Astronomy 203 Problem Set #3

Due 12 October 1999

1. **(Cancelled! You need not do this problem.)** Derive the *third-order* part of Equation 7.20,

$$TSA3 = -\frac{1-n^2\varepsilon^2}{n^2}\kappa^2 y^3 \quad ,$$

for the transverse spherical aberration at the focus of a convex conic-section dielectric surface of index n , apex curvature κ and eccentricity ε, embedded in vacuum, and illuminated with on-axis rays, as in Figure 1.

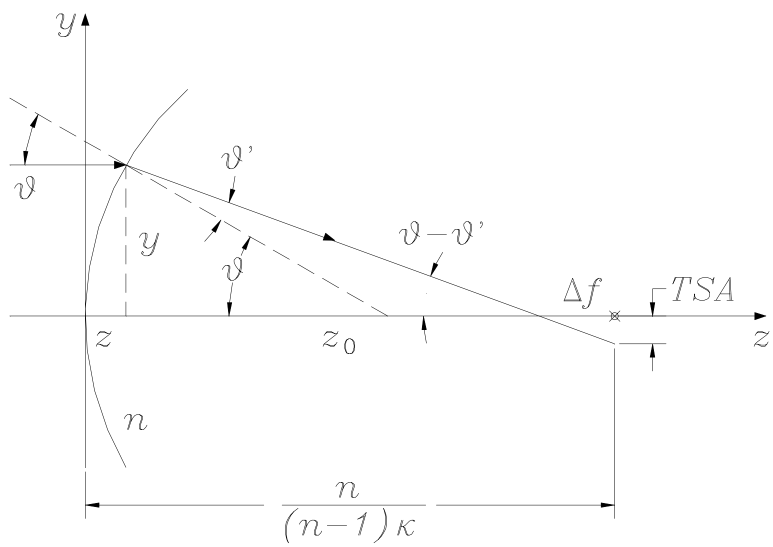

Figure 1: setup for Problem 1.

Hint: along with the binomial theorem, you will need to use the following power-series expansions:

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \ldots$$

$$\sin^{-1} x = x + \frac{x^3}{6} + \frac{3x^5}{40} + \ldots$$

$$\tan\theta = \theta + \frac{\theta^3}{3} + \frac{2\theta^5}{15} + \ldots$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} + \ldots$$

Hint II: use Mathcad (or Mathematica, if you prefer) to do the algebra and the expansions.

2. **(Solo)** Show that the position and diameter of the exit pupil and the plate scale of Ritchey-Chrétien telescopes, in terms of the radii of curvature and eccentricities of the mirrors, are given by

$$\ell = \frac{r_s L}{2\varepsilon_s} \frac{1}{L - \frac{r_s}{2\varepsilon_s}} \quad ,$$

$$d = \frac{r_s D}{2\varepsilon_s} \frac{1}{L - \frac{r_s}{2\varepsilon_s}} \quad ,$$

and

$$\frac{1}{f} = \frac{1}{f_p} \frac{f_1}{f_2} = \frac{2\varepsilon_p}{r_p} \frac{\varepsilon_s - 1}{\varepsilon_s + 1} \quad .$$

where D is the diameter of the primary, L is the separation of the apexes, and ℓ is the distance behind the secondary's apex to the exit pupil. Use the thin-lenses-in-contact model for hyperboloid mirrors; this will give results acceptable as a first approximation, but not exact results, for the plate scale of R-C telescopes.

3. **(Solo)** *What went wrong with the Hubble Space Telescope, and how they fixed it.* The Hubble Space Telescope is a Ritchey - Chrétien. Its nominal parameters are: diameter 2400 mm, apex radius of curvature 11040 mm, and eccentricity 1.00114859 for the primary mirror; diameter 266.88 mm, apex radius of curvature 1358 mm, and eccentricity 1.223110788 for the secondary. The apexes are separated by 4905.97 mm. Provision was made in the satellite to adjust the position of the secondary mirror along the telescope's axis, so that small errors in mirror parameters or detector placement could be corrected, and a sharp focus obtained. A detector is placed in the nominal position of this focus.

   a. Calculate the diameter and position of the exit pupil, and the plate scale. (See problem 2.)

   b. Using RayTrace 5.0, obtain an RMS width and spot diagram for the on-axis focus. Include the shadow of the secondary mirror in your prescription, and *measure* the plate scale for comparison to your calculated value, and to convert the RMS width into an angular width. What is the position of the best focus? (That's where the detector is.)

   c. Suppose an error were made in the fabrication of the primary mirror, and that its eccentricity turned out to be 1.006578, with all of the other dimensions of the telescope, *including the focal-plane position*, remaining the same. Using RayTrace 5.0, adjust the primary-secondary spacing and the back-focal distance, in such a way as to move the secondary mirror without moving the primary or the detector, to find the minimum RMS spot size on the detector. What is the angular size of the best focus? Plot a spot diagram for on-axis rays.

   d. What is the dominant aberration introduced through the eccentricity error? How would you try to correct the aberration without changing the primary or secondary?

4. **(Solo)** ***A s****ingle-element Gregorian spherical-aberration corrector.* You will do three ray traces in this problem; make sure they're all done with the same number of rays.

   a. Consider a 200 cm diameter, spherical mirror with focal length 400 cm and a 40 cm, circular hole in the center. (The role of the hole will be made clear below.) Use RayTrace, with parallel on-axis rays (far field, DX=DY=0), to plot a spot diagram and to compute the RMS spot size at the position of best focus. Calculate the plate scale for the mirror, and use this to find the angular spread on the sky corresponding to this blur.

b. Next, consider a 200 cm diameter *paraboloid* mirror with the same focal length and central-hole size as the sphere in part a. Use with this an ellipsoidal secondary mirror, 40 cm in diameter, with on-axis focal lengths 80 cm and 40 cm, but instead of using the usual Gregorian arrangement, place the *far* focus in coincidence with the paraboloid's focus, so that the final image is formed closer to the ellipsoid's apex than the prime-focus image, as shown in Figure 2. Calculate the apex radius of curvature and eccentricity this mirror must have, and the plate scale at the final focus. Now use RayTrace and on-axis rays to plot a spot diagram and to compute the RMS spot size at the position of best focus, and calculate angular size on the sky corresponding to this blur.

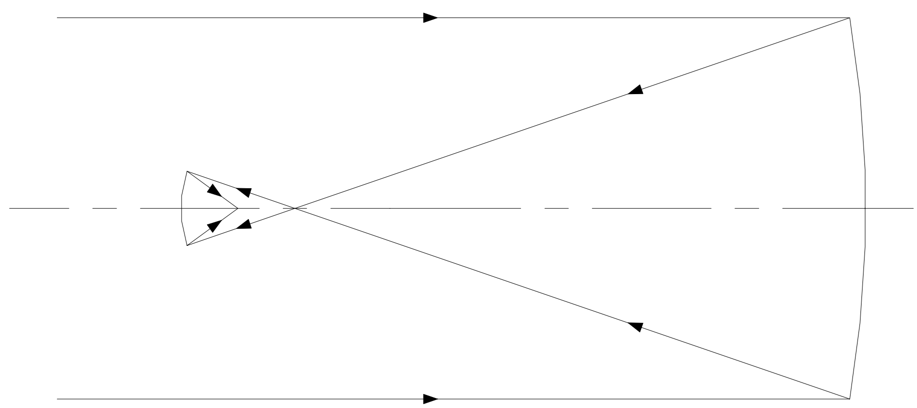

Figure 2: setup for Problem 4.

c. Now, bend the mirrors in the telescope from part b: leave the apex curvatures and distances fixed, but change the eccentricity of the primary to zero (thus transforming it into the spherical mirror of part a), and calculate the eccentricity the secondary must have in order that third-order spherical aberration is corrected. Use these new eccentricities in RayTrace to plot a spot diagram and compute an RMS spot size. Calculate the angular size on the sky corresponding to this blur.

d. By what factor has the blur decreased from part a to part c? Account for the secondary's magnification in your answer.

e. Why isn't the blur in the telescope of part c as small as that of the telescope in part b?