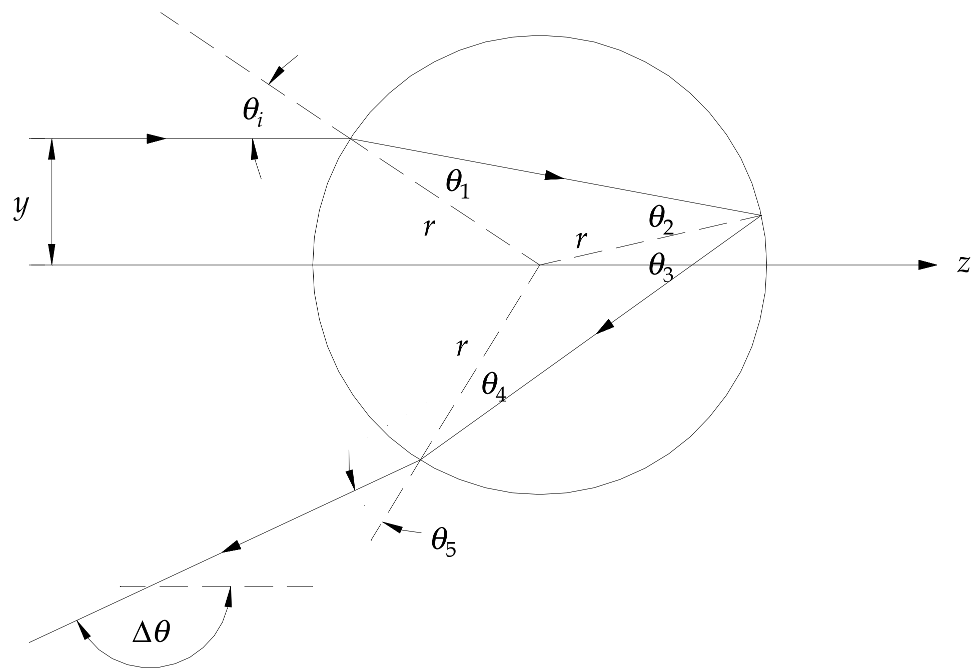
Astronomy 203 Problem Set #2

Due 28 September 1999, in class

1. *Caustics and rainbows.* Consider light incident a distance y from the center on a dielectric sphere with radius r and index n, that is refracted at the surface, reflects internally at the next, and then refracts out of the sphere, as shown in the figure below.



a. Show that as one varies y from zero to r, the angle $\Delta\theta$ by which the ray is deviated by the sphere passes through a minimum. Obtain expressions for the minimum value of $\Delta\theta$ and for the value of y that corresponds to this minimum.

   (**Hint**: prove first that all the angles of incidence, reflection and refraction within the sphere, $\theta_{1\text{-}4}$, have the same magnitude.)

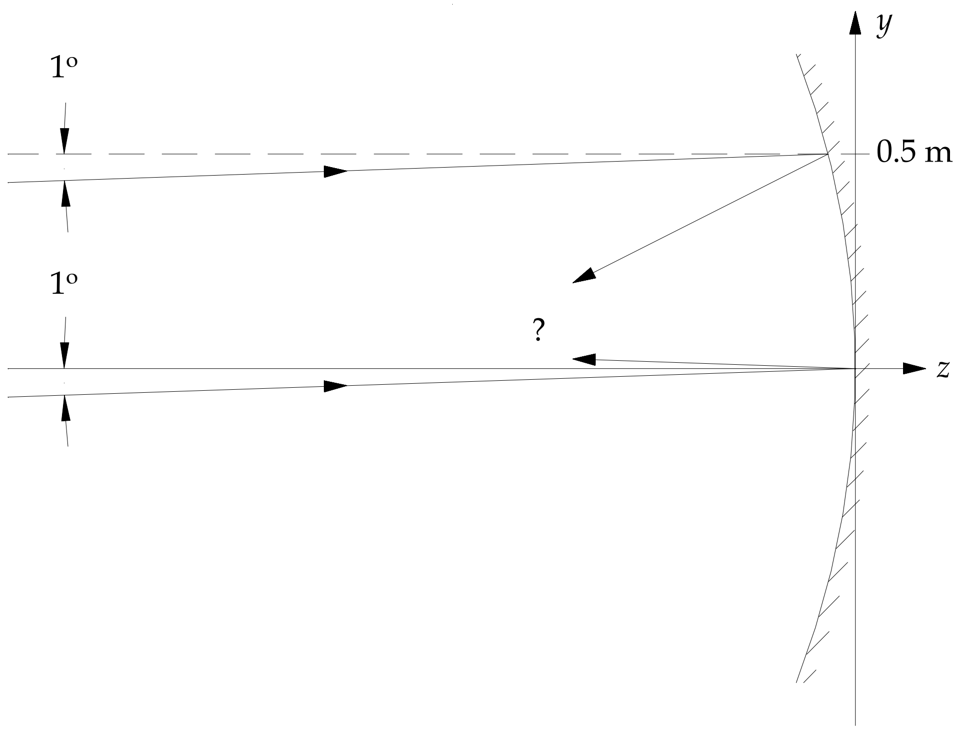
   (**Hint II**: don't try to apply the matrix formulation of ray-tracing all the way through.)

b. Make a sketch of the situation for rays incident on the sphere at several different values of y between zero and r. Use the result to argue that a uniformly illuminated sphere appears brightest in scattered light when viewed near the direction of this minimum-deviation ray, because the deviation angle passes through a minimum.

   Such minima (or maxima) in deviation angles, and corresponding intensity peaks, produced by optical elements are called *caustics.*

c. Water is a dielectric with refractive index $n \approx 4/3$; raindrops are spherical blobs of water. Use these facts with the results from parts a and b to explain how rainbows work. Calculate the angle from the direction of incident sunlight at which one should look to see a rainbow.

2. A concave paraboloidal mirror has on-axis focal length 4 m, and is illuminated by two parallel rays intersecting its surface at its apex and at a point 0.5 m from its apex, as shown in the figure below. The incident rays are inclined by 1° from the paraboloid's optical axis; both rays and axis lie in the same plane. Where do the reflected rays intersect each other? Where do the reflected rays intersect the paraxial focal plane?



3. For the paraboloidal mirror of problem 2, use RayTrace 5.0, with a bullseye ray pattern, to plot spot diagrams for bundles of parallel rays incident

   a. parallel to the paraboloid's axis, and

   b. inclined at an angle of 1° to the paraboloid's axis.

   Compare these results to those obtained in class (Example 4.1, for the on-axis rays) and in problem 2.

Choose and work out only one of the following three problems. Share your results with your classmates as indicated.

4. *A telescope with coma.* Consider a Cassegrain telescope with a paraboloidal primary mirror with diameter 2 m and focal length 4 m. The secondary mirror is a hyperboloid with eccentricity 2.0, 0.5 m in diameter.

   a. Compute the radius of curvature of this mirror at its apex, and its position relative to the primary, if it produces a Cassegrain focus at the apex of the primary, with a plate scale 1/3 that of the prime focus.

   b. What is the Cassegrain plate scale, in arcsec mm^{-1}?

c. Using RayTrace 5.0, generate spot diagrams in the Cassegrain focal plane for ray bundles incident on the primary mirror. and use the RMS diameter of the spot pattern at the best focus (computed by the program as **`Sm_RMS`**) to identify the off-axis angle θ_{UD} that produces a distorted image equal in size to the atmospheric-turbulence-broadened image (the "seeing disk"), taken to be 1 arcsec in diameter. One usually calls $2\theta_{UD}$ the *unblurred field of view* of the telescope. Plot a spot diagram for the offset angle θ_{UD}.

d. Ask some of your classmates who did the other two problems to show you their results for $2\theta_{UD}$. Compare the performance of this telescope with the other two. Which one offers the widest unblurred field of view?

An example optical prescription called CASS.RAY is provided for your convenience in setting up this problem; from inside RayTrace 5.0, one can gain access to this prescription by use of the **`Fetch`** command.

5. *A telescope without coma.* Same as problem 4, but a Ritchey - Chretien telescope: same dimensions and apex curvatures as the Cassegrain but with primary eccentricity 1.0368 and secondary eccentricity 2.2389.

6. *A telescope with spherical aberration.* Same as problem 4, but with spherical mirrors (eccentricity = 0) having the same size and apex curvature, and replacing parts c and d with the questions below.

   a. What is the position of the best focus? (That is, where does the "circle of least confusion" lie along the optical axis, judging from the RMS diameter of the focal-plane spot diagram?) Why is it different from the Cassegrain and Ritchey - Chretien telescopes?

   b. Plot a spot diagram for the central focus (i.e. on-axis rays, DX = DY = 0). What is the angle on the sky that corresponds to the RMS diameter of this distorted image? Compare the performance of this telescope with the other two.