Astronomy 203 Problem Set #8

Due 10 December 1999

1. *A pressure-scanned Fabry-Perot interferometer.* At normal atmospheric pressure and room temperature, the index of refraction of CO_2 is 1.0045. At the same temperature and a pressure of 4 atmospheres, the index is 1.0180. For constant temperature, the index varies linearly with pressure between these extremes. Using a few complete sentences, suggest a way of using this effect to tune a Fabry-Perot interferometer. For a pressure varying between 1-4 atmospheres, and an interferometer with spacing 0.3 cm and finesse $Q = 30$, operating at a wavelength 0.55 μm, what range of wavelengths is covered by the scan? Which Fabry-Perot order is used? How many FWHM resolution elements are contained in the scan? (Many high-resolution, visible-wavelength Fabry-Perot spectrometers employ this principle.)

2. *Beam size and spectral resolution of a Fabry-Perot.* An incoherent detector looks through a Fabry-Perot interferometer at normal incidence, with a beam of small angular radius θ.

   a. Show that the detector is therefore sensitive to a range of wavelengths, varying from $\lambda = 2nd/m$ to $\lambda' = 2nd(1 - \theta^2/2)/m$, and that a wavelength resolution element can therefore be no smaller than

   $$\Delta\lambda = \lambda\frac{\theta^2}{2} \quad .$$

   We will refer to this result as the beam-divergence limit to the spectral resolution of a Fabry-Perot interferometer.

   b. Suppose you wanted to have the beam be 0.1 radian (5.7°) in radius. For a Fabry-Perot with a finesse of 20, what is the highest order number you can use before the beam-divergence limitation on the spectral resolution is equal to the reflectance-limited resolution?

   c. Suppose further that you really need better spectral resolution than that. Suggest an optical configuration for the Fabry-Perot that will overcome the beam-divergence limitation.

3. *Diffraction grating measurements of the sodium* D*-lines* (λ = 0.58959, 0.58900 μm).

   a. A diffraction grating has 10^4 rulings uniformly spaced over 2.5 cm. It is illuminated by yellow light from a low-pressure sodium-vapor lamp, at normal incidence. At what angles will the first-order maxima occur for these lines?

   b. How many rulings must a diffraction grating have in order barely to resolve them in third order?

   c. In a particular grating the *D*-lines are viewed in third order at 80° to the normal and are barely resolved. How far apart are the grating rulings?

4. *Grating spectrometer design.* Taking the limits of the visible spectrum to be λ = 0.43 - 0.68 μm, design a grating that will spread the first order spectrum through an angular range of 20°. Use any incidence angle you like. Report the incidence angle, the range of diffracted angles, and the proper blaze angle.

5. *The entrance slit of a grating spectrometer***.** Consider a telescope with a grating spectrometer, as shown schematically in Figure 1. The telescope has diameter D and focal length F, the grating-spectrometer

collimating mirror has diameter d and focal length f, and the light can enter the spectrometer through a slit of width x. The spectrometer is in Littrow mode $(\theta_m = -\theta_i)$.

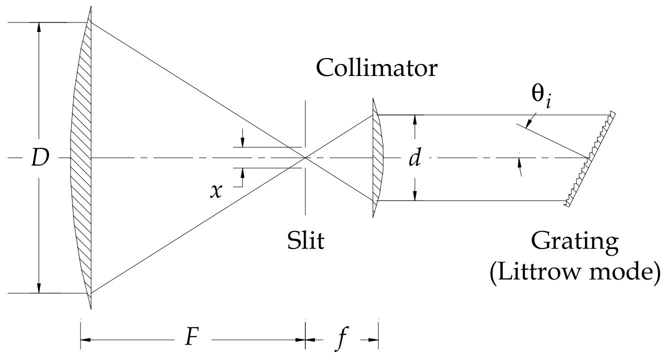

Figure 1: optical setup for Problem 5.

a. If x is not zero, the "collimated" light hitting the grating has some angular spread. Explain why, and show that the angular spread is given by $\Delta\theta = x / f$, and thus that the resolution of the spectrometer is

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{\lambda}\frac{x}{f}\frac{2a\cos\theta_m}{m}$$

— that is, you can make the resolution better by making x smaller.

b. Of course, diffraction prevents you from making x arbitrarily small. Show that the smallest x is allowed to be is $1.2 f\lambda / d$, and therefore that the smallest $\Delta\lambda / \lambda$ can be is

$$\frac{\Delta\lambda}{\lambda} = \frac{2.4}{mN} \quad ,$$

c. Suppose you want the slit to match the image size from the telescope's beam, an angle ϕ in size. Find an expression for the resolution, in terms of ϕ, D, and d. (This, and the previous result, show how one "matches" a grating spectrometer to a telescope.)

