11 Lecture, 7 October 1999

One needs additional *small* optics, besides the telescope, to accommodate the special requirements of instruments. Most of the functions of these additional optics will be obviously necessary, such as those that prepare beams properly for spectrometers or polarimeters, or those that change the plate scale at the final focus in order to make the resolution a better match for the detector size. The extra optics can also make it easier for the telescope and instrument to achieve ideal performance, for instance by correction of remaining aberrations, or rejection of stray light. Today we will discuss a few concepts that are important in the design of these auxiliary optics.

11.1 Petzval field curvature and field lenses

The fifth (and last) of the third-order aberrations is *Petzval field curvature.* It is evident most clearly in the spherical focal surface of an SA-corrected spherical-primary telescope like a Schmidt camera (see Figure 10.1 or 10.9), since to third order all the off-axis aberrations are absent *ab initio*, and third-order SA is corrected. It is possible to show that all the focal surfaces in any given optical system are related by simple transformation to a single *invariant* surface, called the optical system's *Petzval surface*; this result is of course called *Petzval's theorem.* The curvature κ_P of the Petzval surface in a system with k optical surfaces is given by

$$\kappa_P = -n_k \sum_{j=1}^{k} \kappa_j \left(\frac{1}{n_j} - \frac{1}{n_{j-1}} \right) \quad , \tag{11.1}$$

where κ_j is the apex curvature of the jth optical surface, n_j the index following the jth optical surface, and the usual raytracing sign convention on n is assumed – that is, that n changes sign on reflection. The surface is considered invariant in the sense that the result does not depend upon the distances between the optical surfaces, only upon the straight algebraic sum of the powers of these surfaces. To prove Petzval's theorem is complicated, and we will not attempt to do so here * . However, the result is simple enough to apply: all of the focal surfaces are flat if the curvature of the Petzval surface vanishes:

$$\sum_{j=1}^{k} \kappa_j \left(\frac{1}{n_j} - \frac{1}{n_{j-1}} \right) = 0 \quad . \tag{11.2}$$

Since it doesn't matter what the separations of these optical surfaces are, judicious choices of the indices and apex curvatures can result in a flat field *and* a finite focal length.

The most common astronomical application of *Petzval's condition*, Equation (11.2), is the use of a *field lens* to flatten the Petzval surface of a telescope. Note that the sum in Equation (11.2) for a system of k mirrors in an all-reflecting telescope is

* See Born and Wolf, *Principles of optics*, sixth edition, pp. 225-226, or Schroeder, *Astronomical optics*, pp. 82-88, for a proof.

$$\sum_{j=1}^{k} \kappa_j \left(\frac{1}{n_j} - \frac{1}{n_{j-1}} \right) = \sum_{j=1}^{k} \frac{1}{f_j} \quad , \tag{11.3}$$

where f_j is the focal length of the *j*th mirror; thus the field will be flattened by placing a lens with focal length *f* and index *n* given by

$$\frac{1}{nf} = -\sum_{j=1}^{k} \frac{1}{f_j} \tag{11.4}$$

very close to the focus, as in Figure 11.1. Placing such a lens at the focus does almost nothing to the focal position or other aberrations, if the lens is thin. However, it flattens the field for the detector right behind this lens, allowing the image received by this detector to be in sharp focus.

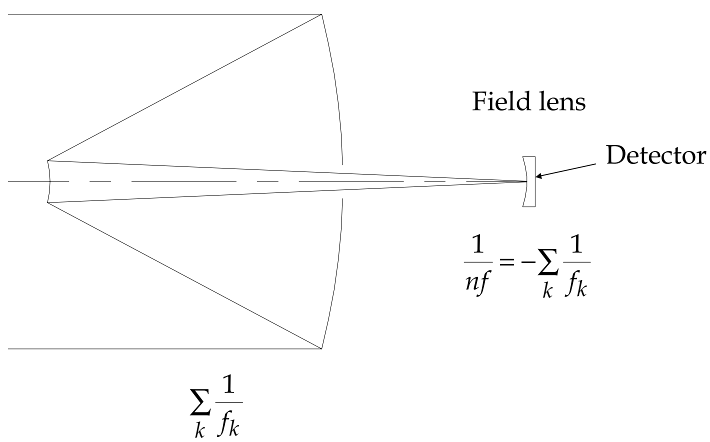

Figure 11.1: field-flattening lens for Cassegrain telescope. The size of the lens is exaggerated for clarity.

11.2 Re-imaging optics, collimators, cameras and zooms

Rarely are the pixels on available detectors a good match for the resolution of a given telescope, so even if broad-band imaging is the project one has in mind, one will often need a lens or mirror system that will convert the plate scale to a more convenient value. Such *reimaging* optics are usually part of the instrument, rather than the telescope. Individual instruments are generally intended to cover rather smaller ranges of wavelengths than the telescopes on which they are used; the optics are also quite small as well. Thus the structural and chromatic difficulties that apply to refracting telescopes abate, and one often sees lenses used, especially at visible and near-infrared wavelengths.

The job can obviously be done with a single lens. One usually uses more, after considering the optical requirements on the filters that precede the detectors (see Lecture 25), and the rejection of stray light and thermal background light (see below). A common lens or mirror configuration is simply a pair of converging optics, one (the *collimator*) placed a focal length away from the telescope focal plane and the other (the *camera*), as in Figure 11.2. Note that since the diameters necessary for these optical elements are relatively small, lenses with spherical surfaces are often good enough despite their spherical aberration; since for a single surface $TSA \propto y^3$ and y is small, the SA can still be much smaller than the telescope aberrations.

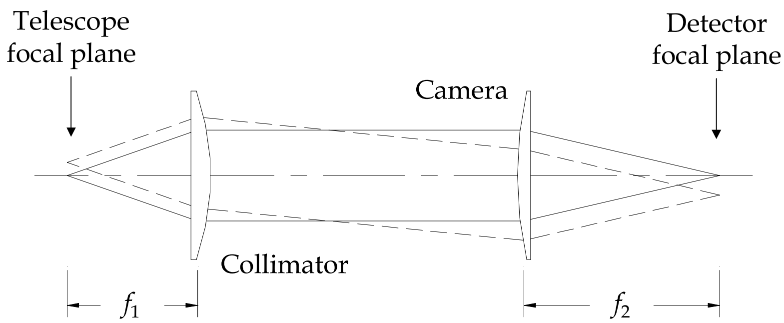

Figure 11.2: collimator and camera lenses for reimaging; on-axis (solid lines) and off-axis (dashed lines) shown.

Homework problem 11.1.

a. Show that the plate scale at the detector, in Figure 11.2, is f_1/f_2 times the plate scale at the telescope focus.

b. Suppose the two ray bundles in Figure 11.2 represent light from two point objects separated by a small angle θ in the sky, and the effective focal length of the telescope is *f*. Show that the (small) angle between the two bundles of rays in the collimated portion of the beam is $\theta' = (f/f_1)\theta$.

Homework problem 11.2. The arrangement of the lenses in Figure 11.2, planoconvex with the curved surfaces facing the collimated light, was chosen on purpose, to minimize the SA of these lenses. Use RayTrace to demonstrate that this is true. Take the lens focal lengths to be 20 cm and 30 cm and to be separated by 50 cm. Plot spot diagrams at the detector focus, and measure the RMS spot size at the paraxial focus, for the lens shapes shown in the figure, for the lenses reversed (flat sides facing the collimated beams) and for the lenses replaced with equiconvex lenses of the same focal lengths.

It is also possible to produce optics with continuously variable plate scale for a fixed detector and telescope position, by replacing the second lens in Figure 11.2 with a *zoom lens*. A simple example of a three-element zoom, the Pan-Cinor Zoom invented by Cuvillier in 1949, is illustrated in Figure 11.3. It works by motion of the first and third lenses by equal distances*, with the second lens fixed in position. Over a fairly wide range of displacements the position of the best focus changes very little, but the plate scale (or equivalently the reciprocal of the system's effective focal length) can change substantially, usually by a factor of three or more.

Homework problem 11.3. The numerical inputs for this problem were taken from the example Pan-Cinor treated in Kingslake's *Lens design fundamentals*, pages 63-66.

a. The focal lengths of the lenses in Figure 11.3 are f_a, f_b, and f_c, from left to right. Show that the back focal distance i_c is given for $\Delta = 0$ by

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* Thus it requires only one translation mechanism. Such a system is called an *optically compensated* zoom lens. There are other zooms for which the movable lenses have to be translated at different rates or in different directions, which is done with systems of gears and cams driven by one control "knob." These are called *mechanically compensated* zoom lenses.

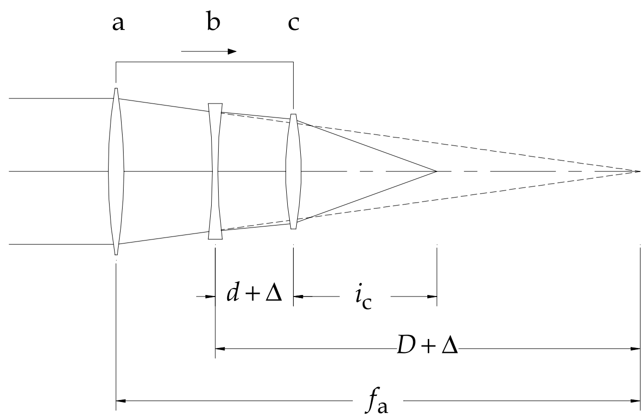

Figure 11.3: the Pan-Cinor zoom.

$$i_c = \frac{\left[dD - f_b(D-d)\right]f_c}{dD - f_b(D-d) - f_c D - f_b f_c} \quad .$$

b. Show similarly that the plate scale is

$$PS = -\frac{dD - f_b(D-d) - f_c D - f_b f_c}{f_a f_b f_c} \quad .$$

c. A certain zoom has $f_a = 7.15959$ cm, $f_b = -1.95959$ cm, $f_c = 3.35410$ cm, D = 4.15959 cm and d = 1.69451 cm. Replace d and D by $d+\Delta$ and $D+\Delta$ in the equations above, and plot the image displacement,

$$\delta(\Delta) = i_c(\Delta) + d + \Delta - \left[i_c(0) + d\right] \quad ,$$

and the plate scale as a function of Δ from –0.5 to 2.5 cm. Show thereby that the image is displaced by at most 0.068 cm, and that the plate scale increases by a factor of 3.0, as Δ runs from zero to 2 cm.

11.3 Stops and pupils

"Stop" is the name given to apertures or masks within an optical system that determine how much of the input light gets through it. The *aperture stop* determines how much light enters, and the *field stop*, how much leaves or is detected. In astronomy, the aperture stop is most often the edge of the primary mirror, but we have already seen one major exception to this rule, in the entrance aperture of the Schmidt camera. Another exception is provided by infrared-optimized telescopes, in which the secondary is slightly undersize (or the primary oversize, depending upon your attitude), and for which the secondary's edge is the aperture stop. The effective field stop of a complete astronomical telescope instrument system is usually the edge of the detector or detector array in the final focal plane.

"Pupil" is the name given to images of the aperture stop. The *exit pupil* is the image of the aperture stop as seen through all of the following optics (i.e. from the final image's point of view), and the *entrance pupil* is the image of the aperture stop as seen through all of the *preceding* optics – that is, as seen by the *object.* Thus for most astronomical telescopes (non-Schmidt, non-infrared), the entrance pupil and the aperture stop are identical, there being no preceding optics.

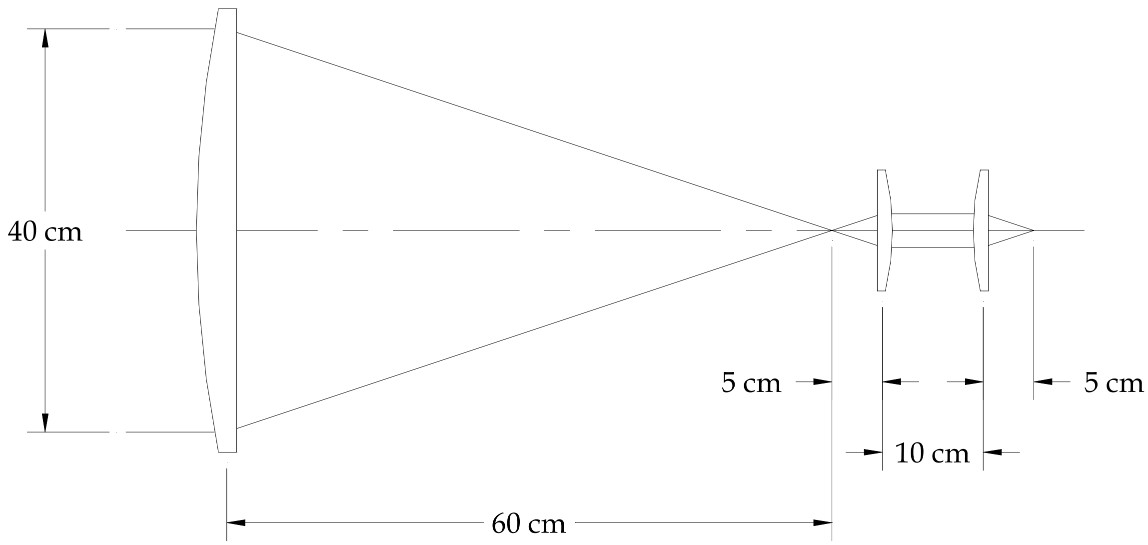

Figure 11.4: optical setup for Example 11.1.

It is very useful to keep track of stops and pupils in optical systems. By calculating their sizes and doing some simple ray tracing one can estimate the sizes necessary for optical elements to catch all the light they are intended to reflect or refract. (For an optic to do otherwise is called *vignetting*.) Also, pupils that are *real* images of the aperture stop are good places to put *baffles*, opaque masks that reject stray light. Geometrically, all the light that deserves to get through all the optics has to be contained within the images of the aperture stop, and thus none of *this* light is affected by placement of an opaque screen with a hole the same size as the pupil, at the location of the pupil. Such a screen may, however, block light from other sources that could reflect around and find its way to the detector. Such *stray light* can be produced in a variety of obvious ways, such as city lights, lamps left on within the telescope dome. It can even come from the optics and their enclosures themselves, at long wavelengths: blackbody emission from the optics would dwarf the signal from celestial sources at infrared wavelengths, without the use of cryogenic baffles and optics. A mask the size and shape of a pupil, located at the pupil's position, is commonly called a *Lyot stop*, after B. Lyot, who discovered its utility while designing coronagraphs.

Example 11.1
a. *A D = 40 cm diameter, f = 60 cm focal length lens is used as a telescope, with two 5 cm focal length lenses, set up as a collimator and camera, as shown in Figure 11.4. The lenses are separated by the sums of their focal lengths: 65 cm for the first two, 10 cm for the next two. Find the location and diameter of each pupil, and indicate the best pupil to use as a Lyot stop.*

The first pupil lies a distance

$$i_1 = \frac{o_1 f_1}{o_1 - f_1} = 5.42 \text{ cm}$$

behind the first 5 cm lens, and has diameter

$$d_1 = D|m_1| = D\frac{i_1}{o_1} = 3.33 \text{ cm} \quad .$$

This pupil lies $o_2 = 4.58$ cm in front of the second 5 cm lens, so

$$i_2 = \frac{o_2 f_2}{o_2 - f_2} = -55 \text{ cm}$$

$$d_2 = D|m_1||m_2| = d_1 \frac{-i_2}{o_2} = 40 \text{ cm} \quad .$$

The second pupil is the exit pupil of the system, but is virtual, so it can't be used as a Lyot stop. The other one will do nicely, though.

b. *We want the telescope to have a 2 degree field of view. How large in diameter do the 5 cm lenses have to be?*

This means that rays as far off axis as $\theta = 1^\circ$ are to be transmitted through the system. In between the small lenses, such rays are $\theta' = (f/f_1)\theta = 12^\circ$ off axis. Consider two marginal rays, which having come in on the edge of the primary must intersect the edge of the Lyot stop too. An inbound one must have originated at a point $i_1 \tan\theta' + d/2$ from the center of the collimator lens in order to intersect the Lyot stop a distance $d/2$ from its center, as in Figure 11.5. Similarly an outbound marginal ray leaves the edge of the Lyot stop and meets the camera lens a distance $o_2 \tan\theta' + d/2$ from its center. Thus the smallest that the diameters of these lenses can be are

$$D_1 = 2i_1 \tan\theta' + d = 5.64 \text{ cm}$$
$$D_2 = 2o_2 \tan\theta' + d = 5.28 \text{ cm}$$

in order that no light be lost.

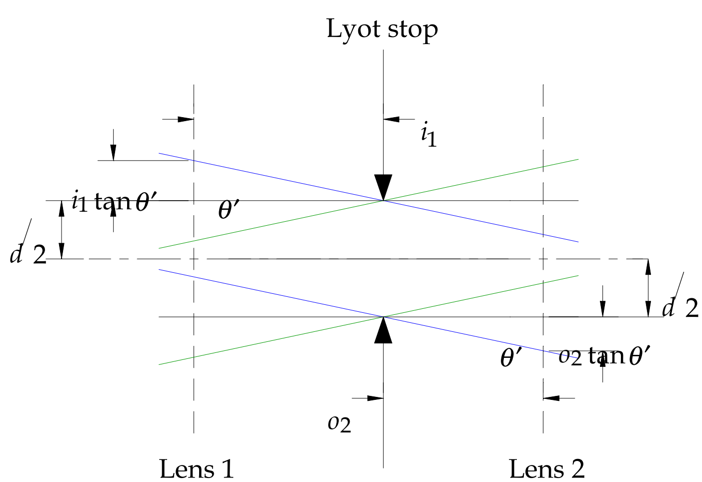

Figure 11.5: geometry of off-axis marginal rays between collimator and camera in Example 11.1.

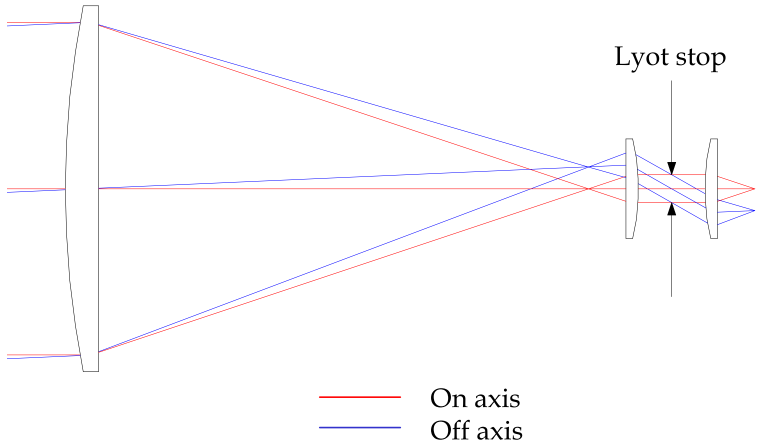

Figure 11.6: tracing chief and marginal rays all the way through the telescope of Example 11.1, both on axis (red) and off axis (blue). For clarity we use here a larger off-axis angle, and larger –diameter collimator and camera, than used in Example 11.1.

Homework problem 11.4. UR infrared astronomers frequently use their newest infrared camera, built by Profs. Bill Forrest and Judy Pipher, at the Wyoming Infrared Observatory. WIRO has a classical Cassegrain optimized for infrared performance. The primary mirror is a 2340 mm diameter paraboloid with focal length 4800.1 mm. The secondary has diameter 202.8 mm and focal lengths 431.7 mm and 5380.0 mm. Its edge comprises the aperture stop. 269.5 cm past the Cassegrain focus there is an achromatic doublet lens with focal length 76.37 mm. The detector array sits at the final focus.

a. Calculate the diameter and position of the entrance pupil.

b. Calculate the diameter and position of the exit pupil. Is it real or virtual? Would it make a good Lyot stop?

