5. Lecture, 16 September 1999

5.1 Ray-surface intersections and ray propagation

It would be time-consuming, even for computers, always to locate intersections between rays and surfaces algebraically from the equations describing them. Fortunately, most practical optical systems possess symmetries that can be exploited to expedite the location of intersections, and the propagation of rays from one surface to the next. The case of the propagation of *meridional* rays* through an axisymmetric optical system provides a good illustration of the simplification that symmetry affords.

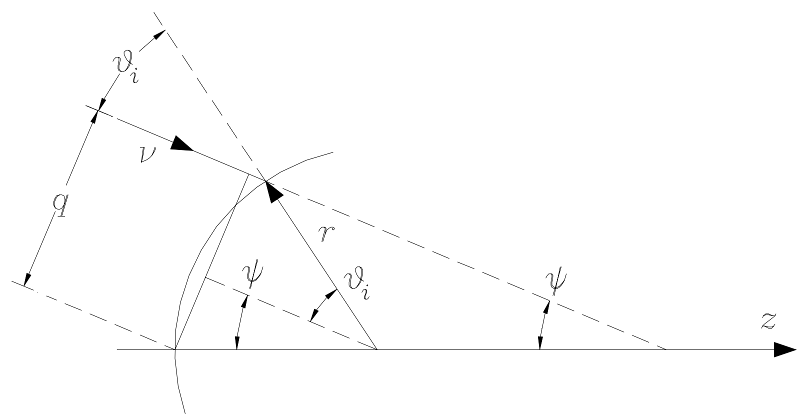

Figure 5.1: meridional ray and spherical surface. The vertical direction here is the y direction, the axis omitted for simplicity.

First, let's consider a spherical surface with its apex and center on the optical axis (Figure 5.1), and the origin of the coordinate system placed on the apex. We are presumed to know the direction cosines and point of origin of the incident ray. From this we can calculate the distance, q, of closest approach of the ray to the apex; it is an easy matter to show that the minimum distance between the coordinate origin and the line $y = mz + y_0$ is

$$q = \frac{y_0}{\sqrt{1+m^2}} \quad . \tag{5.1}$$

This "impact parameter" bears a simple relation to the point of intersection and, in this case, the incidence angle at the intersection, as can be seen in Figure 5.1:

$$q = r\sin\theta_i + r\sin\psi \quad . \tag{5.2}$$

From this equation, the incidence angle is obtained directly, and the location of the intersection from it: $y = r\sin(\theta_i + \psi)$, $z = r - r\cos(\theta_i + \psi)$ -- a much quicker route to the answer than the direct one.

* Meridional rays are those for which the plane defined by the unit vectors $\boldsymbol{\nu}$ and $\boldsymbol{\nu}''$ also contains the optical axis. Those that do not fit this description are called *skew* rays.

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With this result the direction cosines of the refracted or reflected ray $\boldsymbol{\nu}''$ can be computed by solution of Equation 5.7, as discussed at length last lecture. The distance of closest approach of $\boldsymbol{\nu}''$ to the present apex is

$$q'' = r\sin\theta_r + r\sin\psi'' \quad . \tag{5.3}$$

Now this ray can be propagated to the next surface, and we start over; with our recent result for $\boldsymbol{\nu}''$ as the new $\boldsymbol{\nu}$. If the distance between the apices of the two surfaces – usually called the following thickness from the point of view of the first surface – is d, then the new value of q is just

$$q = q'' - d\sin\psi'' \quad , \tag{5.4}$$

as shown in Figure 5.2.

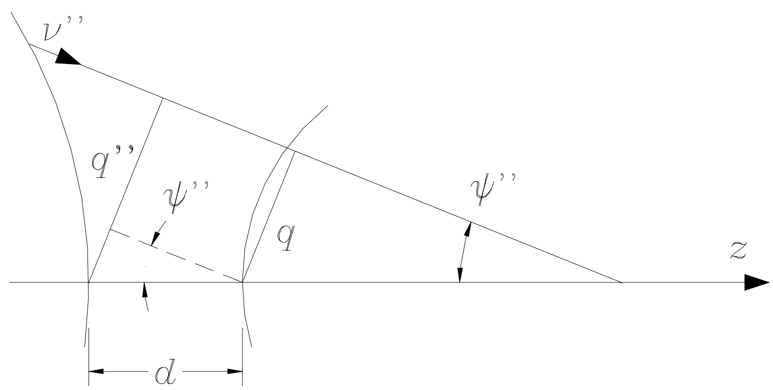

Figure 5.2: propagation of output ray to next surface.

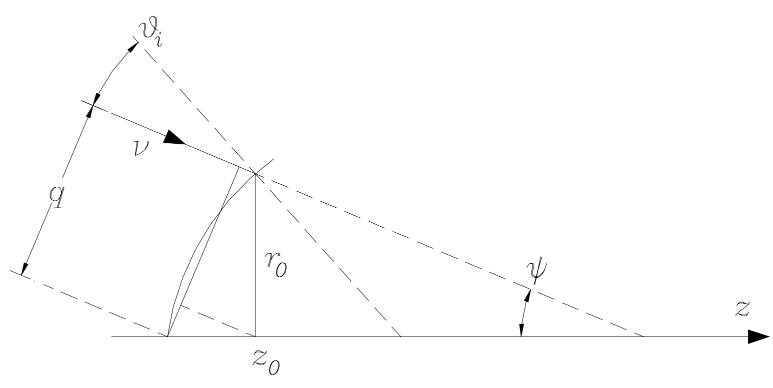

Figure 5.3: meridional ray and general axisymmetric surface. The coordinate origin is at the surface apex, and the y axis, not shown, is vertical.

It is only slightly more complicated to deal with nonspherical axisymmetric surfaces. For a general conic section surface with apex curvature κ and eccentricity ε, given in Equation 3.14 and illustrated in cross section in Figure 5.3, we can express the distance of closest approach of a meridional ray to the apex as

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$$q = z_0 \sin\psi + r_0 \cos\psi$$
$$= \frac{\kappa r_0^2 \sin\psi}{1+\left[1-\kappa^2 r_0^2\left(1-\varepsilon^2\right)\right]^{1/2}} + r_0 \cos\psi \quad . \tag{5.5}$$

We are presumed to know what q and ψ, and the properties of the surface, are, so this expression needs only to be solved for r_0 in order to determine the intersection. This can be done in a straightforward fashion because it is a simple quadratic, but it is not usually done that way in raytracing programs, because they usually allow specification of non-conic section surfaces as well, and it's easier to have one routine that can solve for r_0 in all cases. A general "asphere" is usually written in cylindrical coordinates as

$$z = \frac{\kappa r^2}{1+\left(1-\kappa^2 r^2\right)^{1/2}} + a_4 r^4 + a_6 r^6 + \ldots \quad . \tag{5.6}$$

The solution for r_0 is usually carried out iteratively, as follows. Starting with Equation 5.5, we make a rough guess for r_0 (say, $r_0 = q$), and put it in the formula

$$R = z_0 \sin\psi + r_0 \cos\psi - q \quad . \tag{5.7}$$

The object here is to pick r_0 to make $R = 0$. We can use Newton's method to find the root of Equation 5.7:

$$\text{a better } r_0 = \text{ previous } r_0 - \frac{R}{R'} \quad , \tag{5.8}$$

where

$$R' = \left.\frac{dR}{dr}\right|_{r_0} = \frac{dz}{dr}\left(r_0\right)\sin\psi + \cos\psi \quad . \tag{5.9}$$

One can simply continue to substitute the latest "better" result for the previous value of r_0, recompute R and R', and make R as small as desired.

Once r_0 is know to the required accuracy, the slope of the normal to the asphere at that point is dz/dr, and thus the incidence angle is determined (see Figure 5.3):

$$\tan\left(\psi + \theta_i\right) = \frac{dz}{dr}\left(r_0\right) \quad , \tag{5.10}$$

after which Equation 5.7 can be solved as usual for the output ray.

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