

6. Lecture, 21 September 1999

6.1 Computer ray tracing

RayTrace version 5.0 (by J.R. Houck and T.L. Herter, professors of astronomy at Cornell) is a computer program that traces rays through optical systems in the manner discussed above. It is a powerful program, capable of tracing systems with very complicated surfaces, off-axis mirrors and lenses, and even dispersive elements such as gratings and prisms. Despite its power, it's still easy to use for simple lenses and mirrors.

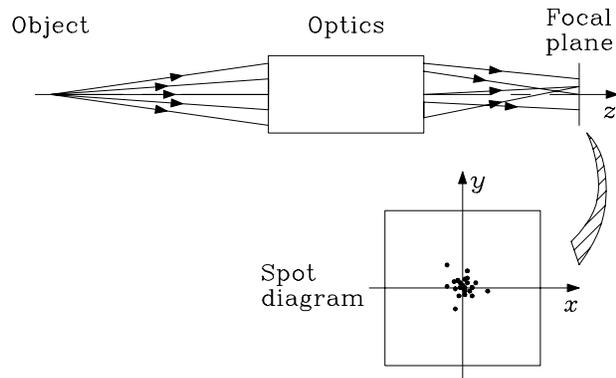


Figure 6.1: schematic diagram of ray bundle, optical system and focal-plane spot diagram.

RayTrace allows one to specify an optical system prescription (lenses and mirrors) and a “bundle” of rays originating in a point – either lying a finite or an infinite distance from the first surface. Tracing the bundle through the system, it ends by showing the points at which the rays intersect a final surface, the focal plane (Figure 6.1). This plot of intersection points is called a *spot diagram*. Since the rays always originate in a point, a perfect optical system would have a point image, so there would be only one spot in the spot diagram. The actual spread of spots in the focal plane is most often what an optical designer is concerned with minimizing, by modification of the optical prescription. Many optical-design programs can even perform this optimization automatically, subject to whatever constraints or free parameters are chosen by the designer.

RayTrace 5.0, its manual, a tutorial, and several example optical prescriptions, are provided, but the easiest way to learn to use it is by example; let's therefore demonstrate the program on some simple lenses and mirrors.

Example 6.1

Enter the RayTrace prescription, and obtain the focal-plane spot diagram and position of best focus, for an equiconvex lens made of flint glass ($n = 1.5$) and used in vacuum, with an infinitely distant object. Take the focal length to be 10 cm, the diameter 1 cm, and the thickness between apices 0.2 cm.

First we will need the radii of curvature for the surfaces of the lens. Since they're equally curved in this case, $r_1 = -r_2$, and from the lensmaker's equation we get

$$r_1 = 2(n - 1)f = +10 \text{ cm} \quad .$$

Then we can turn on RayTrace by typing **Ray <CR>** at the command prompt in a DOS window. (Here, and henceforth, **<CR>** means to type a carriage return.) This should produce the screen shown in Figure 6.2; selection of the **P** option then produces the optical prescription entry screen shown in Figure 6.3. The front surface of our lens is spherical, has radius +10 cm and diameter 1 cm, and is followed by a medium of thickness 0.2 cm and index 1.5; these values are entered as follows:

```

RayTrace by  J.R. Houck and T. Herter

Version 5.0; Last Update: 29-Aug-90

Options:
                Current
P : Prescription mods  (None)
S : Save Prescription
F : Fetch Prescription
C : Color Selections
D : Directory
I : Initialize
L : List Prescription on printer

T : Trace
B : File to write trace info (None)
Q : Quit

Select :
    
```

Figure 6.2: RayTrace startup screen

```

SN: Surface No. .... 1          NA: Name ..

RA: Radius (0 for flat) .      0.00000      AP: Aperture .....      1.00000
EC: Eccentricity .....        0.00000      FT: Following Thickness .  0.00000

Indices                Aconics                Tips
-----                -----                -----
FI: Follow'g          0.00000      FA: Fourth .. 0.0E+0000      AT: Alpha ...  0.00000
SI: Short ...        1.00000      SA: Sixth ... 0.0E+0000      BT: Beta ....  0.00000
LI: Long ....        1.00000      EA: Eighth .. 0.0E+0000      GT: Gamma ...  0.00000

Decenters              Spec. Surface              Miscellaneous
-----              -----              -----
XD: X-Decntr          0.00000      SS: Special Conic Surf      NS: Next Surface
YD: Y-Decntr          0.00000      S1: Coef'nt1  0.00000      PS: Prev. Surface
ZS: Z-Shift           0.00000      S2: Coef'nt2  0.00000      +1: Insert Surface
                   S3: Coef'nt3  0.0E+0000      -1: Delete Surface
                   PN: Perscript Name
                   EX: Exit Routine

Select :
    
```

Figure 6.3: RayTrace optical prescription entry screen.

```

RA <CR> 10 <CR>
AP <CR> 1 <CR>
FT <CR> 0.2 <CR>
FI <CR> 1.5 <CR>

```

All of the other parameters can be left with their default values, including the eccentricity EC, for which the default value is that of a sphere. RayTrace uses the same units throughout; since we have started entering lengths in centimeters we must now continue to do so. The next surface of the lens has radius -10 cm (RayTrace's sign convention is the same as ours), diameter 1 cm (still), and is followed by vacuum. In the spirit of the paraxial approximation we may expect the focus to be 10 cm from the lens, or 9.9 cm from this surface. Thus

```

NS <CR>          (to move RayTrace's attention to the next surface)
RA <CR> -10 <CR>
AP <CR> 1 <CR>
FT <CR> 9.9 <CR>
FI <CR> 1 <CR>

```

The next surface is the focal plane. By default the following index FI of RayTrace's surfaces is zero; RayTrace will trace rays until it finds a surface with FI=0, so the default surface setup will suit us just fine for the focal plane. Now type

```

EX <CR>          to get out of prescription-entry mode,
S <CR>           to save the prescription (always a good idea), and
T                to enter ray-specification mode.

```

This last step calls up a new menu, shown in Figure 6.4. Some of the variables listed here can be entered as numbers; some allow one to toggle through a list of options; others still are looked up from the previous data-entry steps, such as the value of BF, which is set equal to the last value of FT in the optical prescription. Now type

```

FI <CR>          until the Field of Object is "Far Field" (actually the
                  default value); this indicates that an infinite object
                  distance (i.e. parallel rays) are required.
RP <CR>          until the Ray Pattern is "bullseye" (a circularly-
                  symmetric pattern of rays covering the first surface in
                  the prescription, with the number of rays given by NR).
CR <CR>          until it reads "List;" this will produce a list of the
                  intersections and direction cosines of the central ray in
                  the pattern (the chief ray) before a spot diagram is drawn.
PS <CR> 0.01 <CR> (actually the default value) to draw the spot diagram in
                  a box 0.02 cm on a side, centered on the point where the
                  chief ray intersects the focal plane.

```

We're now ready to generate a spot diagram; type **SD <CR>**. First, the program will type a list of positions and direction cosines (γ , δ , ϵ) for the chief ray. Note that the coordinate system in RayTrace is the same that we have been using, with the z axis pointing along the optical symmetry axis. This list will be useful in the future for debugging more complicated optical systems. Type any key to continue to the spot diagram; the result is shown in Figure 6.5.

```

FI : Field of Object ..... Far Field           None
                                                None

DI : Object Distance ..... 100.00000
DX : X Offset (Degrees or Units) .. 0.00000
DY : Y Offset (Degrees or Units) .. 0.00000
SI : Starting Index ..... 1.00000
WC : Wavelength Code (1, 2 or 3) .. 1

NP : Number of Panels ... 1
NR : Number of Rays ..... 100  RP : Ray Pattern ..... Circmscrbd Sqr

BF : Back Focal Distance ..... 9.90000
PS : Plot Scale (Units/100 pix.) .. 0.01000

CR : Central ray      No List
RX : X coord ...     0.00000  RY : Y coord ...     0.00000 (Sngl
Ray)

SD : Spot Diagram    MT : Meridional Trace    TS : Trace Save (FALSE)
PA : Plot Again      BE : Bend surfaces       MP : Modify Prescription

SP : System Plot     AX : X-Axis (FALSE)    EX : Exit
SS : Start Surf ( 1)  VH : Ver/Hor (1.00)

Select :
    
```

Figure 6.4: RayTrace ray-specification menu.

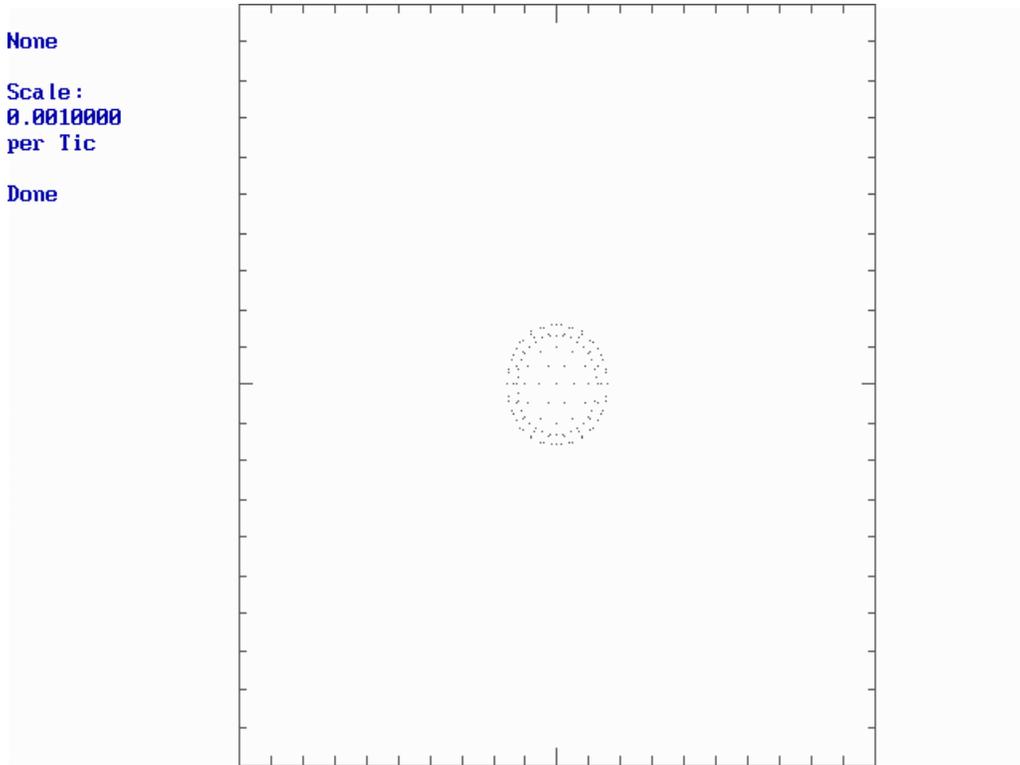


Figure 6.5: spot diagram for equiconvex lens, 9.9 cm from second surface.

The spot diagram is a plot of the intersection between approximately NR (=100 at present) rays and the plane perpendicular to the z axis, a distance BF (=9.9 cm) from the apex of the last surface. Note that it appears as a series of nested circles, about 0.004 cm in diameter at the widest. Type a key to proceed.

The next screen is a collection of statistics on the spot diagram just generated; it is shown in Figure 6.6. It includes the coordinates of the spot center, that of the chief ray (with its direction cosines), and the root-mean-square (RMS) width of the collection of spots, which turns out to be 0.001394 cm. This RMS width varies along the optical path, of course, and it turns out that our present position is not the sharpest focus. RayTrace, however, has already worked the distance and size of the smallest RMS diameter, and lists this under the terms Sm_RMS (0.000398 cm) and Delta_Z (0.035592 cm; further from the lens). The program can automatically move the focal plane to this new position; and will do so if you simply type 1. This brings back the ray specification menu, with the value of BF changed from the original; it is now 9.93559 cm.

Now type PA <CR> ("plot again"). If all that was done since the last SD was a re-focus to change BF, then RayTrace does not need to recalculate anything to plot the new set of intersections between rays and focal plane. The new spot diagram is shown in Figure 6.7. Note that indeed the spot is considerably smaller than before; this new position is a better focus. It is still not a point image, though. As we will see, this remaining blur is due to *spherical aberration*. Hit <CR> twice to return to the ray-specification menu.

Example 6.2

Now put into the lens from Example 6.1 a parallel bundle of rays inclined 0.01 radian with respect to the optical axis, and find the position and shape of the spot diagram in the same focal plane as before.

In the ray-specification screen, the parameters DX and DY function as angular offsets (in degrees) from the axis for the ray bundles; let us take the tilt to be about the y axis, and enter DX <CR> 0.572958 <CR> (that's 0.01 radians, in degrees). Type SD <CR> to generate a new spot diagram, and another key to

```

File : None      Name: None

      Average Spot Position:  X =  -0.000010   Y =   0.000002

Central Ray Values:
      X =   0.000000   Y =   0.000000   Z =   0.000000
      Cx =   0.000000   Cy =   0.000000   Cz =   1.000000

Spot Statistics:  ( 126 rays traced in 126 tries) :
      RMS      = 0.001394  Sm_RMS   = 0.000398  Delta_Z   = 0.035592
      RMS_X    = 0.000982  Sm_RMS_X = 0.000280  Delta_Zx  = 0.035718
      RMS_Y    = 0.000989  Sm_RMS_Y = 0.000283  Delta_Zy  = 0.035469

Bull's Eye pattern traced.

Do you want to refocus?
  1 - To the minimum RMS
  2 - To the minimum RMS_X
  3 - To the minimum RMS_Y
    - Any character for not refocus

Select :
```

Figure 6.6: spot statistics screen.

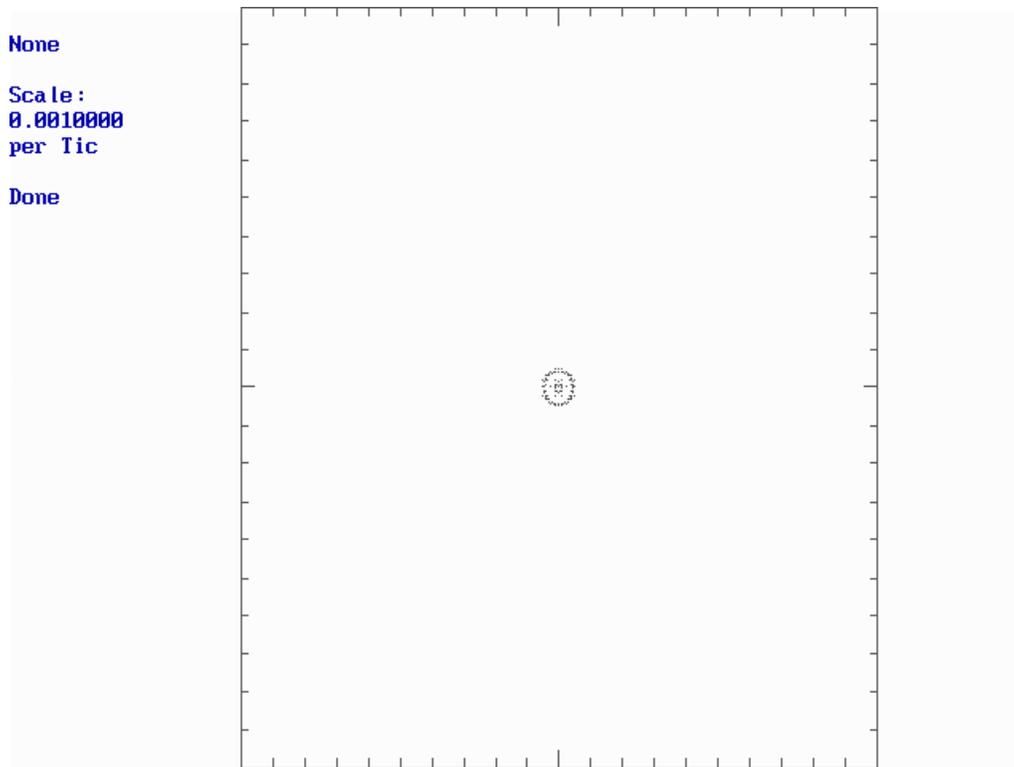


Figure 6.7: spot diagram for the equiconvex lens at the position of best focus, 9.93559 cm from the apex of the second surface. Compare with Figure 6.5.

get the spot statistics (Figure 6.8). Three features of this solution are worth noting. First, the spot diagram is no longer circularly symmetrical, and is slightly larger than our last result. Next, the displacement of the chief ray from the optical center, winds up very close to that predicted by the paraxial estimate of the plate scale: 1.0003 mm, compared to $f\Delta\theta = 1$ mm. Finally, the position of the best focus for the present incidence angle is slightly closer to the lens ($\Delta Z = -0.001382$ cm) than that for the on-axis rays in Example 6.1.

Example 6.3

With the same lens, place an object 20 cm in front of the lens; find the focus of the on-axis point, and in the same focal plane plot spot diagrams for images of points on the object 1 cm above and below the axis.

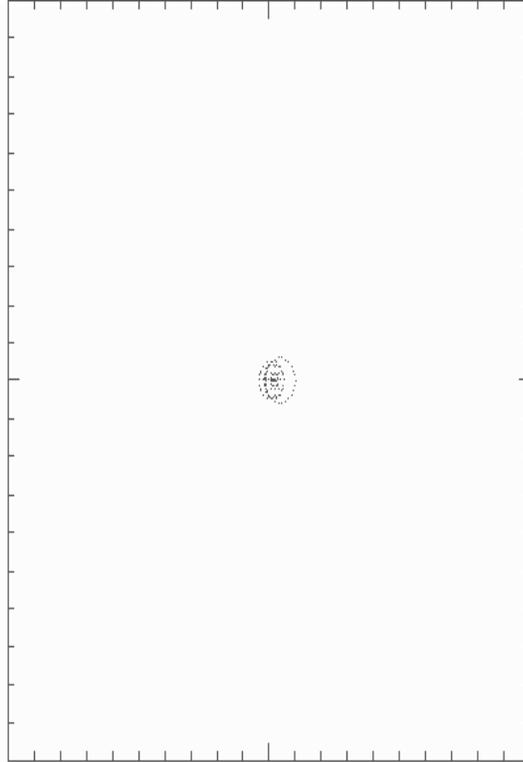
Type

FI <CR>	until the Field of Object is "Near Field"; this indicates finite object distance.
DI <CR> 20 <CR>	to set the distance to the front of the lens to 20 cm.
SD <CR>	for a spot diagram. Only one spot will appear, because the old value of BF is very far from the new focal plane, but we need only refocus:
<CR> 1 <CR>	Refocus. Note that the position of the new focal plane is 19.91557 cm, close to but not quite the paraxial value.

None

Scale:
0.0010000
per Tic

Done



```
File : None      Name: None

      Average Spot Position:  X =   0.100215   Y =  -0.000000

Central Ray Values:
  X =   0.100030   Y =   0.000000   Z =   0.000000
  Cx =   0.009933   Cy =   0.000000   Cz =   0.999951

Spot Statistics: ( 126 rays traced in 126 tries) :
  RMS   = 0.000431  Sm_RMS   = 0.000428  Delta_Z  = -0.001382
  RMS_X = 0.000315  Sm_RMS_X = 0.000311  Delta_Zx = -0.001827
  RMS_Y = 0.000294  Sm_RMS_Y = 0.000293  Delta_Zy = -0.000950

Bull's Eye pattern traced.

Do you want to refocus?
  1 - To the minimum RMS
  2 - To the minimum RMS_X
  3 - To the minimum RMS_Y
  - Any character for not refocus

Select :
```

Figure 6.8: spot diagram and spot statistics for Example 6.2.

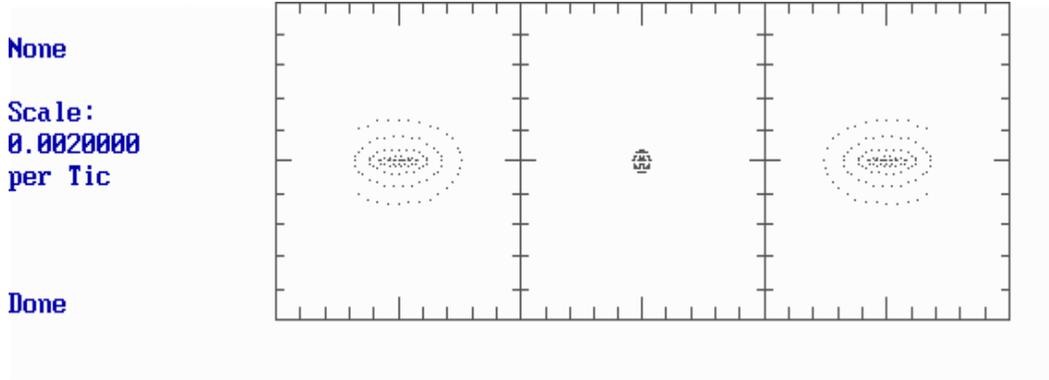
Now we have the best BF for on axis points. We can plot spot diagrams and obtain statistics for all three object points requested in one ray trace, by typing

```

DX <CR> 1 <CR>    DX and DY are lengths, measured in the current distance
                    units, when FI is set to near field.
NP <CR> 5 <CR>    to set the number of panels to plot. This will produce
                    spot diagrams for points at +DX, 0 and -DX. It should be
                    NP = 3; this represents a minor bug in the program.
SD <CR>           for a spot diagrams, and
<CR>             for statistics.
    
```

The results are shown in Figure 6.9. The spot diagrams for the off-axis points are larger and oblong compared to central spots; this stretched-out appearance is due primarily to *astigmatism*.

Note from the statistics that the central rays originating at $x = \mp 1$ cm wind up at $x = \pm 0.996$ cm - the image is inverted, and the lateral magnification is very close to -1, as expected.



```

File : None      Name: None

                Panel Mode:   3 panels computed.

126 rays attempted in bull's eye pattern.

      DX      DY  NumRays  Avg. X   Avg. Y   Z-Center   RMS   Delta_Z
1  -1.000000  0.000000  118   0.996116 -0.000000  0.000000  0.002447 -0.121745
2   0.000000  0.000000  126   0.000005 -0.000001  0.000000  0.000544 -0.000000
3   1.000000  0.000000  117  -0.996077 -0.000004  0.000000  0.002419 -0.120653

Done with Stats
Press <cr> to continue
    
```

Figure 6.9: finite imagery with the equiconvex lens in Example 6.3. Left to right: $DX = -1, 0, 1$ cm.

Example 6.4

Obtain spot diagrams and the best focal position for a Cassegrain telescope with a primary mirror 20 cm in diameter 10 cm and apex curvature 60 cm, and a secondary with eccentricity 1.66667, apex curvature 18.66660 cm, and diameter 3 cm, for incident light from a distant object at angles $0, \pm 0.1$ degree, and ± 0.2 degree.

First, we need to know the focal lengths. That of the paraboloid is 30 cm, half the apex curvature radius (see Equations 3.8 and 3.9). For the hyperboloid we have Equations 3.11 and 3.12,

$$r = \frac{b^2}{a} \quad ,$$

$$\varepsilon = \frac{\sqrt{a^2 + b^2}}{a} \quad .$$

Using the first of these to eliminate a , we get

$$\varepsilon = \frac{r\sqrt{\frac{b^4}{r^2} + b^2}}{b^2} \Rightarrow b = \frac{r}{\sqrt{\varepsilon^2 - 1}} \quad , \quad (6.1)$$

whence

$$a = \frac{b^2}{r} = \frac{r}{\varepsilon^2 - 1}$$

$$c = \varepsilon a$$

$$f = c \pm a = (\varepsilon \pm 1) \frac{r}{\varepsilon^2 - 1} = \frac{r}{\varepsilon \mp 1} = 28 \text{ and } 7 \text{ cm} \quad . \quad (6.2)$$

This convex mirror faces the concave paraboloid in a “nesting” fashion, with its nearer focal point coincident with that of the paraboloid; thus the apices are 23 cm apart.

Type **EX** to escape back to the startup menu, **I** to initialize (throw away the lens prescription), and **P** to enter the prescription. First the primary mirror:

RA <CR> -60 <CR> -- negative, because its center of curvature lies at smaller z than the surface (i.e. it's concave from the point of view of the incident light);
EC <CR> 1 <CR> for a paraboloid;
FI <CR> -1 <CR> to signal reflection rather than refraction;
AP <CR> 10 <CR>
NS <CR> for the next surface.

RayTrace allows specification of the shadow cast by the secondary mirror on the primary; we have left the FT of this first surface zero because it is conventional to place this shadow right on the primary. It is specified as follows:

SS <CR> CO <CR> to specify the special “central obscuration” surface;
S1 <CR> 3 <CR> when a central obscuration is specified, this parameter gives its diameter; all rays inside this diameter are blocked;
AP <CR> 10 <CR> all rays outside this diameter are blocked;
FI <CR> -1 <CR> rays still travel the same way they did after reflection from the primary mirror;

FT <CR> -23 <CR> reflected rays travel toward negative z, so the following thickness is negative;
NS <CR> for the next surface.

Now the secondary mirror:

RA <CR> -18.6666 <CR> -- still negative, because its center of curvature lies at smaller z than the surface (same as the primary mirror);
EC <CR> 1.66667 <CR> for a paraboloid;
FI <CR> 1 <CR> change the sign of the index again, to signal reflection rather than refraction;
AP <CR> 3 <CR>
FT <CR> 28 <CR> -- positive, because the light travels in the positive z direction again;
NS <CR> for the next surface.

The default values are fine for the next surface, which is the focal plane. Type **EX <CR> T** to get to the ray-specification menu; choose Far Field with **FI**, and a bullseye ray pattern

EX <CR> T to leave prescription entry and enter ray specification;
FI <CR> to choose Far Field;
DX <CR> 0 <CR> to restrict ourselves to on-axis rays for now;
NP <CR> 1 <CR>
RP <CR> until the bullseye pattern is chosen;
SD <CR> 1 <CR> to focus.

The best focus turns out to be at $BF = 28.00030$ cm, slightly different from the nominal 28 cm because of the finite precision with which we entered the optical prescription. Now to do the off axis rays: type

DX <CR> 0.1 <CR>
DY <CR> 0.2 <CR>
NP <CR> 9 <CR>
PS <CR> 0.001 <CR>
SD <CR> for the spot diagram,
<CR> for the statistics,

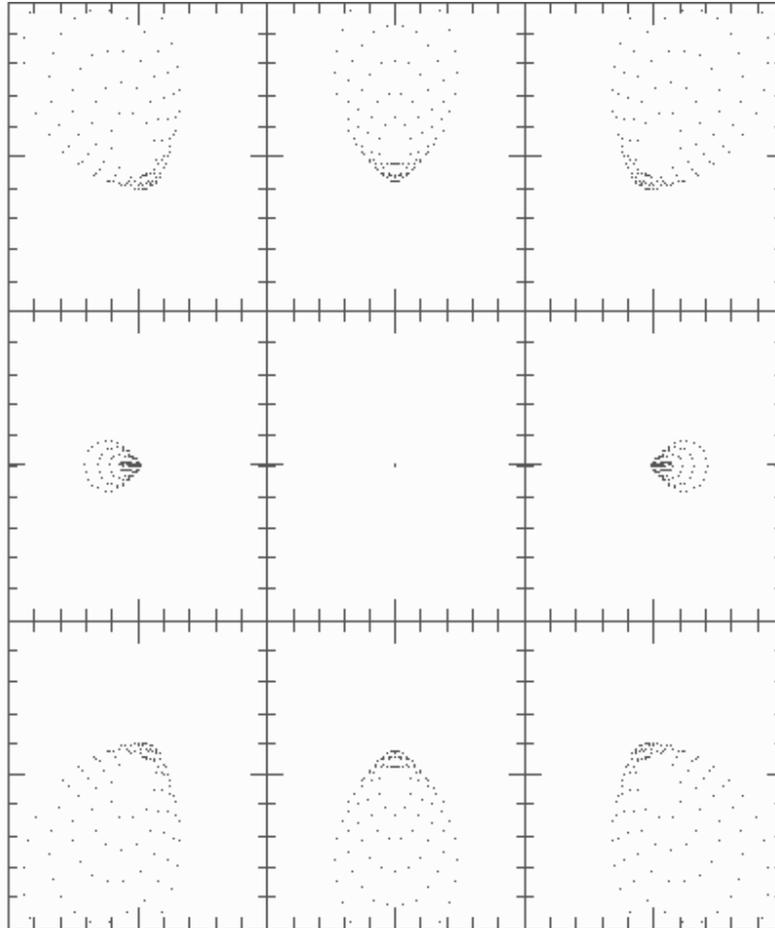
With the use of the axial symmetry of the system, and both **DX** and **DY**, we get the values we were asked for and one more besides. The results are shown in Figure 6.10. Worthy of note here are the following features of the calculation:

1. On-axis rays still focus perfectly.
2. Off-axis rays do not. With a bullseye-shaped ray bundle to start with, the resulting spot diagrams appear as concentric circles of spots with a range of diameters, the larger ones lying further away from the center of the focal plane. It turns out that these larger circles of spots come from rays originally incident further out toward the edge of the primary. The "cometary" shape of these spot diagrams and the tendency for the "tail" to point away from the center of the focal plane gives rise to the name of the aberration that is dominant here: *coma*.

CASS. RAY

Scale:
0.0002000
per Tic

Done



File : None

Name: None

Panel Mode: 9 panels computed.

126 rays attempted in bull's eye pattern.

	DX	DY	NumRays	Avg. X	Avg. Y	Z-Center	RMS	Delta_Z
1	-0.100000	0.200000	119	-0.209554	0.419121	-0.000000	0.000508	-0.014345
2	0.000000	0.200000	119	0.000004	0.419118	0.000000	0.000415	-0.011538
3	0.100000	0.200000	119	0.209564	0.419123	-0.000000	0.000512	-0.014475
4	-0.100000	0.000000	119	-0.209550	-0.000000	0.000000	0.000124	-0.002821
5	0.000000	0.000000	119	0.000000	-0.000000	0.000000	0.000000	-0.000000
6	0.100000	0.000000	119	0.209553	0.000000	0.000000	0.000127	-0.002926
7	-0.100000	-0.200000	119	-0.209556	-0.419123	-0.000000	0.000510	-0.014406
8	0.000000	-0.200000	119	0.000003	-0.419120	0.000000	0.000416	-0.011576
9	0.100000	-0.200000	119	0.209563	-0.419124	-0.000000	0.000512	-0.014488

Done with Stats

Press <cr> to continue

Figure 6.10: spot diagrams and statistics for the Cassegrain telescope of Example 6.4.

3. Judging from the values of ΔZ , the surface of best focus is curved; the minimum RMS spot-diagram width for the off-axis points lie closer to the secondary mirror the further off axis the rays are. This is an example of yet another aberration: *Petzval field curvature*.

The prescription we just generated is included with your copy of RayTrace, and is called CASS.RAY.