

## Astronomy 203 Problem Set #6

Due 16 November 1999

1. A cryogenic spectrometer exposes a far-infrared detector that has resistance  $5 \times 10^6 \Omega$ , quantum efficiency  $\eta = 1$ , and photoconductive gain  $G = 1$ , to radiation from a 300 K blackbody, in a diffraction-limited beam and a relative bandwidth of  $\Delta\nu/\nu = 1/100$  at a wavelength of 100  $\mu\text{m}$ . The detector's temperature is 4.2 K. Calculate the signal current through the detector from the blackbody radiation, the RMS shot noise current per square root bandwidth, and the RMS current per square root bandwidth due to Johnson noise. What is the largest source of noise?
2. *Noise in a random walk.* A friend of yours has just left your party, in a world-record state of drunkenness. She can't drive home, because you've wisely hidden her car keys, but her walking suffers from the following restriction: to hold herself up she leans her back against the wall of the building, so all she can do is stagger along in one dimension. Every lurching step she takes is one meter long. The directions of her steps are of course completely random; suppose the probability of any given step being in her right-hand direction is  $q$ . You wish to estimate where she's likely to be as a function of how many steps she's taken.

- a. Argue that the probability that she has travelled  $N$  steps to the right out of  $n$  total steps is governed by the binomial distribution,

$$p_n(N) = \frac{n!}{N!(n-N)!} q^N (1-q)^{n-N} .$$

- b. Derive an expression for her average distance from your door after  $n$  total steps. *Hint:* You will probably want to use the binomial theorem in this form:

$$(x+y)^k = \sum_{j=0}^k \frac{k!}{j!(k-j)!} x^j y^{k-j} .$$

- c. Derive an expression for the RMS deviation of her distance from your door, after  $n$  total steps.
- d. Suppose she is equally likely to step to the left or the right. How far from your door is she likely to be after 120 steps? (Report this in the form  $x \pm \Delta x$ .) What is the probability that she'll be falling through your neighbor's open door, 40 meters to her right of yours, after 120 steps? *Hint:* remember Stirling's approximation for the factorial of a large number  $n$ :

$$\ln(n!) \approx n \ln n - n + \frac{1}{2} \ln(2\pi n) .$$

- e. Suppose now that the weight of her backpack, slung over her right shoulder, leans her over enough that she is twice as likely to take a step to the right as to the left. Now how far from your door is she likely to be after 120 steps? (Report it in the form  $x \pm \Delta x$ .) What is the probability now that she'll fall through your neighbor's door on her 120<sup>th</sup> step (if she hasn't already)?