Lesson 1: instrumental sensitivity

Useful references:

- AST 203 lectures 19 - 24:
  http://www.pas.rochester.edu/~dmw/ast203/

The Deer Lick galaxy group (Mees Observatory image)

Making astronomical images, simplified

Much more complicated than most picture-taking.

- ❑ Identify a date range and a favorite object which is high in the sky at night for at least 3-4 hours on those dates.

- ❑ Compile, or plan to take, the calibration data.

  - Dark-current and bias frames, 32-256 frame sequences each, with the CCD operating at the temperature you intend to use (T = -35 to -20 C).

  - Flat field frames, 32-64 frame sequences in each filter, using the telescope cover's built-in flat field lamp.

  - A few 8-10 mag A stars near your target.

- ❑ Plan to autoguide on all deep-sky targets; identify a 6-14 mag star about 12 arcminutes from your target.

Making images (continued)

- Once at the telescope: for scientifically useful images, take
  - frames in R, G, B, and/or narrowband spectral-line filters, all binned 2x2 pixels. Relative numbers depend on desired sensitivity.
  - shorter-exposure sequences on the calibration standards, every so often.
  - as many frame sequences as you have time for.
  - 5 minute exposures in moonlight; 8-10 minute exposures in dark skies. All will be averaged together in the end.
- For pretty pictures: as above, but
  - 3-4 times as many frames in L as any of the other filters, in 1x1 binning.
  - again, as many frames as you have time for in > 4 hours.

Note that most deep-sky APOD images involve >> 4 hours.

Making images (continued)

- Process the images: for each object and filter,
  - calibrate each image: subtract dark and bias charge, divide by flat field.
  - normalize the images, so that stars and blank sky are the same (corresponding) brightness in each.
  - align the images so the stars are all in the same pixels.
  - remove cosmic rays.
  - remove satellite trails.
  - remove images in which something bad happened (clouds, tracking failure, dome lights inadvertently turned on, etc.).
  - average all the images in the same filter; convert to physical units.
  - extract the quantities you need (e.g. fluxes for each star) and proceed to scientific analysis.

Making images (continued)

- For pretty pictures,

  - Make a composite RGB or false-color image out of the images taken with different filters, with brightness scales (usually) chosen so that A0 stars look white.

  - deconvolve your averaged L image, preferably using the maximum entropy method in CCDStack, IDL, or python.

  - process your deconvolved L image and composite RGB or false-color image in Photoshop, making a composite LRGB image with

    - high contrast and dynamic range (black blank sky, vibrant colors, reduced noise).

    - no flaws such as “dust donuts,” “death stars,” etc.

    - cropping to the desired size and aspect ratio.

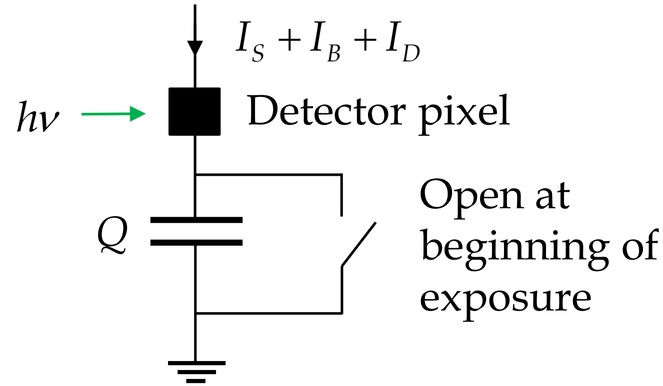
How long an exposure is needed?

The first thing one needs to know, before starting a project, is how long it will take. For astronomical images one needs to know the instrumental sensitivity, and the target flux, to estimate the exposure time necessary.

- Individual **frames** exposed long enough that sky background is larger than dark current, but not so long that there are very many cosmic-ray hits in the frame.
  - Usually 5-12 minutes per frame, depending on whether the moon is up.
- As many frames as it takes to fill the time available.
  - 4-5 hours is a good target for a high-quality color image of a Messier object.
- More is always better, but signal-to-noise ratio of at least 10 on your target is needed.
  - An image barely showing objects half as bright, takes four times as long.

Sensitivity and signal-to-noise ratio

To astronomers, "sensitive" means a large ratio of signal to noise.



Signal = current produced by CCD absorption of light from the celestial object, which we'll call I_S. For **small bandwidth** – that is, filter width $\Delta\lambda \ll \lambda$ – a typical CCD pixel draws

$$I_S = \tau\eta q_e \frac{\lambda P_S}{hc} \quad .$$

The other terms are

τ	Transmission of optics and atmosphere, within $\Delta\lambda$; range = 0-1
η	**Quantum efficiency** of detector; range = 0-1
λ	Wavelength; range = 300-700 nm for visible light
h, c, q_e	Physical constants, by their usual symbols
P_S	(Signal) power incident from object, in watts or erg/sec

Sensitivity and signal-to-noise ratio (continued)

Because of the finite, quantized electron charge, and random arrival time of electrons at given points in a circuit, there are three statistically-independent noise sources in the CCD current. General term for this effect: **shot noise**. In terms of current drawn,

$$I_N^2 = \overline{\left(\Delta I^2\right)} = \frac{q_e I}{\Delta t} = \frac{q_e}{\Delta t}\left(I_S + I_B + I_D\right)$$
$$= \frac{q_e}{\Delta t}\left[\frac{\lambda \tau \eta q_e}{hc}\left(P_S + P_B\right) + I_D\right] \quad .$$

Here, we refer to a single pixel or group of pixels in the CCD, and

P_S	still the power from the object, incident on pixel
P_B	incident background power (e.g. moonlight, city lights)
I_D	dark current: current drawn by pixel even when no light shines; can be made negligible at sufficiently low CCD operating temperature
Δt	exposure time: time over which current is averaged

Sensitivity and signal-to-noise ratio (continued)

The reason for this form of the square of the noise current is that electrons in small currents pass given points in a circuit randomly and at discrete times. This is a process governed by Poisson statistics.

Reminders on Poisson:

- For a Poisson-distributed variable with mean value $\bar{N}$, the variance $\overline{\Delta N^2}$ is

$$\overline{\Delta N^2} \equiv \overline{\left(N - \bar{N}\right)^2} = \overline{N^2} - \bar{N}^2 = \bar{N} \quad \Rightarrow \quad \Delta N_{\mathrm{rms}} = \sqrt{\bar{N}} \quad .$$

- If we're talking about N electrons counted in a time Δt, we can write this in terms of charge or current as

$$\begin{aligned} \overline{\Delta Q^2} &= q_e \bar{Q} \\ \overline{\Delta I^2} &= \frac{q_e}{\Delta t} \bar{I} \end{aligned} \quad \Rightarrow \quad \begin{aligned} \Delta Q_{\mathrm{rms}} &= \sqrt{q_e \bar{Q}} \\ I_N &= \sqrt{\frac{q_e}{\Delta t} \bar{I}} \end{aligned}$$

Sensitivity and signal-to-noise ratio (continued)

So the **signal-to-noise ratio** is

$$\frac{S}{N} \equiv \frac{I_S}{I_N} = \tau\eta q_e \frac{\lambda P_S}{hc}\left(\frac{q_e}{\Delta t}\left[\frac{\lambda\tau\eta q_e}{hc}\left(P_S + P_B\right) + I_D + I_R\right]\right)^{-1/2}$$

$$= P_S\sqrt{\frac{\lambda\tau\eta}{hc\left(P_S + P_B + \frac{hc}{\lambda\tau\eta q_e}\left[I_D + I_R\right]\right)}\Delta t} \quad .$$

Usually one of the terms in the denominator is by far the larger, and the smaller ones can be ignored to good approximation:

- **Background-limited**: $P_B \gg P_S, hc\left(I_D + I_R\right)/\lambda\tau\eta q_e$;

$$\left(\frac{S}{N}\right)_{BL} = P_S\sqrt{\frac{\lambda\tau\eta}{hcP_B}\Delta t} \propto P_S\sqrt{\eta\Delta t} \quad .$$

Sensitivity and signal-to-noise ratio (continued)

- **Source limited**: $P_S \gg P_B, hc\left(I_D + I_R\right)/\lambda\tau\eta q_e$;

$$\left(\frac{S}{N}\right)_{SL} = \sqrt{\frac{\lambda\tau\eta P_S}{hc}\Delta t} \propto \sqrt{\eta P_S \Delta t} \quad .$$

- **Dark-current limited**: $I_D \gg I_R, \lambda\tau\eta P_B/hc, \lambda\tau\eta P_S/hc$;

$$\left(\frac{S}{N}\right)_{DL} = \frac{\lambda\tau\eta P_S}{hc}\sqrt{\frac{q_e}{I_D}\Delta t} \propto \eta P_S \sqrt{\Delta t} \quad .$$

Note that in all cases *S/N* scales with $\sqrt{\Delta t}$. **To increase S/N by a factor of 2, one needs to increase the exposure time by a factor of 4.**

Use these equations to estimate the exposure time necessary for a given project.

Sensitivity and signal-to-noise ratio (continued)

Along with these noise sources related to object, natural backgrounds, and detector, there is additional noise from the readout circuitry, called **read noise**, that appears as a random extra number of electrons added to each pixel in each image, **independent of exposure time.**

- Read noise is statistically independent of the other noise sources. That means the grand-total variance is the sum of the separate variances (**quadrature sum**).
- Including read noise, the grand-total variance in the charge on each pixel is

$$\overline{\left(\Delta Q^2\right)} = I_N^2 \Delta t^2 + q_e^2 N_R^2 \quad ,$$

  where N_R is the root-mean-square (**rms**) number of electrons randomly added to each pixel.

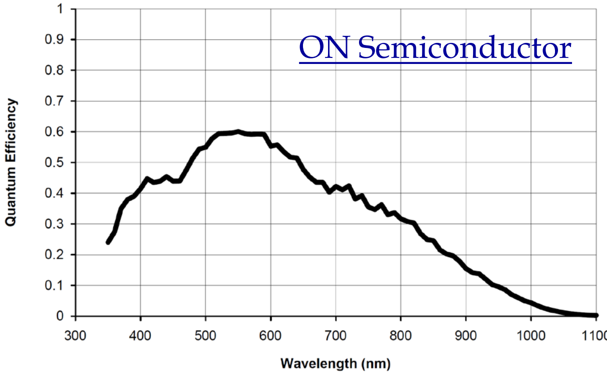
We strive to have sensitivity limited by natural sources of background. Thus we

- operate the CCD at low enough T that $I_D \ll I_B$, and
- take long enough exposures that $I_N^2 \Delta t^2 \gg q_e^2 N_R^2$.

Our imaging system

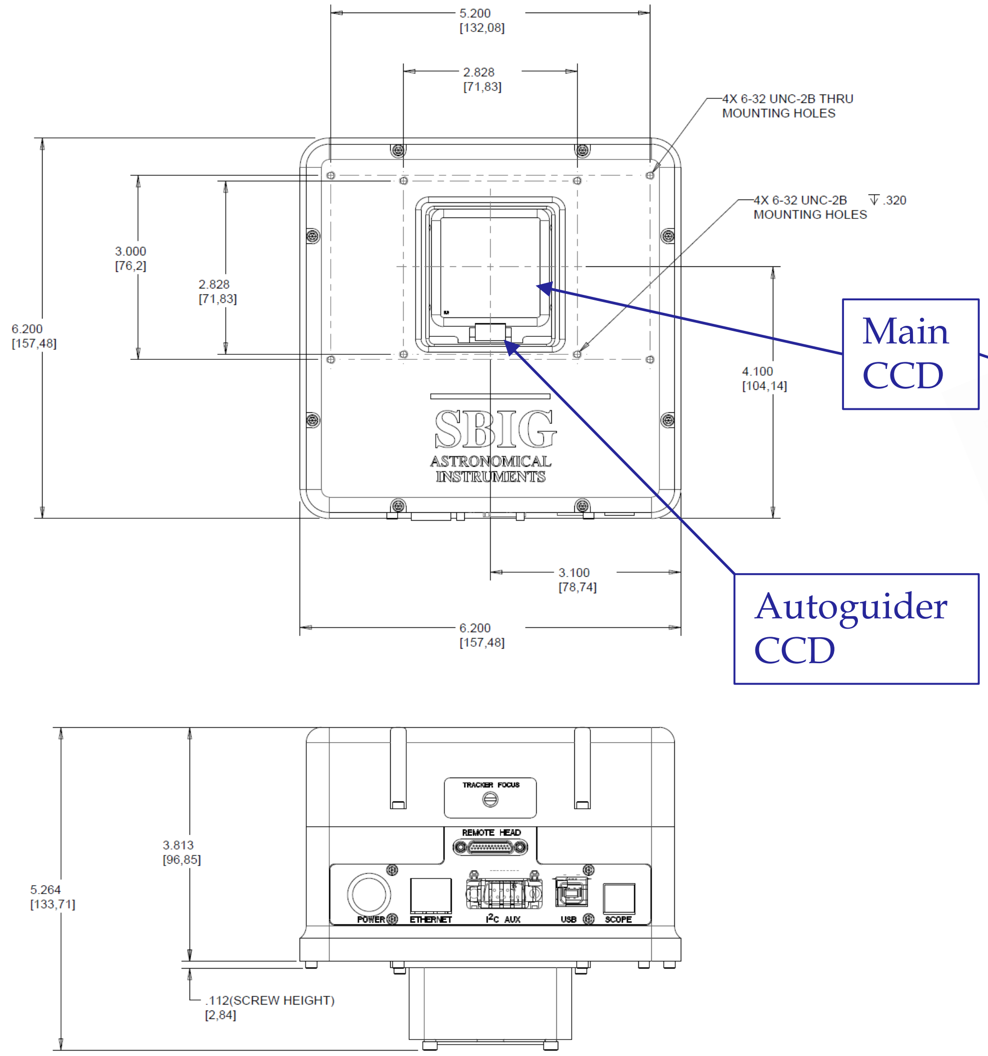
Santa Barbara Instrument Group (SBIG) STX 16803 CCD camera.

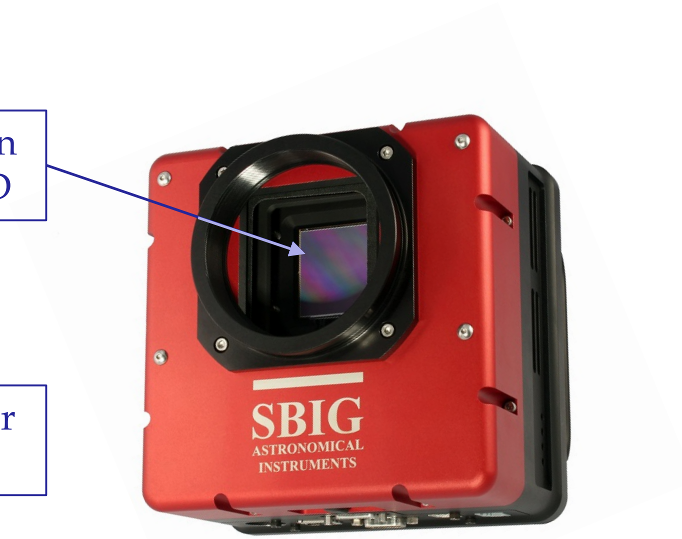
- Frame-transfer CCD, 4096×4096, 9 μm pixels, plus a separate interline CCD, 657×495, 7.4 μm pixels for autoguiding.
- Quantum efficiency η = 0.45-0.6 across the visible band.
- 16-bit output; 1.27 electrons per **data number** (DN).
  - Pixel outputs are recorded in the image files in DN.



- Dark current I_D/q_e = 0.009 electrons per second per pixel at T = -20 C.
- Read noise N_R = 9 electrons (rms) per pixel at T = -20 C. Half that, if the images is binned into 2×2 pixel blocks, as we usually do except with our L filter.

Our system (continued)

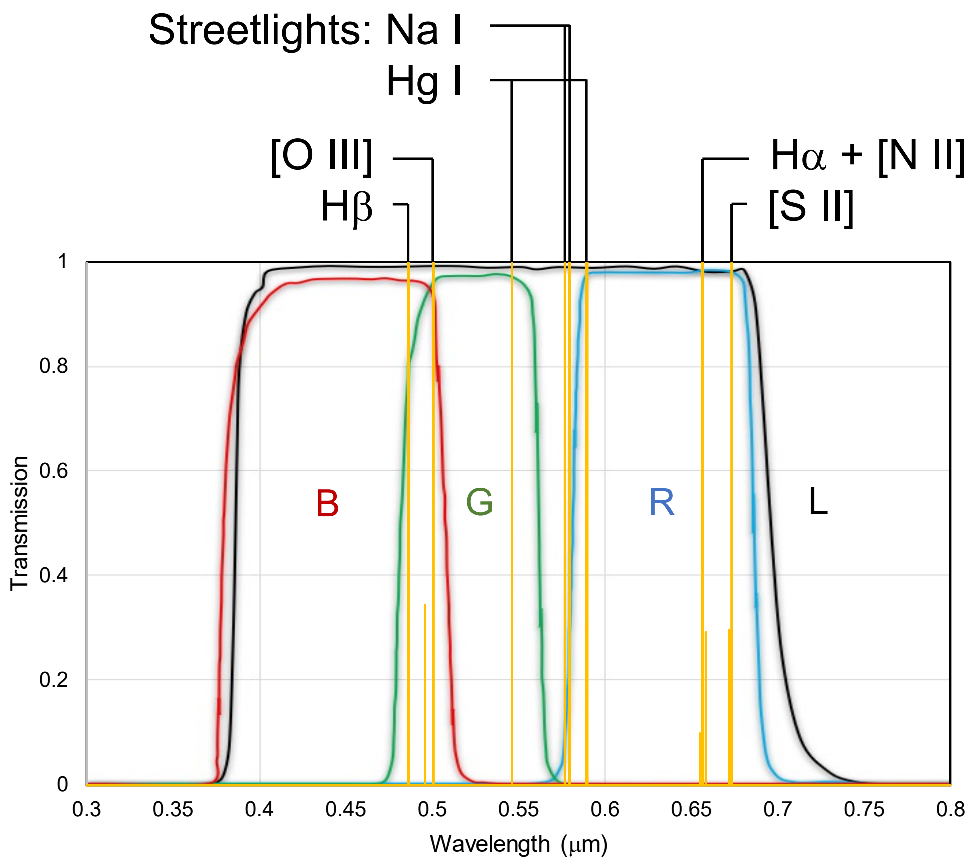




Shutter open

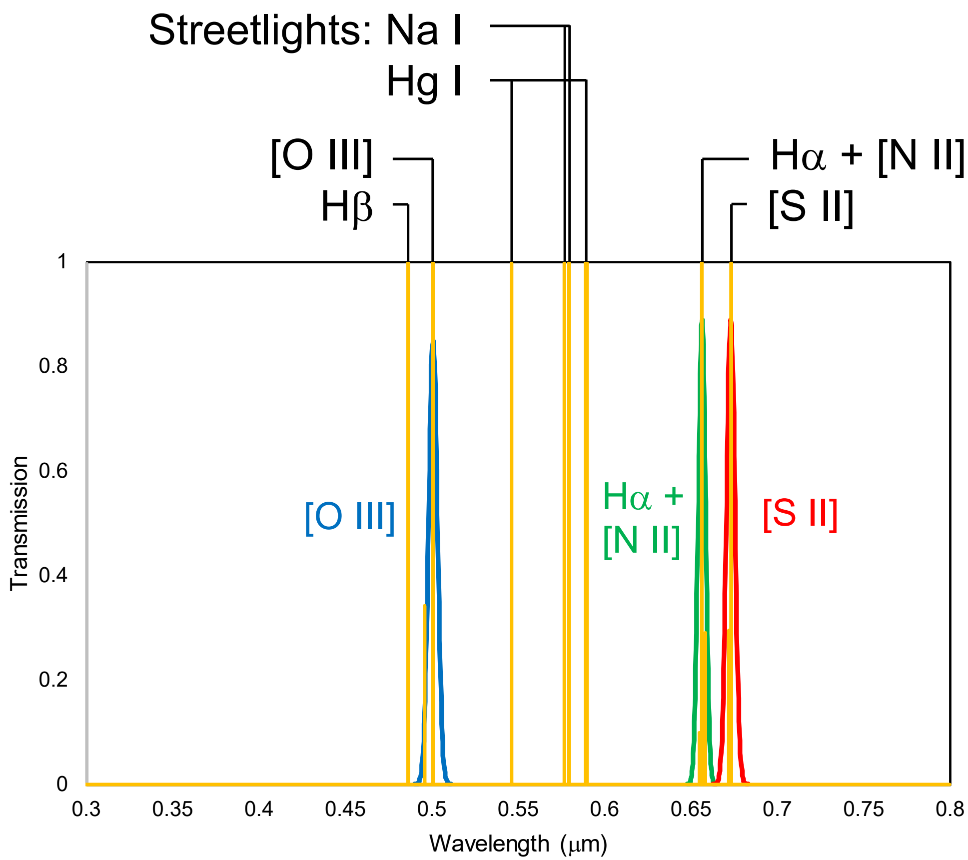
Our system (continued)

Broadband filters for stellar magnitudes and colors: L, R, G, and B, all of which have peak τ
$\tau \gtrsim 0.95$.



Our system (continued)

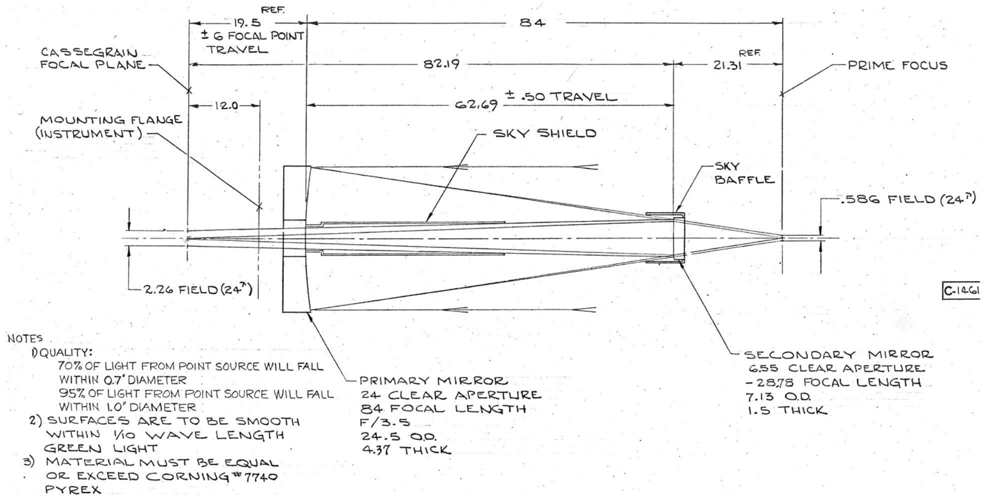
Narrowband filters for spectral lines: [O III], Hα, and [S II], all of which have peak τ = 0.85-0.9. FWHM widths are $\Delta\lambda$ = 8.5, 7.0, and 8.0 nm, respectively.



Our system (continued)

Boller & Chivens/DFM Engineering 24-inch Cassegrain telescope.

- ❑ Primary mirror by Perkin-Elmer, originally for the Stratoscope program.
- ❑ $f/13.5$, **plate scale** 25.1 arcsec per mm in the Cassegrain focal plane.
  - So the big CCD covers 0.224 arcsec/pixel, 15.4 arcmin on a side, 21.7 arcmin diagonal.
  - The autoguider CCD, in turn: 0.185 arcsec/pixel, 2.0 x 1.5 arcminutes.
- ❑ Unvignetted field of view 24 arcmin in diameter.
- ❑ Collecting area 2700 cm^2 = 0.27 m^2.



Brightness of celestial objects

Recall the **magnitude:**

- For two objects A and B, their fluxes F_A and F_B (power per unit area, in real physics units) and magnitudes are related by

$$m_A - m_B = 2.5 \log\left(F_B / F_A\right) \quad .$$

- Past that, one just needs the conversion to/from physical units for a zero-magnitude star, usually Vega. Here is Vega, for the Johnson filters:

Band	λ_0 (μm)	$\Delta\lambda$ (μm)	ν_0 (Hz)	F_λ $\left(\mathrm{W\ cm^{-2}\ \mu m^{-1}}\right)$	F_ν $\left(\mathrm{W\ m^{-2}\ Hz^{-1}}\right)$	$\log F_\nu$ (F_ν in $\mathrm{W\ m^{-2}\ Hz^{-1}}$)
U	0.36	0.07	8.3×10^{14}	4.35×10^{-12}	1.88×10^{-23}	-22.73
B	0.43	0.10	7.0×10^{14}	7.20×10^{-12}	4.44×10^{-23}	-22.36
V	0.54	0.09	5.6×10^{14}	3.92×10^{-12}	3.81×10^{-23}	-22.42
R	0.70	0.22	4.3×10^{14}	1.76×10^{-12}	2.88×10^{-23}	-22.54
I_S	0.80	0.24	3.7×10^{14}	1.20×10^{-12}	2.50×10^{-23}	-22.60

Examples

We do not know what the signal power would be from an arbitrary object in the sky, at an arbitrary wavelength, but we know the magnitudes of lots of stars.

1. What is the power that the Mees telescope collects from a 10th magnitude A0V star, within the bandwidth of the G filter?

- ❑ From the spectrum above we see that the G filter covers wavelengths λ = 501-590 nm. Thus its center wavelength is λ_0 = 546 nm and its bandwidth is $\Delta\lambda$ = 89 nm.

- ❑ This is very much like the Johnson *V* filter in the table above, so we'll assume the same F_λ in G, for the zero magnitude star:

$$\begin{aligned} F_0 &= F_\lambda \Delta\lambda = 3.92\times10^{-12}\ \mathrm{W\ cm^{-2}}\,\mu\mathrm{m}^{-1} \times 0.089\ \mu\mathrm{m} \\ &= 3.5\times10^{-13}\ \mathrm{W\ cm^{-2}}, \end{aligned}$$

Examples (continued)

- Then the tenth-magnitude flux F_{10} is given by

$$10 - 0 = 2.5\log\left(F_0/F_{10}\right) \quad \Rightarrow \quad F_{10} = 10^{-4}F_0 = 3.5\times10^{-17}\ \text{W cm}^{-2}.$$

- The telescope's collecting area is a = 2700 cm^{-2}, so

$$P_S = F_{10}a = \boxed{9.4\times10^{-14}\ \text{W.}}$$

Examples (continued)

2. Atmospheric turbulence (seeing) blurs the images of stars taken with uncorrected ground-based telescopes, typically to a diameter of 2 arcsec at Mees. Suppose for simplicity that this image is uniform in brightness. How many pixels of the array does it cover?

- The solid angle of this 2-arcsec uniform blur is $\Omega_{\mathrm{seeing}} = \pi\left(\frac{2\ \mathrm{arcsec}}{2}\right)^2 = \pi\ \mathrm{arcsec}^2$,
- and that of a pixel is $\Omega_{\mathrm{pixel}} = (0.224\ \mathrm{arcsec})^2 = 5.02\times10^{-2}\ \mathrm{arcsec}^2$,
- so the number of pixels is $N = \Omega_{\mathrm{seeing}}/\Omega_{\mathrm{pixel}} \approx \boxed{63.}$
- In reality, the seeing-broadened stellar image would be gaussian, with typical full width to half-maximum (FWHM) around 2 arcsec.

Examples (continued)

3. Suppose the star in Example 1 produces the image in Example 2. How many electrons are collected in each pixel of the star's image in a Δt = 300 sec exposure? By how many data numbers (DN) does the star's image exceed the background sky level, in the displayed image?

- The total charge in the star's image, collected within Δt, is $Q_S = I_S \Delta t$. Let's ignore atmospheric transmission (for now); then the number n of electrons in each of the N = 63 pixels is

$$n = \frac{Q_S}{Nq_e} = \frac{I_S \Delta t}{Nq_e} = \frac{\tau\eta}{N} \frac{\lambda_0 P_S}{hc} \Delta t$$

$$= \frac{(0.96)(0.6)}{63} \frac{(546 \text{ nm})(9.4 \times 10^{-14} \text{ W})}{hc} (300 \text{ sec}) \boxed{= 72,000 \text{ electrons}}$$

$$= 72,000 e^- \left(\frac{\text{DN}}{1.27 e^-} \right) \boxed{= 57,000 \text{ DN.}}$$

This stellar image would be close to saturation; the maximum output of a camera pixel is 65,536 DN (= 2^{16} DN).

Examples (continued)

4. On a moonless night, in a 300-second exposure in G, the background is measured – in blank, star-free sky – to be 70 DN per pixel. Is this observation background-limited?

- We know from the CCD camera specs that read noise is $\Delta Q_{R,\mathrm{rms}} = 9q_e$; for background and dark current we get

$$\overline{Q_B} = q_e\left(1.27/\mathrm{DN}\right)\left(70\ \mathrm{DN}\right) = 89q_e \quad \Rightarrow \quad \Delta Q_{B,\mathrm{rms}} = \sqrt{89q_e^2} = 9.4q_e$$
$$\overline{Q_D} = q_e\left(0.009\ e^-\ \mathrm{sec}^{-1}\right)\left(300\ \mathrm{sec}\right) = 2.7q_e \quad \Rightarrow \quad \Delta Q_{D,\mathrm{rms}} = \sqrt{2.7q_e^2} = 1.6q_e$$

- So, no, it is not; background shot noise and read noise contribute just about equally.

  - Sure enough, well-behaved parts of the same image have $\Delta Q_{\mathrm{rms}} = 11q_e$, not too far from the $10.3q_e$ expected from the three contributions above.
  - So, for broadband observations on moonless nights, one should take 8-10 minute frames instead of 5 minute frames like this one.

Examples (continued)

5. So suppose we had a moonless night and were taking Δt = 600 second exposures. How many of them would we have to take, barely to detect (S/N = 5) 24th magnitude stars, within the size of the stellar image as in Example 2?

- Suppose that gives a background of n = 140 DN = 178 electrons per pixel on the average, and that the observation is background limited.
- In N = 63 pixels, the noise current is therefore

$$I_{B,\mathrm{rms}} = \sqrt{\frac{q_e \overline{I_B}}{\Delta t}} = \sqrt{\frac{q_e}{\Delta t}\frac{Nnq_e}{\Delta t}} = \frac{q_e}{\Delta t}\sqrt{Nn} \quad \Rightarrow \quad \frac{I_{B,\mathrm{rms}}}{q_e} = 0.18\ \mathrm{sec}^{-1} \quad .$$

- Find the signal power – call it P_{S5} – that is five times noise:

$$\frac{S}{N} = \frac{I_S}{I_N} = \frac{\tau\eta q_e \lambda_0 P_{S5}}{hcI_{B,\mathrm{rms}}} = 5 \quad \Rightarrow \quad P_{S5} = \frac{5hcI_{B,\mathrm{rms}}}{\tau\eta q_e \lambda_0} = 5.7\times 10^{-19}\ \mathrm{W}.$$

Examples (continued)

- According to the chart of flux density for zero magnitude, the power for a 24th magnitude star in the G filter is $P_{G=24} = F_\lambda(V)a\Delta\lambda(2.5)^{-24} = 2.5\times10^{-19}$ W.

- So the noise current needs to be smaller by a factor of $f = P_{S5}/P_{G=24} = 2.25$.

- This, in turn, requires an increase in the exposure time by a factor of $f^2 \approx 5$, since

$$\left(\frac{S}{N}\right)_{BL} = P_S\sqrt{\frac{\lambda\tau\eta}{hcP_B}\Delta t} \quad .$$

- Since other problems would set in were we to try to take a single 3000-second exposure, this result suggests taking a total of five 600-second exposures.

Examples (continued)

6. Repeat problem 5, under the assumption that the seeing is 1 arcsecond instead of 2 arcseconds.

- The background, which is spread uniformly over the image, stays the same, independent of seeing.

- But now the stars are concentrated in $N = 63/4 \approx 16$ pixels, and only 16 pixels worth of background provide noise that may obscure the signal:

$$I_{B,\mathrm{rms}} = \sqrt{\frac{q_e \overline{I_B}}{\Delta t}} = \sqrt{\frac{q_e}{\Delta t}\frac{Nnq_e}{\Delta t}} = \frac{q_e}{\Delta t}\sqrt{Nn} \quad \Rightarrow \quad \frac{I_{B,\mathrm{rms}}}{q_e} = 0.089\ \mathrm{sec}^{-1} \quad .$$

$$\frac{S}{N} = \frac{I_S}{I_N} = \frac{\tau\eta q_e \lambda_0 P_{S5}}{hcI_{B,\mathrm{rms}}} = 5 \quad \Rightarrow \quad P_{S5} = \frac{5hcI_{B,\mathrm{rms}}}{\tau\eta q_e \lambda_0} = 2.9\times 10^{-19}\ \mathrm{W}.$$

Examples (continued)

- Since, as we have seen, the power from a 24th magnitude star is

$$P_{G=24} = F_\lambda\left(V\right) a\Delta\lambda\left(2.5\right)^{-24} = 2.5\times 10^{-19}\ \mathrm{W},$$

we almost get S/N = 5 on such stars in a single 600-sec frame. So, two frames, even though that's a little overkill.

- Takeaways:

  - Better/worse seeing improves/degrades sensitivity substantially.

  - In the days of astronomical photography, 24th magnitude stars were the faintest detected by the world's largest telescopes (4-5 m diameter). With CCD arrays, one can do experiments with a 24-inch telescope that once required a 200-inch telescope.