Astronomy 203 Problem Set #3: Solutions

12 October 1999

1. Because we have been unable to get Mathcad working on the computers in room 407, this problem has been removed from your assignment. However, you should read the following solution anyway.

   Derive an expression for the third-order transverse spherical aberration at the focus of a convex conic-section dielectric surface of index n , curvature κ and eccentricity ε, embedded in vacuum, and illuminated with on-axis rays.

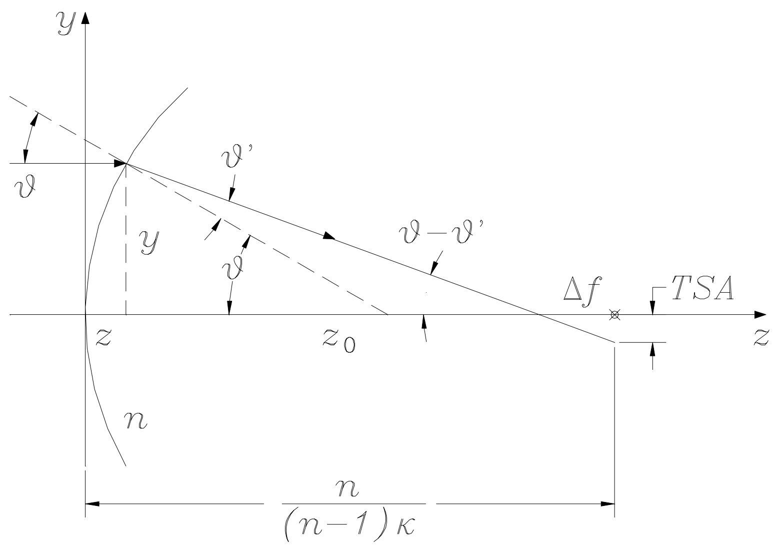


Figure 1: refraction of an on-axis marginal ray by a conic-section dielectric with refractive index n and apex curvature κ.

The setup is shown in Figure 1. Our approach will be similar to that used in the calculation in lecture of the spherical aberration of concave conic-section mirrors. First we note that the position of the point of incidence and the relation between the incidence angle θ and the shape of the surface is the same as given in the previous discussion (Lecture 7), in Equations 7.2 and 7.10:

$$\tan\theta = \frac{dz}{dy} \quad ; \tag{1}$$

$$z \cong \frac{\kappa y^2}{2} + \frac{\left(1-\varepsilon^2\right)\kappa^3 y^4}{8} \quad \text{(to fourth order in } y\text{)}. \tag{2}$$

The angle the refracted ray makes with the optical axis is $\theta-\theta'$, where $\theta' = \sin^{-1}\left[(\sin\theta)/n\right]$ is the angle of refraction; then the distance from the apex to the intersection of the ray and the axis is $z+z_0$, where

$$z_0 = \frac{y}{\tan(\theta-\theta')} \quad . \tag{3}$$

The distance from this intersection to the paraxial focal plane is $\Delta f = f_{\text{paraxial}} - (z + z_0)$, where the paraxial focal length is given by Equation 2.15,

$$f_{\text{paraxial}} = \frac{n}{(n-1)\kappa} \quad . \tag{4}$$

Finally, then, the transverse spherical aberration is given by $TSA = -\Delta f \tan(\theta - \theta')$; the leading minus sign in this expression comes from the intersection with the paraxial focal plane at negative y for positive values of Δf.

In order to obtain results in the appropriate order of approximation it is necessary to obtain expressions for $\tan(\theta - \theta')$ and $1/\tan(\theta - \theta')$ as power series in the off-center distance y. For this purpose we will need to use the following power-series expansions:

$$\begin{aligned}
&\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \ldots \\
&\sin^{-1} x = x + \frac{x^3}{6} + \frac{3x^5}{40} + \ldots \\
&\tan\theta = \theta + \frac{\theta^3}{3} + \frac{2\theta^5}{15} + \ldots \\
&\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} + \ldots \\
&(1+x)^{-1} = 1 - x + x^2 - x^3 + \ldots
\end{aligned} \tag{5}$$

I will assume that everyone used Mathcad to do the algebra (as I did), and give in the following only enough intermediate results for one to check the progress of one's Mathcad document. In Mathcad 6 the expansions can be done automatically by use of the "expand" command in the Symbolics menu; in Mathcad 7 and 8 the use of the "variable"+"expand to series" commands in the Symbolics menu is even easier. In general the use of "simplify" and "collect" (also on the Symbolics menu) after each expansion renders the result easier and facilitates the deletion of unwanted higher-order terms.

Now to the business at hand: starting with Snell's law,

$$\theta' = \sin^{-1}\left[(\sin\theta)/n\right] \quad , \tag{6}$$

we expand the sine, keeping terms through fifth order, and insert the result into the arcsine expansion, once again keeping terms through fifth order. I'll do this one slowly:

$$\begin{aligned}
\theta' &= \sin^{-1}\left[\frac{1}{n}\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!}\right)\right] \\
&= \left[\frac{1}{n}\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!}\right)\right] + \frac{1}{6}\left[\frac{1}{n}\left(\theta - \frac{\theta^3}{3!}\right)\right]^3 + \frac{3}{40}\left[\frac{1}{n}(\theta)\right]^5 \quad ,
\end{aligned}$$

or

$$\theta' = \frac{\theta}{n} + \left(1 - n^2\right)\frac{\theta^3}{6n^3} + \left(\frac{3}{40} - \frac{n^2}{12} + \frac{n^4}{120}\right)\frac{\theta^5}{n^5} \quad . \tag{7}$$

Thus

$$\tan(\theta-\theta')=\tan\left[\frac{n-1}{n}\theta-\left(1-n^2\right)\frac{\theta^3}{6n^3}-\left(\frac{3}{40}-\frac{n^2}{12}+\frac{n^4}{120}\right)\frac{\theta^5}{n^5}\right] \quad . \tag{8}$$

We expand this in its turn, again keeping through fifth order:

$$\tan(\theta-\theta')=(n-1)\frac{\theta}{n}+\left(-\frac{1}{2}+n-\frac{5}{6}n^2+\frac{1}{3}n^3\right)\frac{\theta^3}{n^3}+\left(-\frac{3}{8}+n-\frac{5}{4}n^2+n^3-\frac{61}{120}n^4+\frac{2}{15}n^5\right)\frac{\theta^5}{n^5} \quad . \tag{9}$$

The inverse of this expression is needed for use in Equation 3, which requires substitution of the right-hand side into the binomial expansion for the inverse, the last of Equations 5. Keeping through θ^4 terms, we get

$$\frac{1}{\tan(\theta-\theta')}=\frac{n}{(n-1)\theta}-\frac{\left(-\frac{1}{2}+n-\frac{5}{6}n^2+\frac{1}{3}n^3\right)}{(n-1)^2}\frac{\theta}{n}-\frac{\left(45-45n+15n^2-15n^3+8n\right)}{360(n-1)}\frac{\theta^3}{n^3} \quad . \tag{10}$$

Into these results, Equations 9 and 10, we must insert a power-series expression for θ in terms of *y*. From Equations 1 and 2, we get

$$\begin{aligned}\theta&=\tan^{-1}\left(\frac{dz}{dy}\right)=\tan^{-1}\left(\kappa y+\frac{\left(1-\varepsilon^2\right)\kappa^3y^3}{2}\right)\\&=\kappa y+\left(\frac{1-\varepsilon^2}{2}-\frac{1}{3}\right)\kappa^3y^3\end{aligned} \tag{11}$$

to third order in *y*, and using the binomial expansion of the inverse again, we get the corresponding expression for $1/\theta$:

$$\frac{1}{\theta}=\frac{1}{\kappa y}-\left(\frac{1-\varepsilon^2}{2}-\frac{1}{3}\right)\kappa y-\left(\frac{4}{45}-\frac{1-\varepsilon^2}{6}+\frac{\left(1-\varepsilon^2\right)^2}{8}\right)\kappa^3y^3 \quad . \tag{12}$$

We may now insert Equations 11 and 12 into Equation 10, and multiply it out, rejecting terms of higher order than *three* in *y*, which gives

$$\frac{1}{\tan(\theta-\theta')}=\frac{n}{(n-1)}\left(\frac{1}{\kappa y}-\left(\frac{1-\varepsilon^2}{2}-\frac{1}{3}\right)\kappa y-\left(\frac{4}{45}-\frac{1-\varepsilon^2}{6}+\frac{\left(1-\varepsilon^2\right)^2}{8}\right)\kappa^3 y^3\right)$$
$$-\frac{\left(-\frac{1}{2}+n-\frac{5}{6}n^2+\frac{1}{3}n^3\right)}{(n-1)^2}\frac{1}{n}\left(\kappa y+\left(\frac{1-\varepsilon^2}{2}-\frac{1}{3}\right)\kappa^3 y^3\right)$$
$$-\frac{\left(45-45n+15n^2-15n^3+8n\right)}{360(n-1)}\frac{(\kappa y)^3}{n^3} \quad ,$$

or

$$\begin{aligned}\frac{1}{\tan(\theta-\theta')}=&\frac{n}{(n-1)\kappa y}-\frac{1}{2}\frac{\left(1-n+n^2\left(1-\varepsilon^2\right)\right)}{n(n-1)}\kappa y\\&-\frac{1}{8}\frac{\left(1-n+\left(1-2\varepsilon^2\right)n^2-\left(1-2\varepsilon^2\right)n^3+\left(1-\varepsilon^2\right)^2 n^4\right)}{n^3(n-1)}\kappa^3 y^3 \quad .\end{aligned} \tag{13}$$

Equation 11 may also be inserted into Equation 9, with the result

$$\tan(\theta-\theta')=(n-1)\frac{\kappa y}{n}-\frac{1}{2}\left(1-2n+\left(2-\varepsilon^2\right)n^2-\left(1-\varepsilon^2\right)n^3\right)\frac{\kappa^3 y^3}{n^3} \quad , \tag{14}$$

also to third order in y.

Now we are finally ready to do the calculation. Let's start with

$$\Delta f=\frac{n}{(n-1)\kappa}-\left(z+\frac{y}{\tan(\theta-\theta')}\right) \quad , \tag{15}$$

and insert Equations 13 and 2:

$$\begin{aligned}\Delta f=&\frac{n}{(n-1)\kappa}-\left[\frac{\kappa y^2}{2}+\frac{\left(1-\varepsilon^2\right)\kappa^3 y^4}{8}+y\left(\frac{n}{(n-1)\kappa y}-\frac{1}{2}\frac{\left(1-n+n^2\left(1-\varepsilon^2\right)\right)}{n(n-1)}\kappa y\right.\right.\\&\left.\left.-\frac{1}{8}\frac{\left(1-n+\left(1-2\varepsilon^2\right)n^2-\left(1-2\varepsilon^2\right)n^3+\left(1-\varepsilon^2\right)^2 n^4\right)}{n^3(n-1)}\kappa^3 y^3\right)\right]\\=&\frac{1}{2}\frac{\left(1-n^2\varepsilon^2\right)}{n(n-1)}\kappa y^2+\frac{1}{8}\frac{\left(1-n+\left(1-2\varepsilon^2\right)n^2+\varepsilon^2 n^3+\left(\left(1-\varepsilon^2\right)^2+\left(1-\varepsilon^2\right)\right)n^4\right)}{n^3(n-1)}\kappa^3 y^4 \quad .\end{aligned} \tag{16}$$

Note that this expression is positive for $\varepsilon=0$, which means that the marginal focus lies closer to the apex than the paraxial focus; convex spherical lens surfaces have negative SA, just as concave spherical mirrors do.

At long last, we are ready to invoke

$$TSA = -\Delta f \tan(\theta - \theta') \quad . \tag{17}$$

We have only been asked for the third-order spherical aberration, which we can get simply by multiplying the leading term of Equation 16 by the leading term of Equation 14:

$$TSA3 = -\frac{1}{2}\frac{\left(1 - n^2\varepsilon^2\right)}{n^2}\kappa^2 y^3 \quad . \tag{18}$$

Several remarks are in order concerning this simple result.

- Substitution of $n = -1$ yields the result for the concave spherical mirror, which is nice but a bit accidental. (So does $n = 1$, which you may find confusing; just note that this gives $\theta = \theta'$, so that the ray never intersects the axis.)
- A paraboloid surface does not lead to zero spherical aberration in this case, as it does for concave mirrors. Instead the zero-SA3 surface is an *ellipsoid*, with $\varepsilon = 1/n$.
- *TSA*3 decreases rapidly as *n* increases; thus SA may be reduced simply by making the lens out of glass with higher index. In turn this leads to more light reflected from the surface of the lens, which may be mitigated by applying antireflection coatings to the lens surface. (We will discuss AR coatings later on in this course.)

I intentionally kept more terms in the expansions than were necessary for *TSA*3. The next order, *TSA*5, may be generated by addition of the products of the leading terms of Equations 14 and 16 with the next terms of the other. I know you're just dying to know the answer:

$$TSA5 = -\frac{3}{8}\frac{\left(n^4\left(\varepsilon^4 - \varepsilon^2\right) + n^3\varepsilon^2 + n^2\left(1 - 2\varepsilon^2\right) - n + 1\right)}{n^4}\kappa^4 y^5 \quad . \tag{19}$$

This, too, vanishes for the ellipsoidal surface $\varepsilon = 1/n$.

2. *Show that the position and diameter of the exit pupil and the plate scale of Ritchey-Chrétien telescopes, in terms of the radii of curvature and eccentricities of the mirrors, are given by*

$$\ell = \frac{r_s L}{2\varepsilon_s}\frac{1}{L - \dfrac{r_s}{2\varepsilon_s}} \quad ,$$

$$d = \frac{r_s D}{2\varepsilon_s}\frac{1}{L - \dfrac{r_s}{2\varepsilon_s}} \quad ,$$

and

$$\frac{1}{f} = \frac{1}{f_p}\frac{f_1}{f_2} = \frac{2\varepsilon_p}{r_p}\frac{\varepsilon_s - 1}{\varepsilon_s + 1} \quad ,$$

where D is the diameter of the primary, L is the separation of the apexes, and ℓ is the distance behind the secondary's apex to the exit pupil. Use the thin-lenses-in-contact model for hyperboloid mirrors; this will give results acceptable as a first approximation, but not exact results, for the plate scale of R-C telescopes.

For the hyperboloidal mirrors, the focal lengths are given by the usual relationships in terms of a, c, and the apex radius of curvature r and the eccentricity ε:

$$\begin{aligned} f &= c \pm a \\ r &= \frac{b^2}{a} = \frac{c^2 - a^2}{a} \\ \varepsilon &= \frac{c}{a} \end{aligned} \quad , \tag{20}$$

whence

$$\begin{aligned} f &= \frac{r}{\varepsilon \mp 1} \\ f_{\mathrm{eff}} &= \frac{f_1 f_2}{f_1 + f_2} = \frac{r}{2\varepsilon} \end{aligned} \quad . \tag{21}$$

The ratio of the two focal lengths (short/long) is

$$\frac{f_1}{f_2} = \frac{\varepsilon - 1}{\varepsilon + 1} \quad . \tag{22}$$

Now we apply this to the telescopes. Taking the diameter of the primary to be D and the separation of the vertices to be L, we get the distance ℓ behind the secondary's apex to the exit pupil simply by using the primary as the object:

$$\frac{1}{L} + \frac{1}{\ell} = \frac{1}{f_{\mathrm{eff}}} \qquad \Rightarrow \qquad \ell = \frac{L f_{\mathrm{eff}}}{L - f_{\mathrm{eff}}} = \frac{r_{\mathrm{s}} L}{2\varepsilon_{\mathrm{s}}} \frac{1}{L - \frac{r_{\mathrm{s}}}{2\varepsilon_{\mathrm{s}}}} \quad , \tag{23}$$

where the subscript "s" is used to refer to properties of the secondary mirror. Here the exit pupil is virtual, because the secondary is convex. (A Gregorian telescope, on the other hand, would have a real exit pupil.) The diameter of the exit pupil is

$$d = \frac{D\ell}{L} = \frac{r_{\mathrm{s}} D}{2\varepsilon_{\mathrm{s}}} \frac{1}{L - \frac{r_{\mathrm{s}}}{2\varepsilon_{\mathrm{s}}}} \quad . \tag{24}$$

In terms of the primary mirror focal length f_{p} and the secondary focal lengths f_1 and f_2, the plate scale is

$$\frac{1}{f} = \frac{1}{f_p} \frac{f_1}{f_2} = \frac{2\varepsilon_{\mathrm{p}}}{r_{\mathrm{p}}} \frac{\varepsilon_{\mathrm{s}} - 1}{\varepsilon_{\mathrm{s}} + 1} \quad . \tag{25}$$

The value of the plate scale is approximate, since we assumed the secondary's short focus to be coincident with the effective focus of the primary.

3. What went wrong with the Hubble Space Telescope, and how they fixed it. *The Hubble Space Telescope is a Ritchey - Chrétien. Its nominal parameters are: diameter 2400 mm, apex radius of curvature 11040 mm, and eccentricity 1.00114859 for the primary mirror; diameter 266.88 mm, apex radius of curvature 1358 mm, and eccentricity 1.223110788 for the secondary. The apexes are separated by 4905.97 mm. Provision was made in the satellite to adjust the position of the secondary mirror along the telescope's axis, so that small errors in mirror parameters or detector placement could be corrected, and a sharp focus obtained. A detector is placed in the nominal position of this focus.*

   a. *Calculate the diameter and position of the exit pupil, and the plate scale. (See problem 2.)*

      Plugging into the formulas above, one obtains d = 306.23 mm, ℓ = 625.98 mm (i.e. 5531.95 mm from the primary's apex), and $1/f = 1.8\times10^{-5}$ rad mm^{-1} = 3.75 arcsec mm^{-1}.

   b. *Using RayTrace 5.0, obtain an RMS width and spot diagram for the on-axis focus. Include the shadow of the secondary mirror in your prescription, and measure the plate scale for comparison to your calculated value, and to convert the RMS width into an angular width. What is the position of the best focus? (That's where the detector is.)*

      All lengths will be entered in millimeters. Three surfaces precede the focal plane. In order: the primary mirror, with **`RA = -11040, AP = 2400, EC = 1.00114859, FI = -1, FT = 0`**; the secondary shadow, with **`RA = 0, AP = 2400, EC = 0, FI = -1, FT = -4905.97, SS = CO, S1 = 266.88`**; and the secondary, with **`RA = -1358, AP = 266.88, EC = 1.223110788, FI = 1`**. You can leave **`FT = 0`** if you want your first **`SD`** to work out the position of best focus for you. (In this case, make the focal plane's **`AP`** big enough that some rays get through during the first trace. After that, the default value of **`AP = 1`** is more than sufficient.) The best on-axis focus lies at **`BF = 6417.21`**, or 1511.24 mm behind the primary mirror. For **`DX = 0.027778`** (100 arcsec), the central ray winds up at **`x = 27.969`** mm, so the plate scale is actually PS = 3.575 arcsec mm^{-1}. Tracing 1000 rays, one obtains a focal plane spot with **`RMS = 0.000035`** mm, corresponding to an angular size of $PS\times RMS$ = 0.125 milliarcsec. A spot diagram is shown in Figure 2.

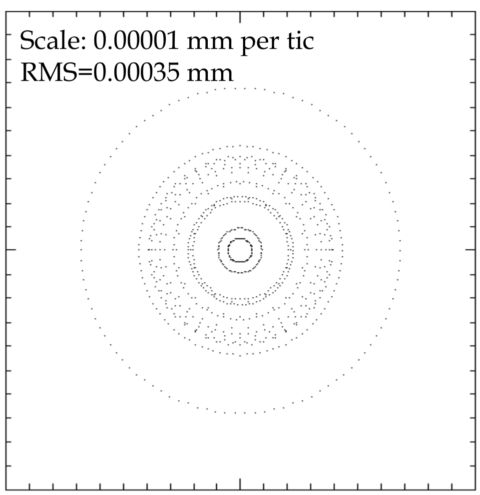


Figure 2: On-axis spot diagram for the "ideal" Hubble Space Telescope. The plot scale is 0.00001 mm per tic, and 1000 rays were traced. The RMS diameter corresponds to an angular diameter on the sky of 0.000125 arcseconds.

   c. *Suppose an error were made in the fabrication of the primary mirror, and that its eccentricity turned out to be 1.006578, with all of the other dimensions of the telescope,* including the focal-plane position, *remaining the same. Using RayTrace 5.0, adjust the primary-secondary spacing and the back-focal distance, in such a way as to move the secondary mirror without moving the primary or the detector, to find the minimum RMS spot size on the detector. What is the angular size of the best focus? Plot a spot diagram for on-axis rays.*

To simulate adjustment of the position of the secondary, one must change the relevant **`FT`**s by equal amounts, so that the *sum* of **`FT`** for the primary (secondary shadow) and for the secondary stays the same as it was for the "ideal" HST, -4905.97 mm + 6417.20648 mm = 1511.23648 mm. An increment of 0.1 mm turned out to be about right for finding the minimum RMS diameter. Table 1 is a display of the focus range and RMS spot diameters obtained from 200 rays, and Figure 3 is a plot of spot diameter against secondary position.

Table 1

Apex separation (mm)	Back focal distance (mm)	RMS spot diameter (mm)
-4905.77	6417.00648	0.600942
-4905.87	6417.10648	0.455675
-4905.97	6417.20648	0.313105
-4906.07	6417.30648	0.179814
-4906.17	6417.40648	0.102594
-4906.27	6417.50648	0.180614
-4906.37	6417.60648	0.314045
-4906.47	6417.70648	0.456661
-4906.57	6417.80648	0.601972

The minimum of the curve in Figure 3 lies at **`FT = -4906.17`** for the primary mirror (secondary shadow), and **`FT = 6417.40648`** for the secondary mirror. There, the RMS spot diameter is 0.102594 mm; the corresponding spot diagram is shown in Figure 4, along with the "ideal" HST on-axis spot diagram from Figure 2, replotted at the same scale. The plate scale in the new focus should not be very different from the ideal case, since the secondary was moved only by a fraction of a millimeter, and indeed a measurement gives a value of 3.575 arcsec mm^{-1}. Thus the angular diameter of the focal-plane spot is $\Delta\theta$ = 0.367 arcseconds, little better than can be done routinely at ground-based observatories, and nowhere near as good as it was supposed to be.

Something like this did, of course, happen to the HST: the primary mirror was made with an incorrect eccentricity. The result of our calculation is in pretty good agreement with the image quality that HST achieved before the repair mission. See the official HST Recovery report, on line, at http://www.stsci.edu/ftp/ExInEd/electronic_reports_folder/recovery.pdf for stellar images taken with the flawed HST, that can be compared with the spot diagram in Figure 4.

d. *What is the dominant aberration introduced through the eccentricity error? How would you try to correct the aberration without changing the primary or secondary?*

Since it shows up on axis, it must be spherical aberration. The easiest way to correct it would be to include lenses or mirrors that possess SA with the opposite sign. Since the primary is a hyperboloid mirror (positive SA), concave spherical or ellipsoidal mirrors or convex spherical lenses would contribute SA in the correct sense.

In the end the HST's original wide-field/planetary camera (WFPC) was replaced by a new version (WFPC2) that contains it own SA-correcting optics. All new instruments installed since then (e.g. NICMOS, STIS) similarly have their own correctors. For the instruments that were not changed out (FOC, FOS) a common set of corrective optics called COSTAR was installed. After putting in WFPC2 and COSTAR the HST finally achieved its design performance.

4. A single-element Gregorian spherical-aberration corrector. *You will do three ray traces in this problem; make sure they're all done with the same number of rays.*

I'll use 1000 rays in a bullseye pattern throughout.

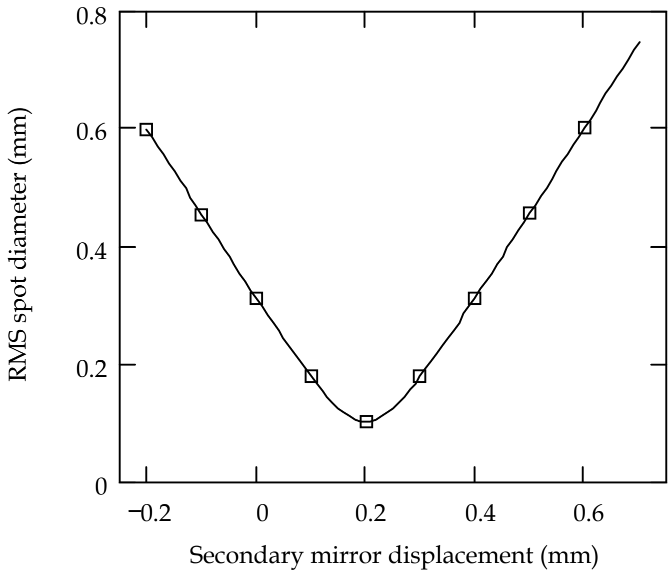
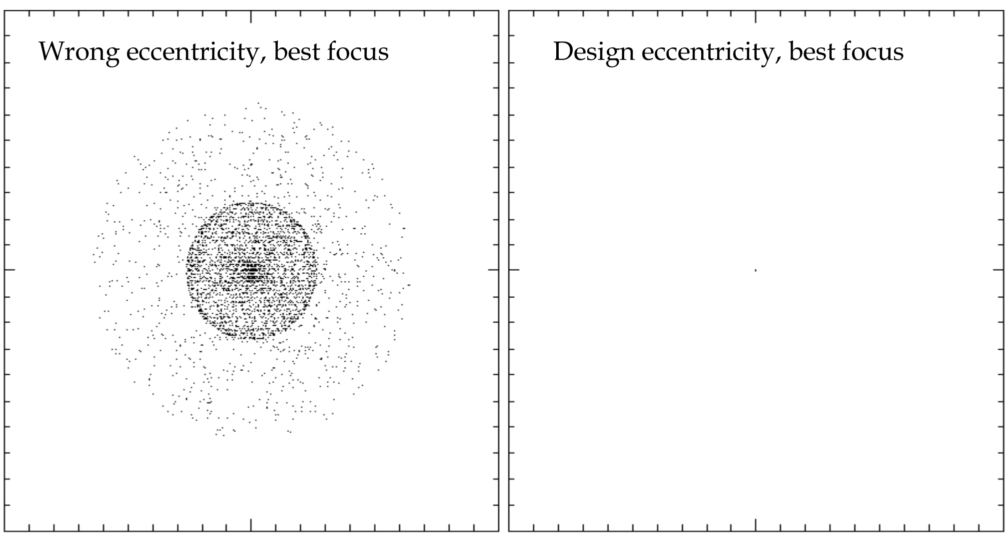


Figure 3: RMS spot diameter as a function of secondary mirror position displacement (back focal distance – 6417.20648 mm) for the Hubble Space Telescope, computed under the assumptions that the primary mirror and focal plane are fixed, and that the eccentricity of the primary mirror is 1.006578, rather than the design value. The boxes mark the results from Table 1, and the smooth curve through them is a cubic spline interpolation.



Scale: 0.05 mm per tic

Figure 4: on-axis spot diagrams for the "real" and "ideal" Hubble Space Telescope, each plotted at a scale of 0.05 mm per tic and representing 4000 rays traced in a random pattern. The RMS diameters correspond to angular diameters on the sky of 0.367 and 0.000125 arcseconds, respectively. The random ray pattern was chosen in order to compare the spot diagram directly with HST images of stars. Note that the image on the left is composed of a small dense core of spots, surrounded by a “plateau” containing most of the spots, which in turn is surrounded by a fainter “halo.”

a. *Consider a 200 cm diameter, spherical mirror with focal length 400 cm and a 40 cm, circular hole in the center. (The role of the hole will be made clear below.) Use RayTrace, with parallel on-axis rays (far field, DX=DY=0), to plot a spot diagram and to compute the RMS spot size at the position of best focus. Calculate the plate scale for the mirror, and use this to find the angular spread on the sky corresponding to this blur.*

In millimeters, that's **`AP=2000, RA=-8000, EC=0, FI=-1`** and **`FT=0`** for the mirror, and **`AP=2000, RA=0, EC=0, FI=-1`**, **`SS=CO`**, **`S1=400`** and **`FT=-4000`** for the central obscuration. We also set **`AP=10`** for the focal plane, to make sure we catch all the rays from the mirror. Then we run a ray trace, and get a best-focus back focal distance of –3980.49039 mm, with an RMS of 1.197803 mm. The plate scale of the mirror is $PS = 1/f = 1/3980.5 \text{ rad mm}^{-1} = 51.8 \text{ arcsec mm}^{-1}$. Thus the angular spread on the sky corresponding to the blurry image is *PS*×*RMS* = 62.1 arcsec. A plot of the spots appears in Figure 6.

b. *Next, consider a 200 cm diameter* paraboloid *mirror with the same focal length and central-hole size as the sphere in part a. Use with this an ellipsoidal secondary mirror, 40 cm in diameter, with on-axis focal lengths 80 cm and 40 cm, but instead of using the usual Gregorian arrangement, place the* far *focus in coincidence with the paraboloid's focus, so that the final image is formed closer to the ellipsoid's apex than the prime-focus image, as shown in Figure 5. Calculate the apex radius of curvature and eccentricity this mirror must have, and the plate scale at the final focus. Now use RayTrace and on-axis rays to plot a spot diagram and to compute the RMS spot size at the position of best focus, and calculate the angular size on the sky corresponding to this blur.*

First we need to calculate the parameters of the secondary. The secondary magnification and secondary/primary diameter ratio are $m = -(f_2/f_1) = -0.5$ and $k = -(40/200) = -0.2$ (note the minus signs, from the Gregorian's sign convention), whence the ratio of apex curvatures is

$$\rho = \frac{mk}{m-1} = -\frac{1}{15} \quad . \tag{26}$$

Thus for RayTrace the apex curvature of the secondary is $r_1 = \rho r_0 = +533.333 \text{ mm}$. Now we need to solve the two-mirror telescope equation (9.12),

$$1 - \varepsilon_0^2 = \frac{k^4}{\rho^3}\left[\left(\frac{m+1}{m-1}\right)^2 - \varepsilon_1^2\right] \quad , \tag{27}$$

for the secondary eccentricity ε_1:

$$\varepsilon_1 = \sqrt{\left(\frac{m+1}{m-1}\right)^2 - \frac{\rho^3}{k^4}\left(1 - \varepsilon_0^2\right)} \quad , \tag{28}$$

and to insert the parameters from above. This gives $\varepsilon_1 = 0.3333333$ for the secondary – suitably ellipsoidal. The plate scale at the Gregorian focus is

$$PS = \frac{1}{f_0}\frac{f_1}{f_2} = \frac{1}{4000 \text{ mm}}\frac{80}{40} = 5\times 10^{-4} \text{ rad mm}^{-1} = 103.1 \text{ arcsec mm}^{-1} \quad . \tag{29}$$

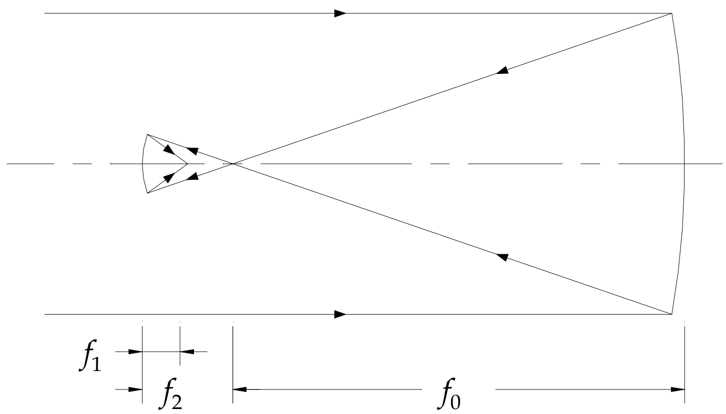


Figure 5: setup for Problem 4.

The secondary replaces the focal plane in the prescription in part a: **`AP=400, RA=53.33333, EC=0.3333333, FI=1`** and **`FT=400`**. We also change the primary's eccentricity to 1, and change the thickness following the central obscuration to the sum of the secondary's long focal length and the primary's focal length, or **`FT=-4800`**. Then we run the trace, and find to nobody's surprise that RMS = 0. There are no on axis aberrations in a Gregorian telescope. This rather boring spot diagram appears in Figure 6.

c. *Now, bend the mirrors in the telescope from part b: leave the apex curvatures and distances fixed, but change the eccentricity of the primary to zero (thus transforming it into the spherical mirror of part a), and calculate the eccentricity the secondary must have in order that third-order spherical aberration is corrected. Use these new eccentricities in RayTrace to plot a spot diagram and compute an RMS spot size. Calculate the angular size on the sky corresponding to this blur.*

The new secondary eccentricity is

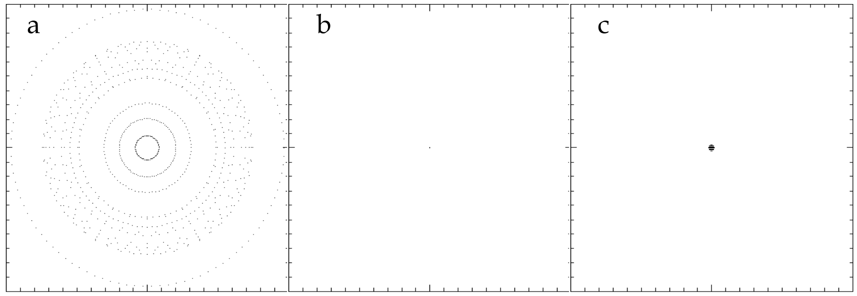
$$\varepsilon_1 = \sqrt{\left(\frac{m+1}{m-1}\right)^2 - \frac{\rho^3}{k^4}\left(1-\varepsilon_0^2\right)} = 0.5443311 \quad . \tag{30}$$

In RayTrace we insert the new primary and secondary eccentricities and run a 100-ray trace to obtain *RMS* = 0.018087 mm. Since none of the distances or paraxial parameters have changed, the plate scale of the modified telescope is the same as that of the classical Gregorian of part c. Thus the blur corresponds to *PS*×*RMS* = 1.86 arcsec. Again, the spot diagram can be seen in Figure 6.

d. *By what factor has the blur decreased from part a to part c? Account for the secondary's magnification in your answer.*

What counts, of course, is the blur on the sky, which has decreased by a factor of 33. The blur in the focal plane has decreased by a factor of 66, but a factor of two of that is due to the "minification" introduced by the secondary.

e. *Why isn't the blur in the telescope of part c as small as that of the telescope in part b?*



Scale: 0.2 mm per tic

Figure 6: focal-plane spot diagrams, with 1000 rays in a bullseye pattern, for the telescopes in Problem 4, parts a-c as indicated. The plots are all on the same scale.

Our procedure only corrects third-order spherical aberration. SA of fifth and higher order remains, and comprises the blur we see. The corrector helps a lot, though.

