Astronomy 203 Problem Set #7

Due 2 December 1999

1. *Gain and gain dispersion in photoconductors.* In photoconductors, the photoconductive gain g is in general different for each photo-generated carrier, and is given by $g = t / t_0$, where t is the length of time the carrier lives before recombining, and t_0 is the *transit time*, the time it takes an unimpeded carrier to travel all the way through the detector. Like most decay times (e.g. radioactive decay), t is distributed exponentially, in the sense that the probability that a carrier lifetime lies between t and $t + dt$, $p(t)dt$, is given by

$$p(t)\,dt \propto e^{-t/t_B}\,dt \quad , \tag{1}$$

   where t_B is called the *mean lifetime* of the carriers.

   a. Normalize $p(t)$; that is, find the proportionality constant in equation 1 that makes the integral of $p(\tau)d\tau$ equal to unity.

   b. Show that the average value of g is $G = \overline{g} = t_B / t_0$; that is, that t_B really is the mean carrier lifetime.

   c. Show that the gain dispersion, $\beta = \overline{g^2} / \overline{g}^2$, is exactly equal to 2.

2. *Coherent detection* vs. *incoherent detection*. Suppose you have five detectors, working at wavelengths 1 cm, 1 mm, 100 μm, and 10 μm, that are capable of quantum-limited heterodyne detection or background-limited direct detection in a diffraction-limited beam and in relative bandwidth $\Delta\nu/\nu = 10^{-4}$. (Never mind whether or not this is *possible*!) Suppose that the product of cold optics transmission and quantum efficiency is 0.2, and that $G = \beta = 1$ in each case. You use each mode of each detector to observe a 1 arcsec diameter, 1000 K object, with a 1 m diameter room temperature (300 K) Cassegrain telescope that has 20% of the primary's aperture blocked by the secondary (that is, its emissivity is 0.2). You observe in each case until a signal-to-noise ratio of 10 is achieved. How long does each of the eight measurements take? Plot the elapsed exposure time, as a function of wavelength. (You'll find it most convenient to display these results on a log-log plot.) Over what range of wavelengths is direct detection significantly more sensitive than heterodyne detection?

3. *Background-limited spectrometers.* You are given a collection of spectrometers that work in the 5-40 μm range with a spectral resolution of $\Delta\nu/\nu = 1/1000$ and (cold) instrumental transmission $\tau = 0.15$, and with incoherent detectors that have quantum efficiency 0.3 and $G = \beta = 1$. You can use them either on the airborne Stratospheric Observatory for Infrared Astronomy (SOFIA) or the satellite-borne Space Infrared Telescope Facility (SIRTF). SOFIA is an ambient-temperature telescope (270 K), 2.5 meters in diameter, with 10% of its area obscured by the secondary. SIRTF is cryogenic (< 5 K), and is in space, so the only source of background is the zodiacal light. The zodiacal light can be thought of as a superposition of two low-emissivity blackbodies, one with $\varepsilon = 3\times10^{-8}$ and temperature 285 K, and the other with $\varepsilon = 2\times10^{-7}$ and temperature 200 K, for this range of wavelengths. SIRTF's primary mirror is 85 cm in diameter. Diffraction-limited beams are used. Compare the NEP of these spectrometers used for observation of point objects on SOFIA and SIRTF, at wavelengths 5, 10, 20 and 40 μm.

4. *The characteristic matrix and energy conservation.*

   a. Show that the determinant of the characteristic matrix is unity.

   b. Show that $|r|^2 + Y_{p+1}|t|^2 / Y_0 = 1$, where the amplitude reflection and transmission coefficients *r* and *t* are given by Equations 25.28 and 25.29.

5. *Antireflection coatings.*

   a. Calculate and plot the transmission of a 5 μm thick wafer of diamond (*n* = 2.4) over the visible wavelength range, $\lambda = 0.35 - 0.7\ \mu\mathrm{m}$.

   b. Repeat the calculation and plot with a layer of $\mathrm{MgF_2}$ (*n* = 1.38) on each face, with thickness equal to a quarter of a wavelength for incident light at $\lambda = 0.5\ \mu\mathrm{m}$.

6. *A pressure-scanned Fabry-Perot interferometer.* At normal atmospheric pressure and room temperature, the index of refraction of CO_2 is 1.0045. At the same temperature and a pressure of 4 atmospheres, the index is 1.0180. For constant temperature, the index varies linearly with pressure between these extremes. Using a few complete sentences, suggest a way of using this effect to tune a Fabry-Perot interferometer. For a pressure varying between 1-4 atmospheres, and an interferometer with spacing 0.3 cm and finesse *Q* = 30, operating at a wavelength 0.55 μm, what range of wavelengths is covered by the scan? Which Fabry-Perot order is used? How many FWHM resolution elements are contained in the scan? (Many high-resolution, visible-wavelength Fabry-Perot spectrometers employ this principle.)