Astronomy 203 Problem Set #5

Due 2 November 1999, in class

1. Using RayTrace 5.0 and the prescriptions developed in Problem Set #3, problem 3, generate spot diagrams, on the same scale, for on-axis rays in the focal plane of the Hubble Space Telescope, as designed and as originally realized. (Or use the plots you generated in that problem set…) On each of these plots, draw circles to indicate the FWHM diameter of the central diffraction maximum, and the diameter of the first dark ring, for a wavelength of λ = 0.55 μm. Comment on the effect the errors in the primary mirror must have had on early observations with HST.

2. *Gaussian beams stay Gaussian as they propagate*. Show that a Gaussian near-field distribution with $1/e$ radius ρ_N,

$$E_N(x',y') = E_0 e^{-i\omega t} \exp\left(-\frac{x'^2 + y'^2}{\rho_N^2}\right) \ ,$$

gives rise to a Gaussian far-field distribution,

$$E_F(x,y,z) = \frac{\pi \rho_N^2 E_0}{\lambda z} e^{i(\kappa z - \omega t)} \exp\left(-\frac{\pi^2 \rho_N^2 \left(x^2 + y^2\right)}{\lambda^2 z^2}\right) \ .$$

that has $1/e$ radius $\lambda z/\pi\rho_N$.

Hint: complete the square in the exponent that you find under the first integral you obtain, before any substitutions are attempted.

3. Most telescope primary mirrors have central obscurations, in addition to being circular, so their diffraction patterns differ somewhat from Equations 13.14-15.

   a. Derive an expression the far-field intensity as a function of $\kappa a\theta$ for an *annular* aperture, with outer half-diameter a and inner half diameter ka ($k < 1$).

   b. Plot the intensity divided by peak intensity, $I(\kappa a\theta, k)/I(0)$, against $\kappa a\theta$, on the same plot with $I(\kappa a\theta)/I(0)$ for the filled circular aperture, as given by Equations 13.14-15, for k = 0.2 – a rather typical value for telescopes, and used in our Cassegrain telescope example in §9.2 – and a more extreme value, k = 0.9. What are the major differences between the diffraction patterns of filled circular apertures and annular apertures?

4. Show that a pair of thin lenses separated by the sum of their focal lengths, with a waist of the input beam at the focal point of one of the lenses, produces an output beam whose waist location is independent of wavelength; then, derive the ratio of the output and input beam waist diameters. This arrangement is usually called a "Gaussian beam telescope," and is useful in radio-astronomy applications in which a large frequency or wavelength range must be used.