Astronomy 142 Recitation #11

12 April 2013

Formulas to remember

Superluminal motion in AGNs

\[ v_{\perp,\text{apparent}} = \frac{v \sin \theta}{1 - \frac{v}{c} \cos \theta} \]

\[ \left( v_{\perp,\text{apparent}} \right)_{\text{max}} = v \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma v \]

Galaxy distribution in clusters (number per unit area on the sky)

\[ N(r) = N(0) \exp \left[ - \left( \frac{r}{r_0} \right)^{-1/4} \right] \]

Relaxation time

\[ t_c \approx \left( \frac{2r_0}{v} \right) \frac{N}{24 \ln(N/2)} \]

Thermal and escape speeds

\[ v_{th} = \sqrt{3kT/m_H}, \quad v_{esc} = \sqrt{2GM/r_0}. \]

Virial mass (of thermalized cluster)

\[ M = \frac{2v_0^2}{G} = \frac{6v_0^2}{G} \]

Mass from Keplerian orbits

\[ M = \frac{rv^2}{G} \]

Atmospheric extinction correction

\[ f = f_0 e^{-\tau} = f_0 e^{-r_0 \sec ZA} \]

\[ \approx f_0 (1 - r_0 \sec ZA) = f_0 - f_0 r_0 \sec ZA \]

Workshop problems

**Warning!** The workshop problems you will do in groups in Recitation are a crucial part of the process of building up your command of the concepts important in AST 142 and subsequent courses. Do not, therefore, do your work on scratch paper and discard it. Better for each of you to keep your own account of each problem, in some sort of bound notebook.

1. Consider the geometry described in class on Tuesday (9 April 2013) for superluminal motion. A clump within a quasar jet moves at speed \( v \) along a straight trajectory at an angle \( \theta \) with respect to an earthbound observer’s line of sight. Said observer records the blob’s position on the sky at two times, \( t = 0 \) and \( t_0 \).

   a. Derive the expression (given above) for the apparent speed \( v_{\perp,\text{apparent}} \) of this blob in the plane of the sky, in terms of \( v \) and \( \theta \).
b. From the resulting expression, show that, for a given jet speed \( v \), there is a maximum value of \( v_{\perp, \text{apparent}} \), given by the expression also listed above.

**Learn your way around the sky, lesson 10.** (An exclusive feature of AST 142 recitations.) Use the lab’s celestial globes, TheSky running on the lab computers, the SIMBAD database at [http://simbad.harvard.edu/simbad/](http://simbad.harvard.edu/simbad/), and any other resources you would like to use, to answer these questions about the celestial sphere and the constellations.

2. Two stars – call them A and B – with declinations 28.0° and 20.0° respectively, are observed every half hour from an observatory with latitude 43.0°. The signals in “data numbers” and the zenith angles of the stars at each time, are as follows:

<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>Star A</th>
<th>Star B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ZA (°)</td>
<td>Signal (DN)</td>
</tr>
<tr>
<td>0</td>
<td>49.98</td>
<td>2924</td>
</tr>
<tr>
<td>0.5</td>
<td>44.50</td>
<td>3262</td>
</tr>
<tr>
<td>1</td>
<td>39.03</td>
<td>3336</td>
</tr>
<tr>
<td>1.5</td>
<td>33.65</td>
<td>3472</td>
</tr>
<tr>
<td>2</td>
<td>28.43</td>
<td>3320</td>
</tr>
<tr>
<td>2.5</td>
<td>23.54</td>
<td>3521</td>
</tr>
<tr>
<td>3</td>
<td>19.28</td>
<td>3598</td>
</tr>
<tr>
<td>3.5</td>
<td>16.18</td>
<td>3602</td>
</tr>
<tr>
<td>4</td>
<td>15.00</td>
<td>3546</td>
</tr>
<tr>
<td>4.5</td>
<td>16.18</td>
<td>3436</td>
</tr>
<tr>
<td>5</td>
<td>19.28</td>
<td>3581</td>
</tr>
<tr>
<td>5.5</td>
<td>23.54</td>
<td>3545</td>
</tr>
<tr>
<td>6</td>
<td>28.43</td>
<td>3512</td>
</tr>
<tr>
<td>6.5</td>
<td>33.65</td>
<td>3408</td>
</tr>
<tr>
<td>7</td>
<td>39.03</td>
<td>3378</td>
</tr>
<tr>
<td>7.5</td>
<td>44.50</td>
<td>3101</td>
</tr>
<tr>
<td>8</td>
<td>49.98</td>
<td>2911</td>
</tr>
</tbody>
</table>

Star A is a well-known, well-behaved star with time-independent magnitude 8.0. The properties of star B are unknown, and for you to figure out here. Use Excel and the numbers above in your work.

a. Determine the minimum value of atmospheric extinction, \( r_0 \), presuming it to be constant through the night. Determine also the signal of star A, corrected for atmospheric extinction.

b. Using the atmospheric extinction as a function of ZA determined for star A, correct the observations of star B for atmospheric extinction. Then determine the magnitude of each star as seen in each observation, and plot the magnitudes as a function of time through the night. What have you learned about Star B?

3. **Artistic astronomy.**
One must still have chaos in oneself to be able to give birth to a dancing star.
-- Nietzsche (Thus spoke Zarathustra, prologue)

The Netherlands produces great astronomers in numbers hugely out of proportion to the nation’s population; it also produces great artists at an enormously disproportionate rate. One of the greatest of all Dutch artists, Vincent Van Gogh, also seemed to be interested in astronomical matters. Five of his best paintings, including the one that must be his most popular (the second *Starry Night*), combine celestial and Earthly scenes, and the combinations are well worth a look. Today we will consider his two *Starry Night* paintings: *Starry Night Over The Rhone* (1888, Figure 1) and *The Starry Night* (1889, Figure 2). Let us henceforth abbreviate their names as SNR and TSN, respectively.

![Starry Night Over The Rhone](https://example.com/starry_night_rhone.png)

**Figure 1: Starry Night Over The Rhone (Orsay Museum, Paris).**

a. Identify the constellations in SNR. Presuming it to be about 10 PM, what time of year is it? How accurately placed are the stars?

b. The view across the river Rhone is of Arles from a point on the east bank of the river, looking southwest; this can be confirmed simply by modern photographs, e.g. the one in the Wikipedia article on SNR. Show that it’s not possible for those stars to be in that view.

c. Why did Van Gogh transplant the star field, placing it behind the view of Arles?
d. Consider now TSN. From the phase of the moon and its relatively low altitude, constrain the direction in which the artist views the sky and the time of night at which he views it, and sketch the direction of the ecliptic.

e. Identify bright objects near the ecliptic which could be planets.

f. If all has gone well, you have shown the ecliptic to cross the nebulous band in the sky. Where in the sky does one find the ecliptic crossing such a nebulous band? What, therefore, is the nebulous band? Is its position in the sky consistent with the direction and time of viewing that was determined in part d?

g. In the light of the results of parts d-f, identify the stars in the picture.

h. Much ink has been spilled over this view by art historians. Apparently the view of the village (Saint-Rémy) is a composite of views to the north or to the south. Is either of these consistent with the view of the sky?

i. Why is the Nietzsche quote appropriate here?
Solutions

1. Here is the geometry, annotated somewhat.

\[ a. \text{ Consider the blob at time } t = 0; \text{ the observer sees the light from it, which gets to him/her at time } t_1 = \frac{r_0}{c}. \text{ In time } t_0, \text{ the star has moved a distance } vt_0, \text{ or distances } vt_0 \cos \theta \text{ along the line of sight and } vt_0 \sin \theta \text{ perpendicular to the line of sight. Once again the light takes time to propagate to the observer, but now it has to travel approximately } r_0 - vt_0 \cos \theta, \text{ so that the object’s position at time } t_0 \text{ is recorded at time } t = t_0 + \left(\frac{r_0 - vt_0 \cos \theta}{c}\right)/c. \text{ The time that the observer says the object spent getting to its new location is thus } \Delta t = t_2 - t_1 = t_0 (1 - \beta \cos \theta), \text{ and it thus looks to him or her as if the object moves across the sky at the speed } \]

\[ \left(\frac{\Delta s}{\Delta t}\right) = \frac{vt_0 \sin \theta}{t_0 (1 - \beta \cos \theta)} \]

This can plainly be greater than \( c \) – that is, the apparent motion across the sky can be faster than light \(-\text{ if } \beta \text{ and } \cos \theta \text{ are both close to unity.}\)

\[ b. \text{ We can find the maximum value of } v_{\perp,\text{apparent}} \text{ in the usual fashion:} \]

\[ \frac{d}{d\theta} v_{\perp,\text{apparent}} = \frac{v \cos \theta}{1 - \beta \cos \theta} - \frac{v \sin \theta}{(1 - \beta \cos \theta)^2} \beta \sin \theta = 0 \quad \text{ for } \theta = \theta_{\text{max}}; \]

\[ \cos \theta_{\text{max}} = \beta \cos^2 \theta_{\text{max}} = \beta \sin^2 \theta_{\text{max}} \]

\[ \cos \theta_{\text{max}} = \beta \left( \cos^2 \theta_{\text{max}} + \sin^2 \theta_{\text{max}} \right) = \beta. \]

\[ \Rightarrow v_{\perp,\text{apparent}}(\text{max}) = \frac{v \sin \theta_{\max}}{1 - \beta \cos \theta_{\max}} = \frac{v \sqrt{1 - \beta^2}}{1 - \beta^2} = \frac{v}{\sqrt{1 - \beta^2}} = \gamma v. \]

For example, if the object’s \textit{real} speed is \( v = 0.99c \), and its velocity is arranged so as to maximize this effect, its \textit{apparent} speed is \( v_{\perp,\text{apparent}}(\text{max}) = 7c \). This is approximately the situation for the 3C 273, as we saw in lecture on Tuesday.

2. The answers are given also in the Excel spreadsheet which can be downloaded from the Recitations page.

\[ a. \text{ The extinction toward the zenith can be determined by plotting the signal } f \text{ against sec } ZA \text{ and fitting a straight line, since } f \equiv f_0 - f_0 \tau_0 \text{ sec } ZA. \text{ Like this:} \]
whence we learn that $f_0 = 4797$ and $\tau_0 = 0.246$.

b. Columns of corrected values are generated by multiplying the observed values by $e^{\tau_0 \sec ZA}$. The average signal for star A is calculated from this; it turns out to be $f_A = 4545$ Then magnitudes are calculated from the corrected signals $f'$ in the usual way, by use of $m - m_A = m - 8 = 2.5 \log (f_A/f')$. Here’s the plot:

We seem to have shown that star B is a pulsating star with period 4 hours and amplitude about 1 magnitude.
In the downloadable Excel sheet you’ll also see how these fake data were generated. I started with a signal of 4500 for star A and 900 for star B, had star A rise a couple of hours earlier than B, set $\tau_0 = 0.25$ and had star B pulsate sinusoidally with a four-hour period. Then I added noise equivalent to 5% of Star A’s signal, peak-to-peak, using a random-number generator, to make it look more realistic.

3. The Big Dipper is obvious. Below it, correctly positioned, are a few of the fainter stars of Ursa Major. To its right we see five of the stars in Auriga, notably Capella and Menkalinan, $\alpha$ and $\beta$ Aurigae. The star to the left of the handle is probably Nekkar, $\beta$ Bootis. Above the Big Dipper the most plausible identifications are probably the three brightest stars in Ursa Minor, Pherkad, Kochab and Polaris ($\gamma, \beta, \alpha$ UMi). Van Gogh seems to have displaced one of the stars (Phecda, $\gamma$ UMa) in the bottom of the dipper, and scrunched the UMi stars to get them in view; otherwise the placement is pretty accurate.

To see the Big Dipper in that orientation at that time of night, it would need to be late summer, probably early September, and you’d have to look north to see it: the right-hand edge of the Dipper will be on the meridian, below the pole. (Sure enough, records show that the painting was produced in September 1888.)

b. The southernmost star in the Big Dipper has a declination of about 50°; one has to look north to see this scene, not southwest, from the latitude of Arles (43.7°). So the scene is a composite of a southwest view of the Earth and a northward view of the Sky.

c. Van Gogh could see the stars by looking over his right shoulder, if he was facing Arles. Surely the reflection of the village’s gaslights over the water, and particularly their number, reminded him of the Big Dipper over his right shoulder, and he couldn’t resist moving the Big dipper and lining up its stars with the gaslights and reflections, even though he needed to displace Phecda to do so. The stars which actually could be seen over Arles at that time are indeed not very inspiring.

d. The moon is crescent, lit on the left, and low in the sky, so the time is probably a couple of hours before sunrise, an hour or two after moonrise, and the viewing direction can be anywhere between NE (midwinter) to SE (midsummer). Since there’s no snow we’re inclined to favor spring or summer, in which case it has to be between E and SE. To first approximation the ecliptic bisects the crescent, so it would lead…

e. …to the two bright objects below and left of the moon; these both could be planets. The brighter (lower, leftmost) one is generally thought by art historians to be Venus, and this seems eminently reasonable. The one closer to the Moon could be Mars or Jupiter, if it’s a planet instead of a star.

f. In the summer sky, the ecliptic crosses the Milky Way very close to the Galactic Center, between Scorpio and Sagittarius. There the Milky Way is at its widest. This sounds just like the nebulous stripe in the picture, so we’ll suppose that Van Gogh is looking at the Milky Way, toward the SE. This would make Antares (“not Mars”; $\alpha$ Scorpii) a good match for one of the two stars near the Moon, but as there are no other “alpha” stars in the neighborhood it may be that one of the two is a planet and the other Antares. The time of year would be about Now (late March/early April), to have the scene in position the right interval before sunrise. The painting was completed in June 1889, which seems about right for a work executed by memory.

The identification of the Milky Way in TSN seems quite solid, considering its appearance and its position relative to the ecliptic. Many art historians – not to mention amateur online
commentators – are on record with other specific identifications, with which I think, after this reasoning, that I’d strongly disagree.

g. The identification of the rest of the stars is less clear. I’m inclined to think that the three stars above Venus and below the Milky Way are the three brightest stars in this neighborhood, Nunki, Kaus and Shaula (σ and ε Sagittarii, λ Scorpii, left to right), but they are not so much brighter than a lot of other second-third magnitude stars of Sagittarius and Scorpio that it is easy to understand how the rest got left out of the picture. As for the five stars on the other side of the Milky Way from Sagittarius, we have about that many: α and β Ophiuchi and Herculis, and κ Ophiuchi; but the arrangement on the painting is not particularly close to the real thing.

On the other hand, we might be reading too much into the precise positions of the stars. Unlike STR, Van Gogh was not at leisure to review his scene while he was painting TSN, and he perhaps wasn’t thinking with the same clarity as before, as he was committed to the asylum in St. Rémy by the time he painted TSN. The striking combination of Milky Way, Moon and planets is much more likely to be a detailed and correct memory.

h. A southward view of the village and hills is more consistent with the sky. The cypress trees would apparently have to be added no matter which way the view really goes. I suppose that Van Gogh was not trying to be photo-realistic here.

i. Indeed Van Gogh still had chaos within himself while he was giving birth to these dancing stars. He died insane, a suicide, less than a year after painting TSN.

So did Nietzsche, who never met his contemporary Van Gogh, but who would have liked him very much. Like Van Gogh’s, Nietzsche’s work was ignored during his lifetime, causing him much stress and depression. Like Van Gogh, Nietzsche died insane (1900, though he broke down early in 1889 and was basically a vegetable after that). And like Van Gogh, Nietzsche became hugely popular shortly after his breakdown.