the star gets smaller, the former because of the added heat.

   d. If fusion adds heat in addition to that brought in by accretion, the temperature of the white dwarf’s gas of nuclei increases in direct proportion. In an ordinary star the increase would not be so great, because the additional pressure would cause the star to expand a bit, dropping the density and temperature.

   e. Thus the stabilizing effects of ideal-gas stars – their ability to expand in response to heat input – are absent, and replaced by destabilizing effects: the white dwarf gets smaller, and the density higher, as the mass increases. So the temperature begins to increase without bound and the white dwarf explodes.

2. a. Consult the solutions to Homework 5 if necessary.

   b. See Figure 1. The difference from the accepted version is not so bad, considering that we only had ten points.

   c. NGC 7790 was the only cluster in our collection for which \( E(U-B)/E(B-V) \) differed substantially from the canonical value of 0.72. (I got \( E(U-B)/E(B-V) = 0.42 \); see the Homework 5 solutions.) So the value of \( A_V \) we derive for this cluster is probably in error; the extinction toward NGC 7790 does not seem to follow the standard interstellar form.

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Figure 1: our very own Cepheid period-luminosity relation. The solid black line in the fit to our data points, $M_V = -2.35 \log \Pi - 1.40$, and the dashed line is the accepted relation (based on a lot more data), $M_V = -2.77 \log \Pi - 1.69$.

3. The geometry is shown in Figure 1.

a. $\ell = d \cos ZA = d \sec ZA$.

b. $\tau = \tau_0 \sec ZA$.

c. Assume $\tau$ and $\tau_0$ to be small, but not $ZA$:

$$f = f_0 e^{-\tau} = f_0 e^{-\tau_0 \sec ZA}$$

$$\approx f_0 (1 - \tau_0 \sec ZA) = f_0 - f_0 \tau_0 \sec ZA$$
d. We can measure \( f \) for a star over a range of \( ZA \) during the course of a night. If we plot our measurements of \( f \) as a function of \( \sec ZA \) we will get a straight line: the \( y \) intercept gives us \( f_0 \), and the slope \( -f_0\tau_0 \).

We'll do a concrete example of atmospheric-extinction determination next week in Recitation.

4. a. *Ulysses* takes place in a single day: 16 June 1904. Anniversaries of this date, called Bloomsday, are celebrated worldwide by Joyce's fans.

b. Thus the Summer Solstice is near: the sidereal time at midnight is within about half an hour of 18:00. The latitude of Dublin is 53.3°. Thus there is no way Bloom and Stephen could have been observing Sirius (\( \alpha = 06:45:08.9, \delta = -16:42:58, J2000 \)) or anything in Orion (e.g. the Orion Nebula region near \( \alpha = 5^h, \delta = -5^\circ \)); these objects are within an hour of RA of the Sun in mid-June. This very same mistake was made in the fifth *Harry Potter* book. I wonder if Rowling, who certainly knows her Joyce, was copying Joyce's description of the summer sky.

They could have seen Arcturus (\( \alpha = 14:15:39.7, \delta = 19:10:57, J2000 \)), though: assuming they were out at \( ST = 18:00 \), which must be pretty close, Arcturus has \( HA = 3:45 \) and \( ZA = 63^\circ \), and won't set for quite a while.

c. **Wrong:** The volume of Sirius is more like 6 million times that of Earth, not 900; about 17 million solar systems would fit within the Orion Nebula, not 100; eyes can only see 5-6th magnitude stars under good conditions, not 7th and not from the middle of a big city like Dublin, even in 1904; Martian Libya (by which he probably means Syrtis Major) is not flooded. **Right:** Sirius is only 8.6 ly away, but Joyce is close enough with 10; the motion of the Sun with respect to the LSR is indeed directed toward Hercules; the sky probably does look darker from the bottom of a deep hole.

d. As we know, the spiral nebulae are galaxies, not stars in formation, and in fact most astro-aware people of 1904 would have considered them to be island universes of stars. Nevertheless it had been popular since Kant to consider a star in the act of formation to look like a miniature version of a spiral nebula; this was called the Nebular Hypothesis.

e. We can gather from the order of presentation that Stephen is a young man, younger than Bloom, so S Andromedae (SN 1885A, type Ia) must have been that star. This enables a more precise estimate of Stephen's age: S Andromedae went off in February 1885, so Stephen is 19 on Bloomsday.

We have mentioned S Andromedae in class, but not the other ones, two of which were ordinary novae. T Aurigae is the star that appeared and disappeared from Auriga in attendance of Rudolph Bloom -- son of Leopold and Molly Bloom, the tragedy of whose death within two weeks of birth is a red thread through the whole book, and explains the Blooms's liking for Stephen. It is often called Nova Aurigae 1892, though it went off in December 1891, because no astronomer drew attention to it until 23 January. The nova in Corona Septentrionalis (= Corona Borealis) attending Leopold Bloom's birth is T Coronae Borealis (Nova Coronae Borealis 1866; 12 May). So Bloom is 38 on Bloomsday.

Oh, and the very bright star which appeared "about the period of the birth of William Shakespeare over delta in the recumbent neversetting constellation of Cassiopeia " can be taken to be SN 1572 (Tycho's supernova; type Ia) because of the good positional match, even though Shakespeare was born almost a decade earlier.
f. “A new luminous sun generated by the collision and amalgamation in incandescence of two nonluminous exsuns” is a very nice way to describe the coalescence of a neutron-star binary and creation thereby of a short gamma-ray burst. I think I’ll remind some of my friends who work on GRBs to quote Joyce.

g. Sidereal origin of meteors? No. Meteors do not come from the stars. On the other hand, Joyce knows about the Perseid meteor shower, which peaks during 10-12 August, and he seems to know the form of Kepler’s third law (cube of radius, square of period).