Periodically-variable objects

By which we mostly mean pulsating stars, spotted stars, or eclipsing systems, but from which we also learn to characterize any sort of time variation.

One-stop shopping for periodogram calculations:

the NStED periodogram tool,

which calculates Lomb-Scargle, box-fitting least squares, and Plavchan periodograms on your input data.

M 15 from Mees, true color with color saturation increased, blinking two images taken 1.5 hours apart.

Cadence

The rate at which photometric observations are made on potentially variable objects is called the **cadence**.

❑ Cadence has to be sufficiently fast to sample the variability thoroughly.

❑ If you know what the range of time durations is, a cadence can be designed.

❑ If the range is HUGE, like from millisecond flares to years-long orbits, the design can be a challenge.

❑ The Vera Rubin Telescope (nee LSST) will use many hybrid cadences, with a baseline of two back-to-back short exposures per filter per visit, and two visits per week to every object that's up.

Cadence (continued)

❑ The main *Kepler* survey had roughly
half-hour exposures every half hour for
about 90 days at a time, for several
years.

- • So it was very good at detecting
  exoplanetary orbits and most
  forms of stellar pulsation.

- • We'll use *Kepler* data in today's
  examples.
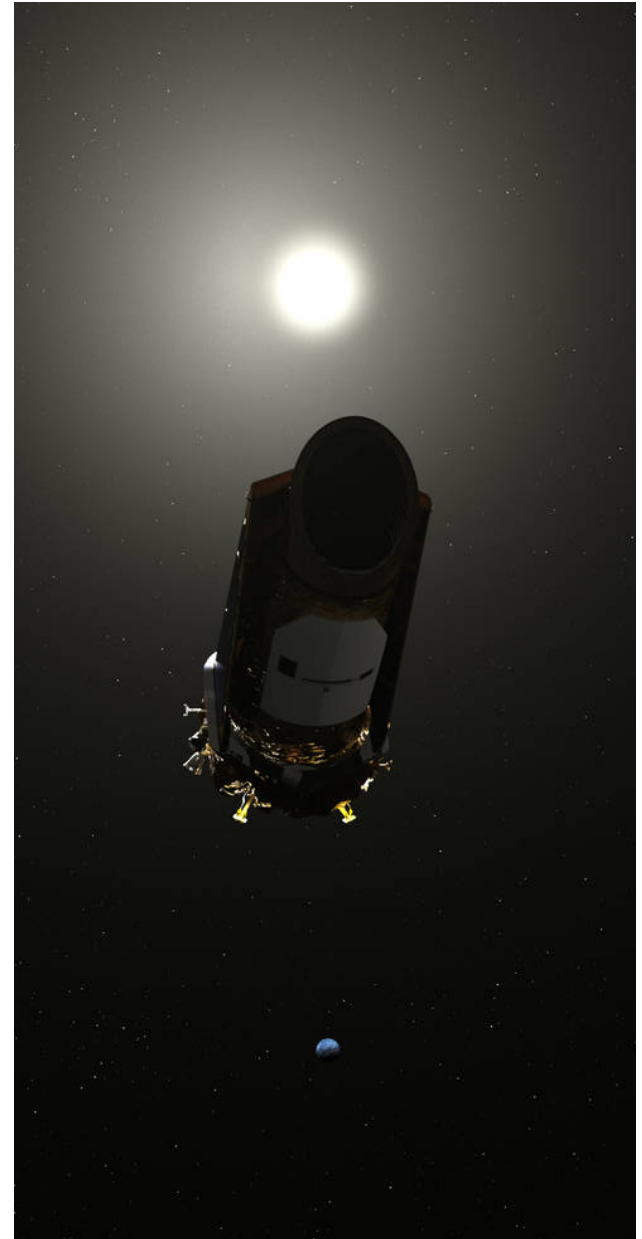
Finding variables: the simple way

In AST 244/444, we look for periodic variability in globular cluster members, and in transiting exoplanetary systems, and seek to measure the periods.

Most practical way to spot the variation:

❑ For RR Lyr stars in globular clusters: blink the images, as at right. Look hard.

❑ For exoplanetary systems: normalize the images for all stars except for the target, then perform aperture photometry on every image for the target star. Look for the dip.

M 15 from Mees, true color with color saturation increased, blinking two images taken 1.5 hours apart.

Finding variables in long-time-series data

The precise and accurate ways to identify variables and measure periods are predominantly these.

❑ **Periodograms**: power spectra derived from specialized forms of the Fourier transform, particularly the Lomb-Scargle periodogram (Scargle 1982, VanderPlas 2017).

❑ **Phase-dispersion minimization**: light curves folded and phase-shifted to minimize dispersion of the brightness at a given phase, relative to a prior or to a smoothed version of the light curve (Stellingwerf 1978, Plavchan et al. 2008).

❑ Both work better, the better the photometric accuracy and the longer the continuous time series.

❑ It's pointless to use either one unless the time series spans at least a few periods.

   • But that sometimes doesn't matter, e.g. if all you're after is the magnitude and dispersion of a population of variables, as in the baseline globular-cluster experiment.

Fourier transforms

A few reminders from MTH 281 or its equivalent:

❑ The Fourier transform of an evenly-spaced infinite time series of data $f(t)$ is the amplitude spectrum $F(\omega)$:

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \quad \Leftrightarrow \quad f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} d\omega$$

❑ If the data $f(t)$ were a sinusoid with (angular) frequency ω_0, the transform $F(\omega)$ would be a pair of delta functions: infinitely narrow spikes at $\pm\omega_0$.

❑ If the time series were to last only for a time T, the amplitude spectrum for a pure sinusoid would not be infinitesimally narrow. For example,

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-T}^{T} e^{i\omega_0 t} e^{-i\omega t} dt = \sqrt{\frac{2}{\pi}} \frac{\sin(\omega - \omega_0)T}{(\omega - \omega_0)T}$$

❑ the sinc function, with central peaks at $\pm\omega_0$, ~$2\pi/T$ wide (FWHM).

Discrete Fourier transforms

❑ In real life, transforms are discrete, as our time-series data are series of averages over intervals of time: **samples**.

❑ If the sampling interval is uniform at Δt for $N+1$ total samples, then the transform becomes

$$F(\omega_k) = \sum_{n=0}^{N} f_n e^{-i\omega_k n \Delta t}$$

❑ or, with $\Delta\omega = 2\pi/N\Delta t$ and $\omega_k = k\Delta\omega$,

$$F_k = \sum_{n=0}^{N} f_n e^{-2\pi i k n / N}$$

Power spectra

❑ "Periodogram" is another name for power spectrum, apart from an extra factor of the inverse of the number of samples:

$$P_0(\omega) = |F(\omega)|^2 = \left| \sum_{n=0}^{N} f_n e^{-i\omega n \Delta t} \right|^2$$

(Discrete) power spectrum

$$P_S(\omega) = \frac{1}{N} \left| \sum_{n=0}^{N} f_n e^{-i\omega n \Delta t} \right|^2$$

Classical (Schuster) periodogram

❑ To be precise, the periodogram is an estimator of the power spectrum, as the power spectrum is a continuous function of the parameters of the star or other emitting region, but most of us ignore the distinction.

Aliasing

❑ With discrete sampling, say $N+1$ samples at interval Δt, one cannot take seriously a frequency higher than $\omega_0 = \pi/\Delta t$, as power is incorrectly mapped to these frequencies. This called **aliasing**.

❑ In the frequency domain instead of the time domain: to represent the frequency content of a signal that is zero outside the range $\pm\omega_0$, one must sample the frequency data at a rate of at least $v_0 = 2(\omega_0/2\pi)$.

❑ This is known as the **Nyquist theorem**, and the minimum sampling rate is known as the **Nyquist rate**.

- So the transform of page 5 has no aliases.
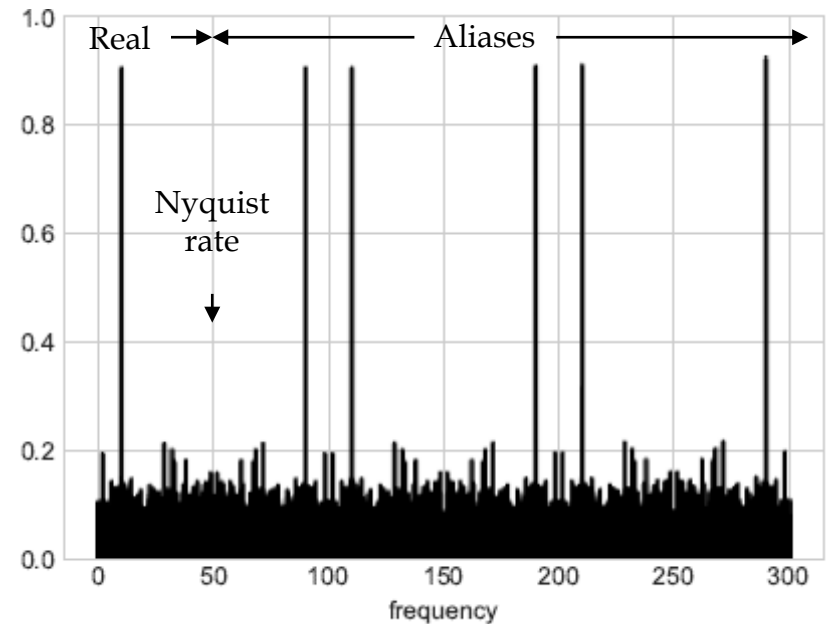
A power spectrum extended to frequencies way too high, to show the aliases. From vanderPlas 2017.

Nonuniform sampling and the Lomb-Scargle periodogram

❑ Sampling intervals in astronomical observations are hardly ever uniform. For a set of samples at times t_n, and upon writing the classical periodogram as

$$P_S(\omega) = \frac{1}{N}\left(\left[\sum_{n=0}^{N} f_n \cos \omega t_n\right]^2 + \left[\sum_{n=0}^{N} f_n \sin \omega t_n\right]^2\right) \, ,$$

Scargle (1982) generalized the expression to

$$P(\omega) = \frac{A^2}{2}\left[\sum_{n=0}^{N} f_n \cos \omega(t_n - \tau)\right]^2 + \frac{B^2}{2}\left[\sum_{n=0}^{N} f_n \sin \omega(t_n - \tau)\right]^2 \, ,$$

and showed that there are unique forms for A, B, and τ such that the new periodogram reduces to the classical one in the limit of uniform sampling interval, and give other desirable properties too, like analytically-computable statistics.

Lomb-Scargle periodogram (continued)

❑ It also turns out that this generalized periodogram is identical to what you get by fitting a sinusoid to the data at every frequency by χ^2 minimization, as was shown by Lomb (1976).

❑ The result is called the **Lomb-Scargle periodogram**:

$$P_{LS}(\omega) = \frac{1}{2} \frac{\left[\sum_{n=0}^{N} f_n \cos \omega (t_n - \tau) \right]^2}{\sum_{n=0}^{N} \cos^2 \omega (t_n - \tau)} + \frac{1}{2} \frac{\left[\sum_{n=0}^{N} f_n \sin \omega (t_n - \tau) \right]^2}{\sum_{n=0}^{N} \sin^2 \omega (t_n - \tau)} \quad , \text{where}$$

$$\tau = \frac{1}{2\omega} \arctan \left(\sum_{n=0}^{N} \sin 2\omega t_n \middle/ \sum_{n=0}^{N} \cos 2\omega t_n \right) \quad .$$

and is much easier to calculate than it looks.

❑ Its use is now universal for measuring periods from time-series data on objects with **large duty-cycle** variability, such as RR Lyr and Cepheid variable stars. Built into the the NStED periodogram tool.

Terminology
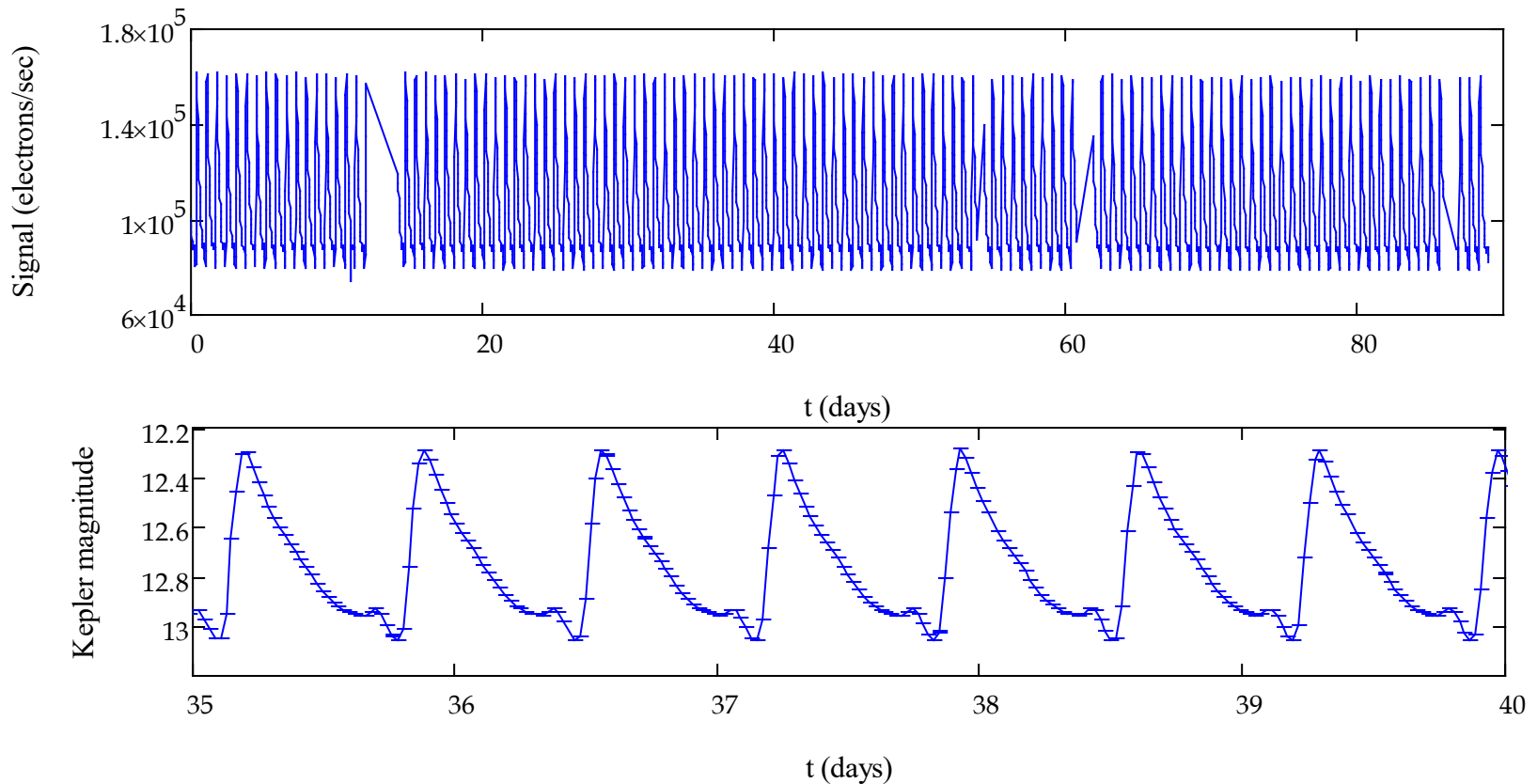
A large duty cycle oscillation:
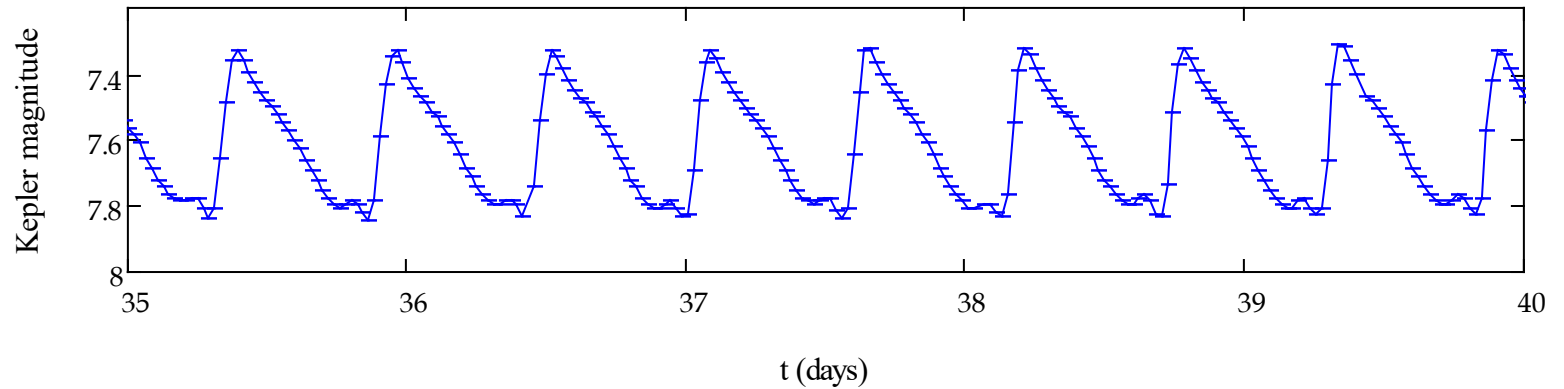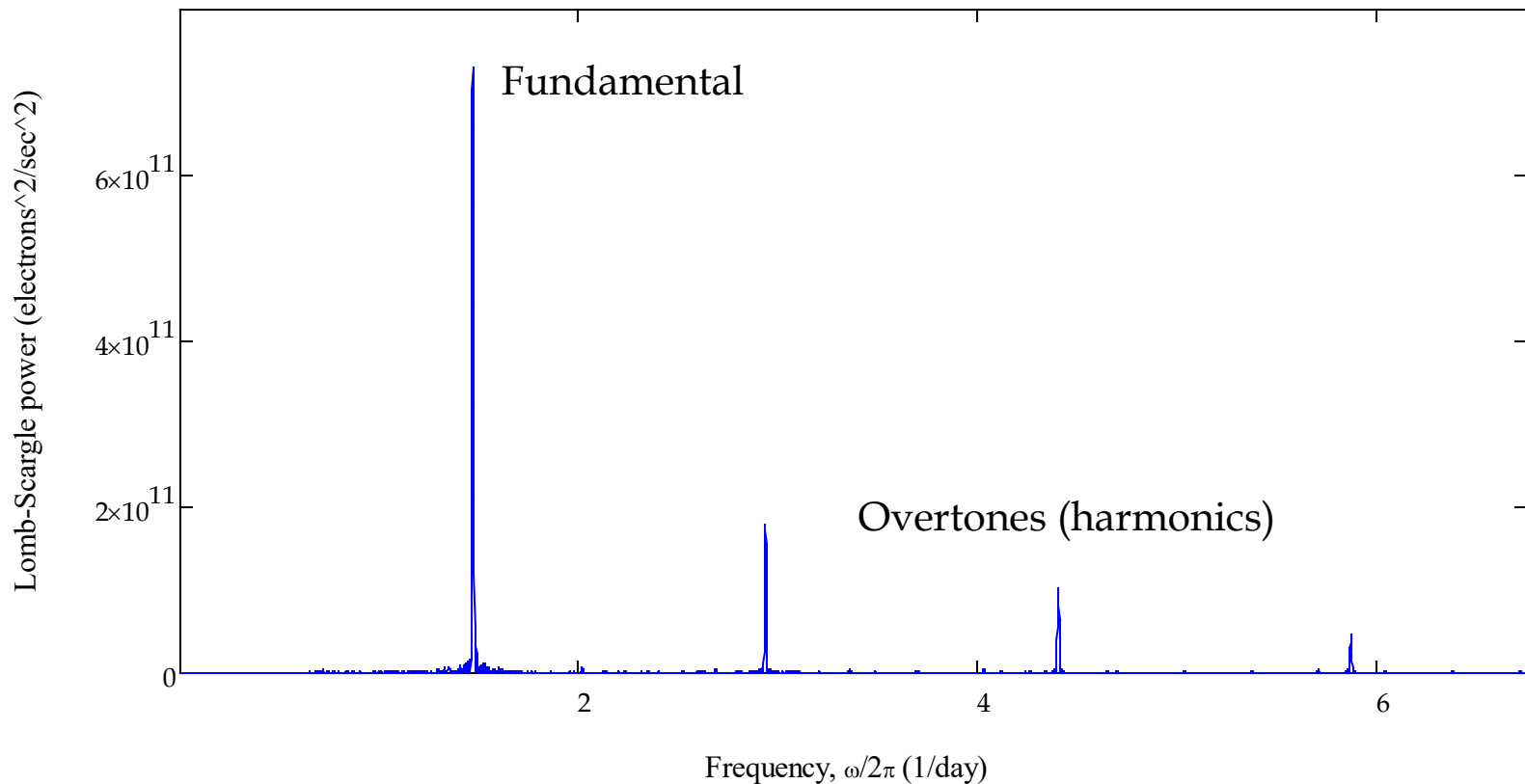

A small duty cycle oscillation:
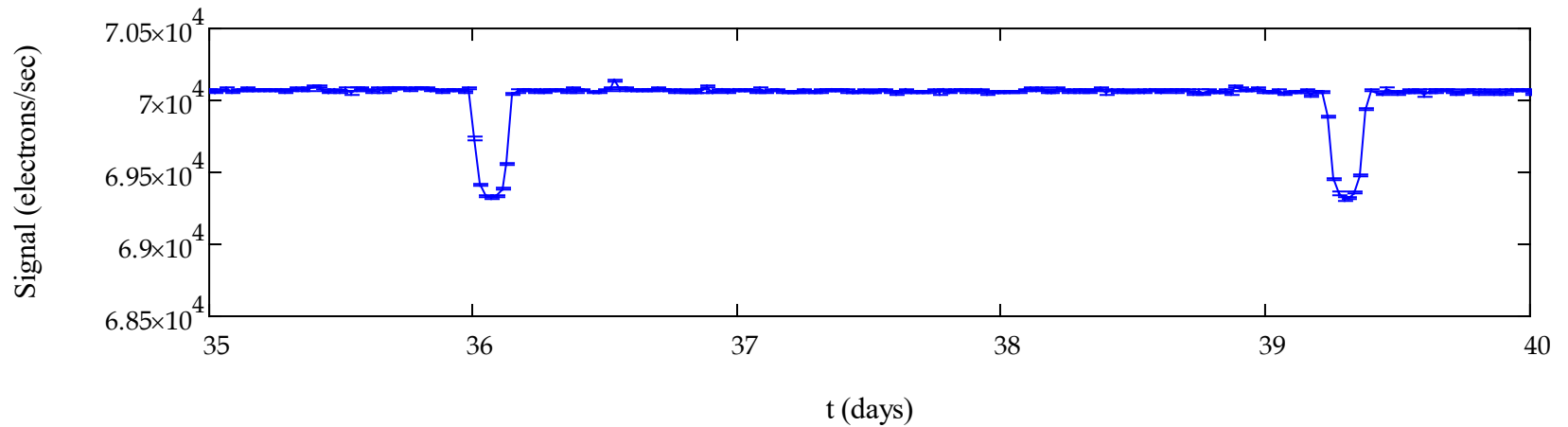
Example 1: NR Lyrae, a perfect RR Lyrae star

Full light curve, and a close up, from *Kepler* Q2. *Kepler* gives really high-quality data.
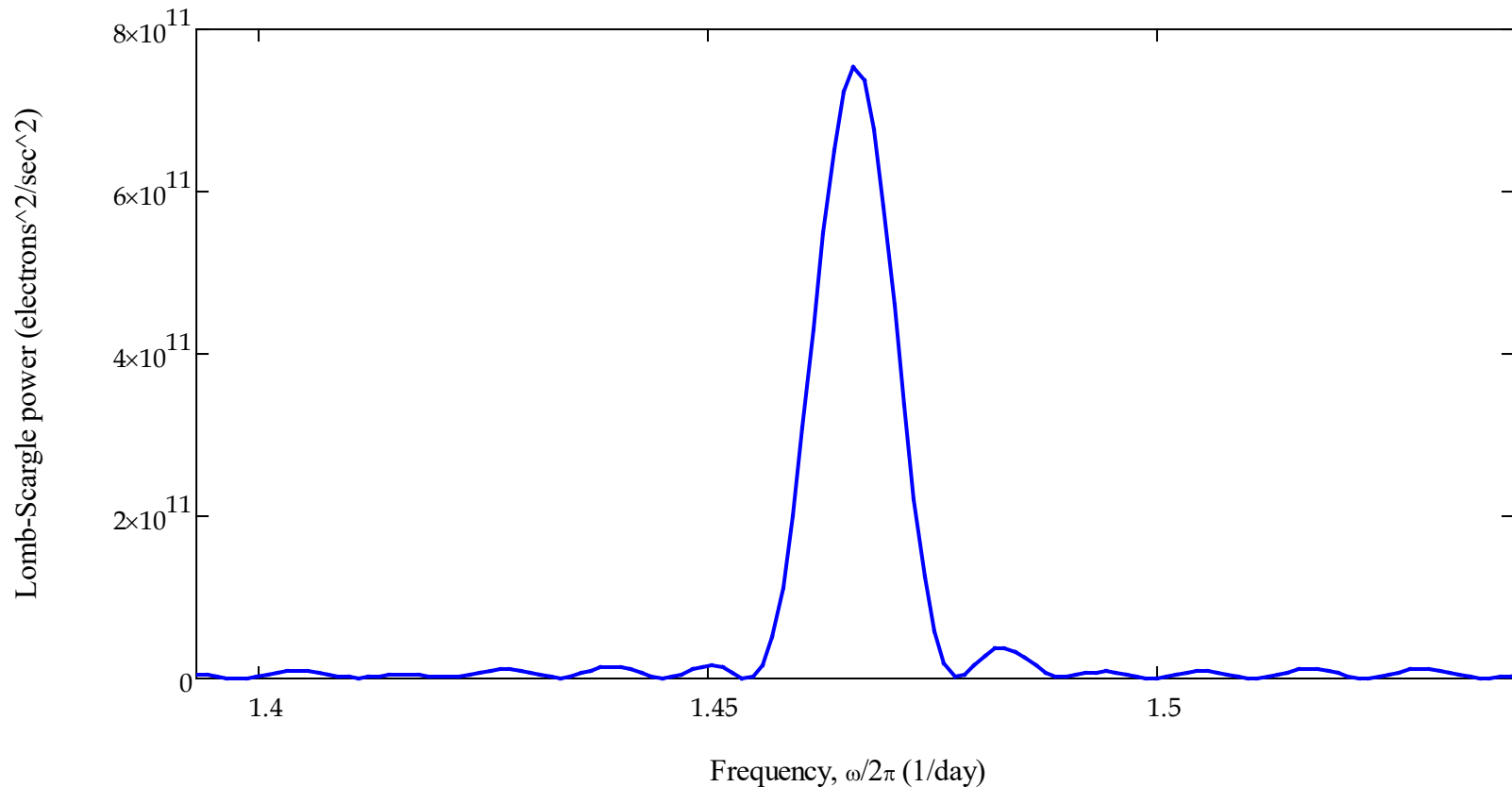
NR Lyrae (continued)

L-S periodogram, after "prewhitening:" basically filtering out low frequencies.
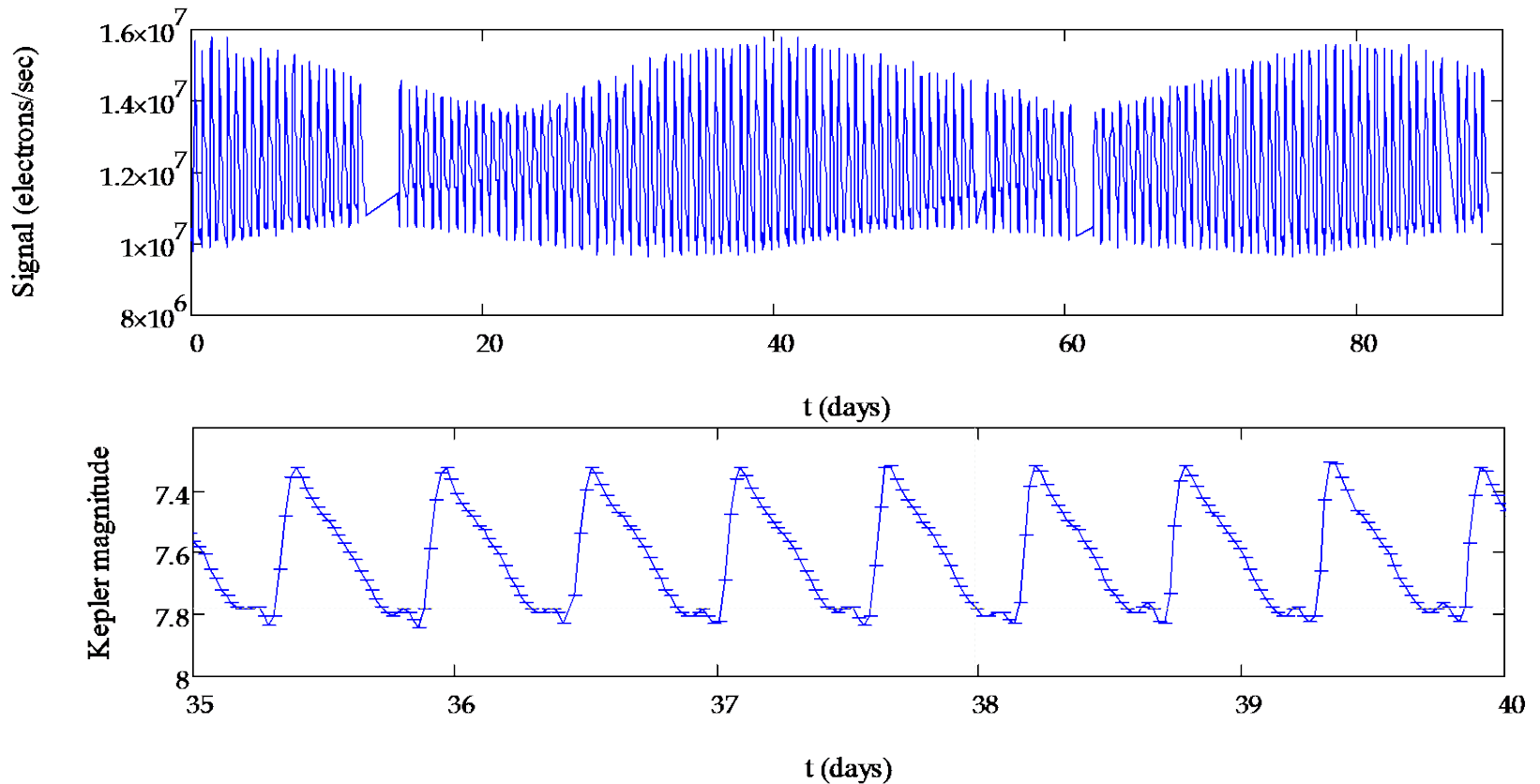
NR Lyrae (continued)

Zoom in on fundamental mode. Note square-sinc-function-like sidebands. The pulsation period is 16.369 hours.
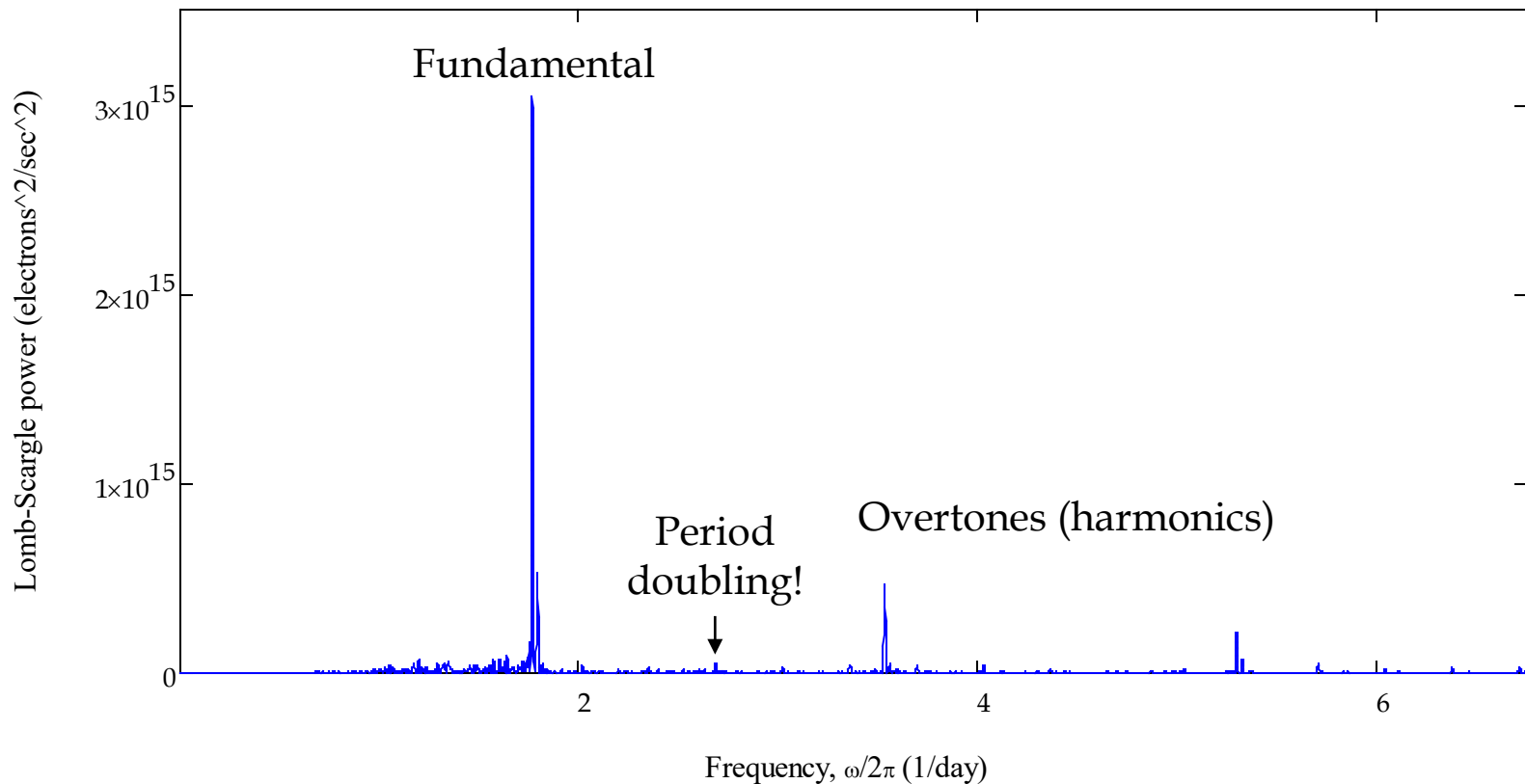
Example 2: RR Lyrae, the archetype

Light curve, from *Kepler* Q2. Note the periodic envelope, classically known as the <u>Blazhko (1907)</u> effect.
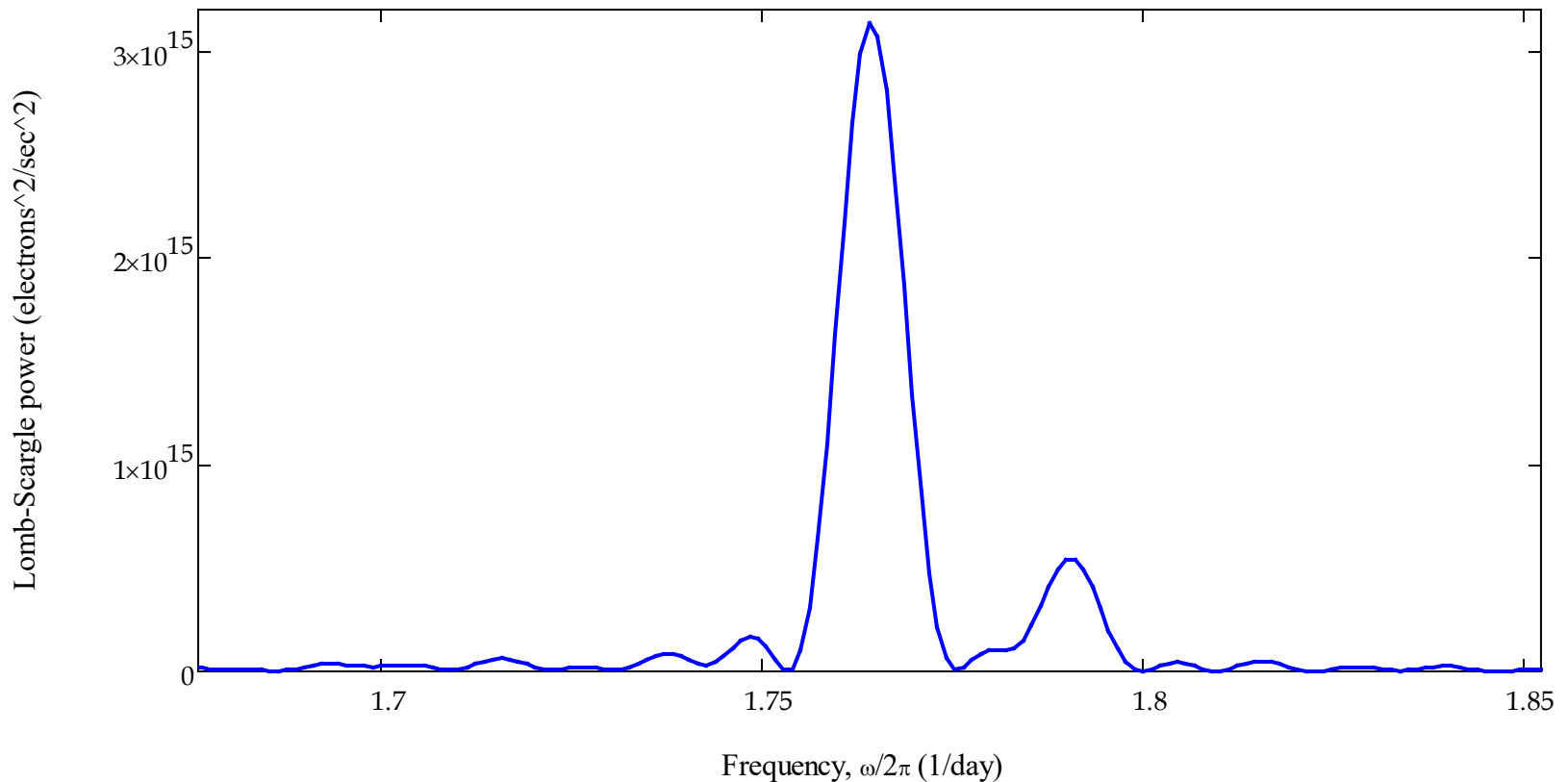
RR Lyrae (continued)

L-S periodogram, after prewhitening. Power at half-integer "harmonics!"

RR Lyrae (continued)

Zoom in on fundamental mode. Note lopsided sidebands, which are the power-spectrum counterpart of the Blazhko effect. The pulsation period is 13.605 hours.

RR Lyrae (continued)

"1.5th harmonic:" sidebands present but not the main pulsation. Half-integer features in the power spectrum – a.k.a. period doubling – shows the origin of the Blazhko effect in nonlinear pulsation (Szabo et al. 2010). No feature like this in NR Lyrae.
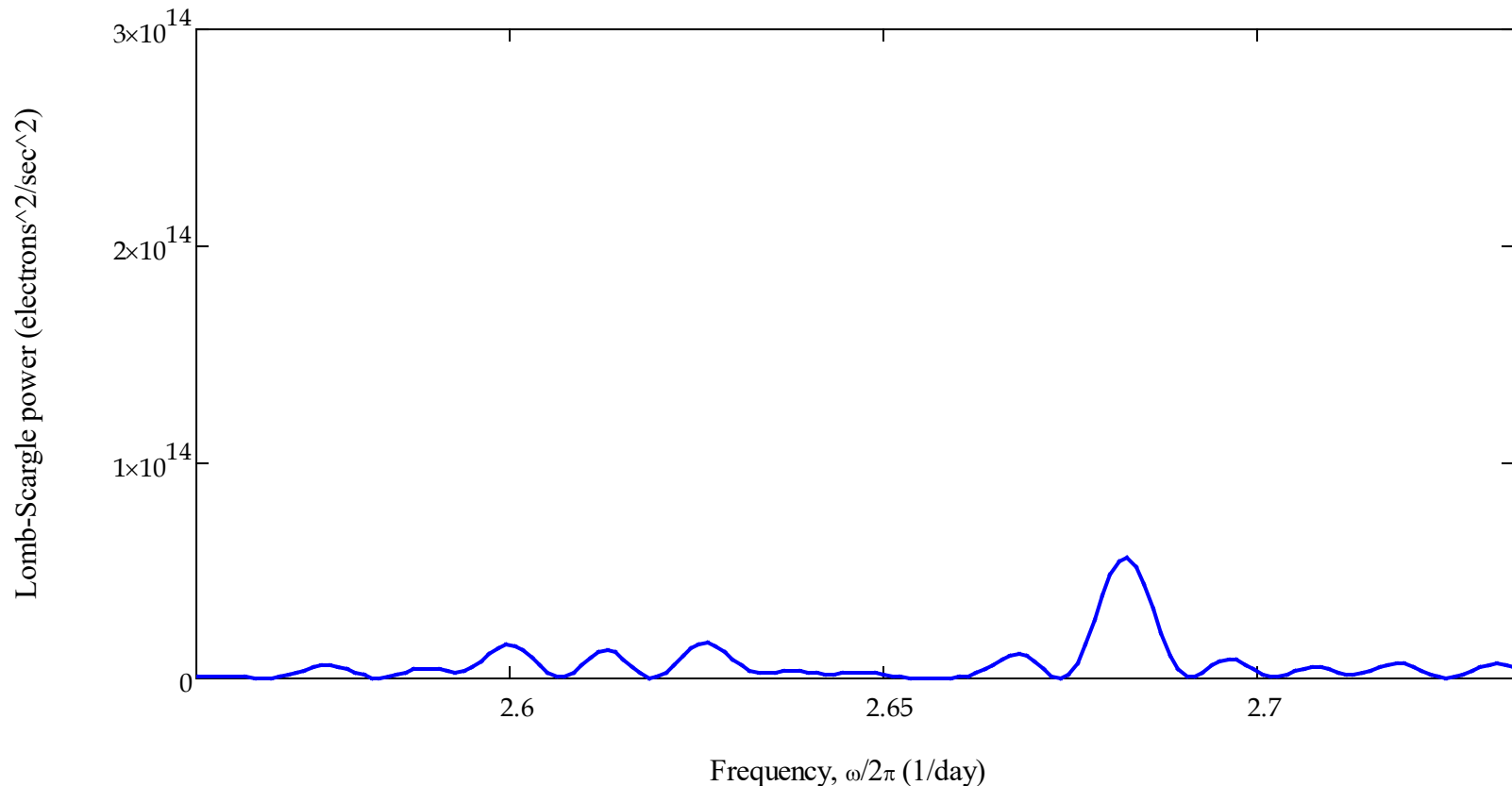
How to make a phase-folded light curve

For long time-series observations, it is useful and customary to phase-fold the light curves, so that all the events in the series are plotted together according to their **phase** within the period.

❑ All programming languages have a built-in function called **mod** that can be used to map time t_n into phase ϕ_n like so:

$$\phi_n = \frac{2\pi}{\delta t} \mathrm{mod}\left(t_n, \delta t\right) + \delta\phi \quad ,$$

where

δt is the period of the variability (pulsation, orbit,...) and

$\delta\phi$ is an initial phase offset.

❑ mod takes every δt segment of the time series and shifts it to earlier times by the amount δt. The other factors just convert the period to the phase scale, 0 to 2π radians, and shifts the scale overall to center it in plots as desired.
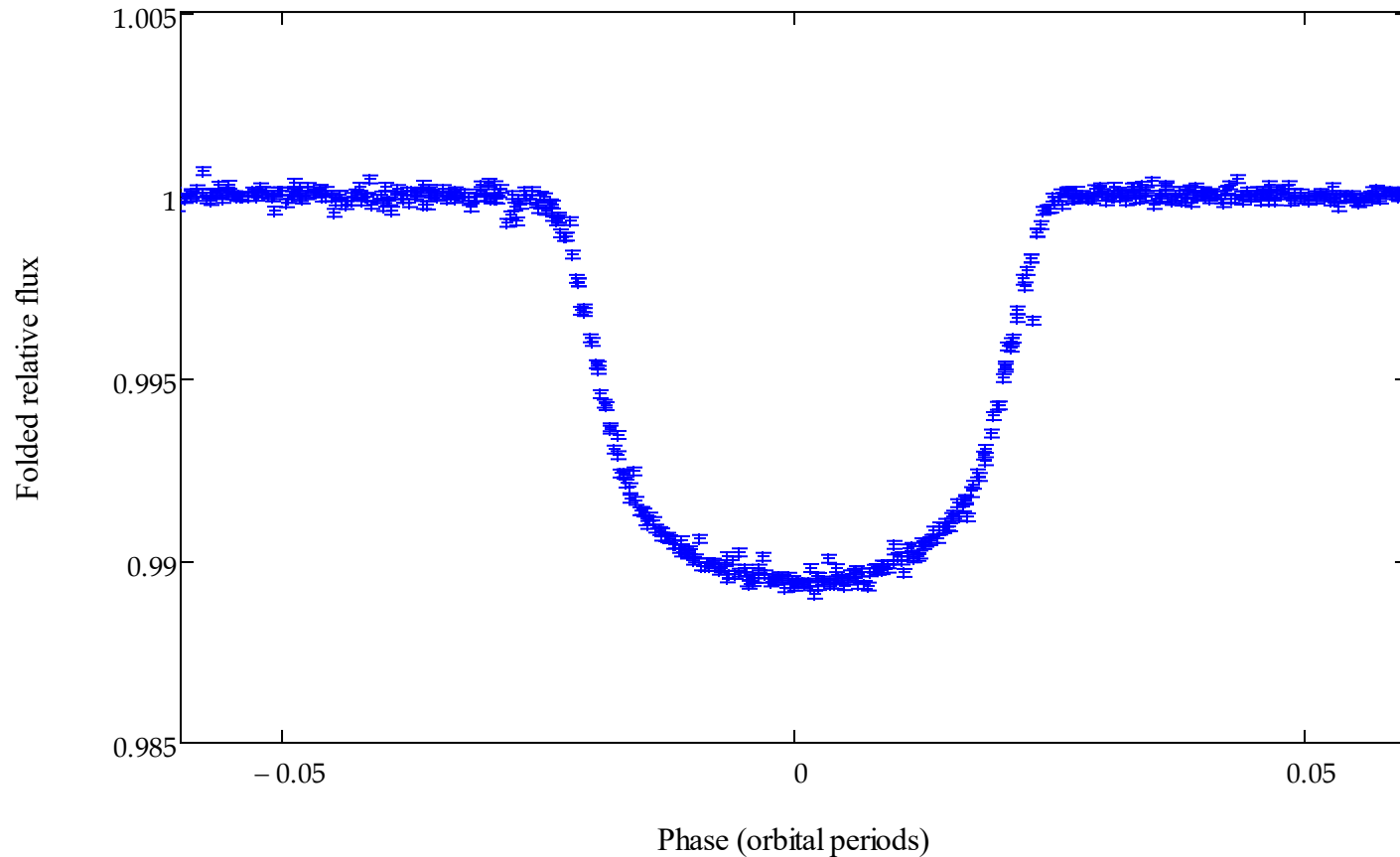
Example 3: Kepler-6 b

Kepler-6 b is a classic hot Jupiter. The early, *Kepler* Q1, data are published in the *Kepler* special issue ([Dunham et al. 2010](#)); this is Q2.
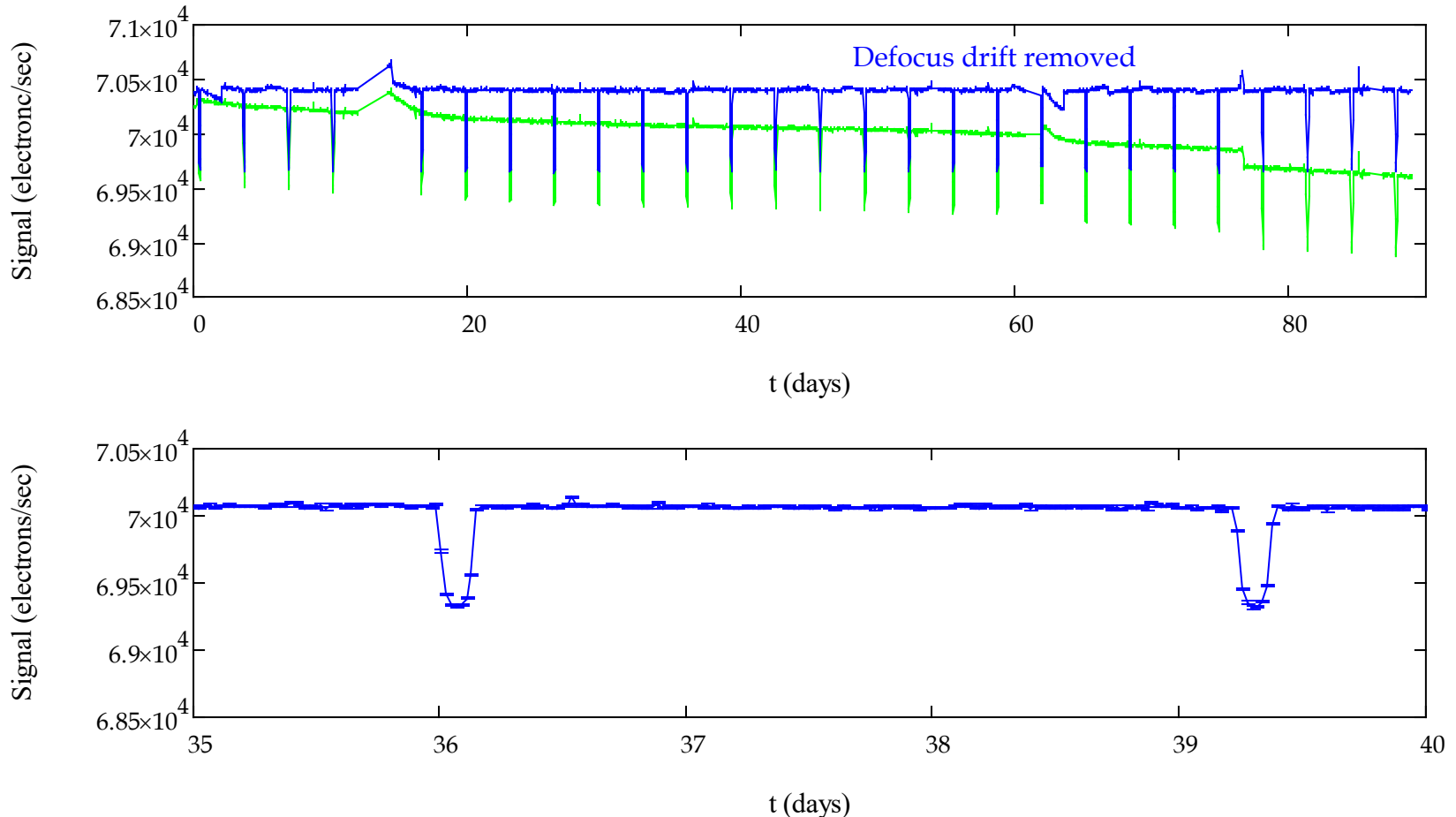
Kepler-6 b (continued)

Phase folded over δt = 3.234723 days, $\delta\phi$ = -0.3π, phase values divided by 2π before plotting.

Phase-dispersion minimization

This procedure immediately suggests an automated, model-independent way to fold the light curve and determine the best value of the period δt:

❑ Start with a value for δt. Generate an initial folded light curve. Calculate its mean and variance,

$$\overline{f} \text{ and } \sigma^2 = \sum_n \left(f_n - \overline{f} \right)^2 \bigg/ (N-1) \quad .$$

❑ Break the folded light curve into M bins, each containing m points. Within each bin j, calculate the mean and variance s_j^2. Sum the variances to create

$$s^2 = \left[\sum_j \left(m_j - 1 \right) s_j^2 \right] \bigg/ \left(\sum_j m_j - M \right) \quad \text{and} \quad \Theta = \frac{s^2}{\sigma^2} \quad .$$

❑ If δt is not the true period, then s^2 will be close to σ^2 and $\Theta \sim 1$.

❑ But as δt approaches the true period, Θ approaches a minimum, with $\Theta \ll 1$.

❑ So vary δt to find the minimum, or incorporate it all into a root-finding routine.

Example 4: minimizing phase dispersion in Kepler-6 b

$$\frac{\delta t}{day} = 3.234723$$

(Plot: Folded relative flux vs. Phase (orbital periods))
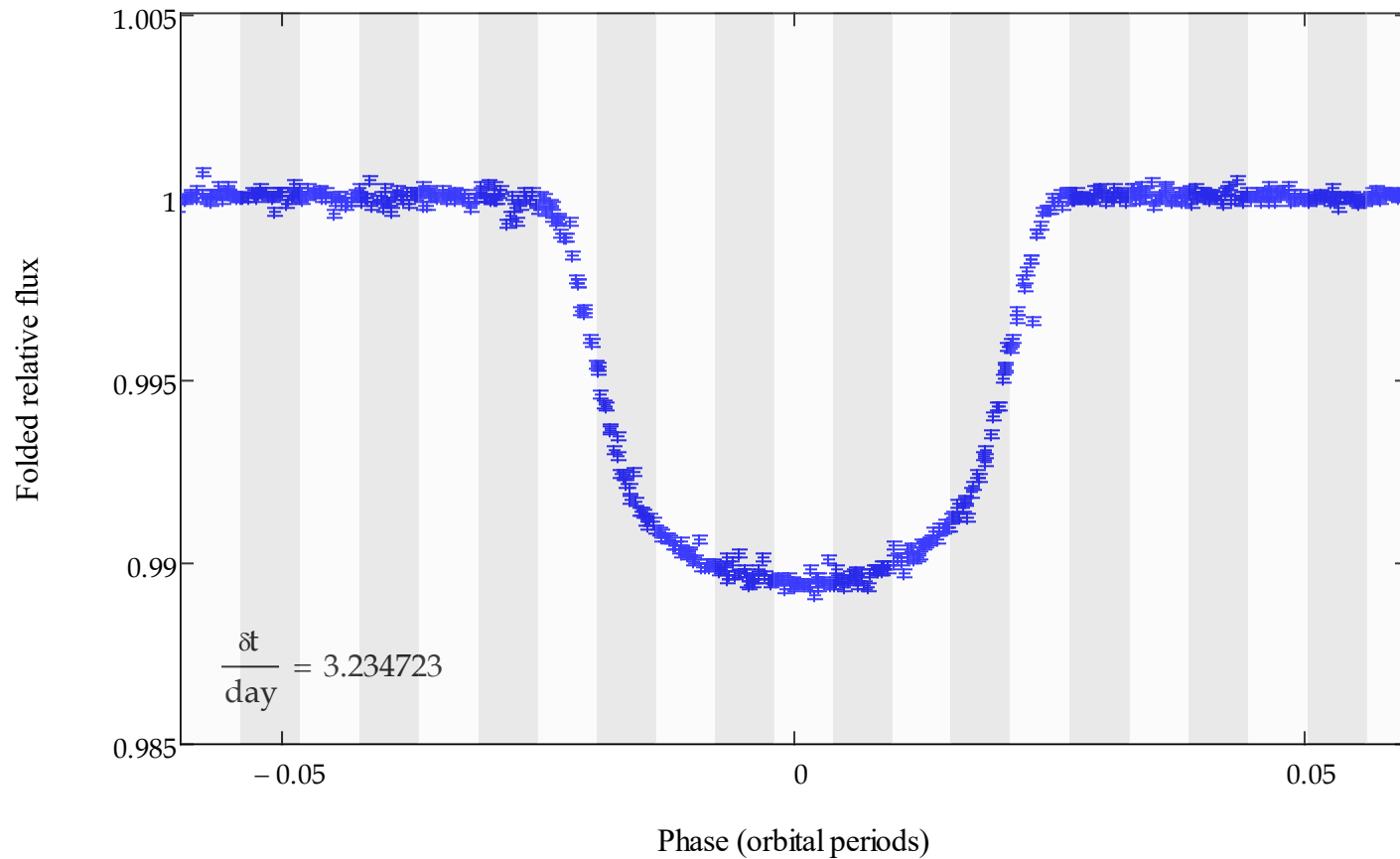
Phase-dispersion minimization (continued)

❑ This means of folding light curves and determining periods was invented by Stellingwerf (1978) and has been in standard use ever since.

❑ A particularly nice version by Plavchan et al. (2008) has been incorporated in the NStED periodogram tool.

Example 5: phase-folded light curve for NR Lyrae

The smooth run of the points and the error-bar ends probably indicates that the tiny uncertainties are still overestimated. (See next lesson.)
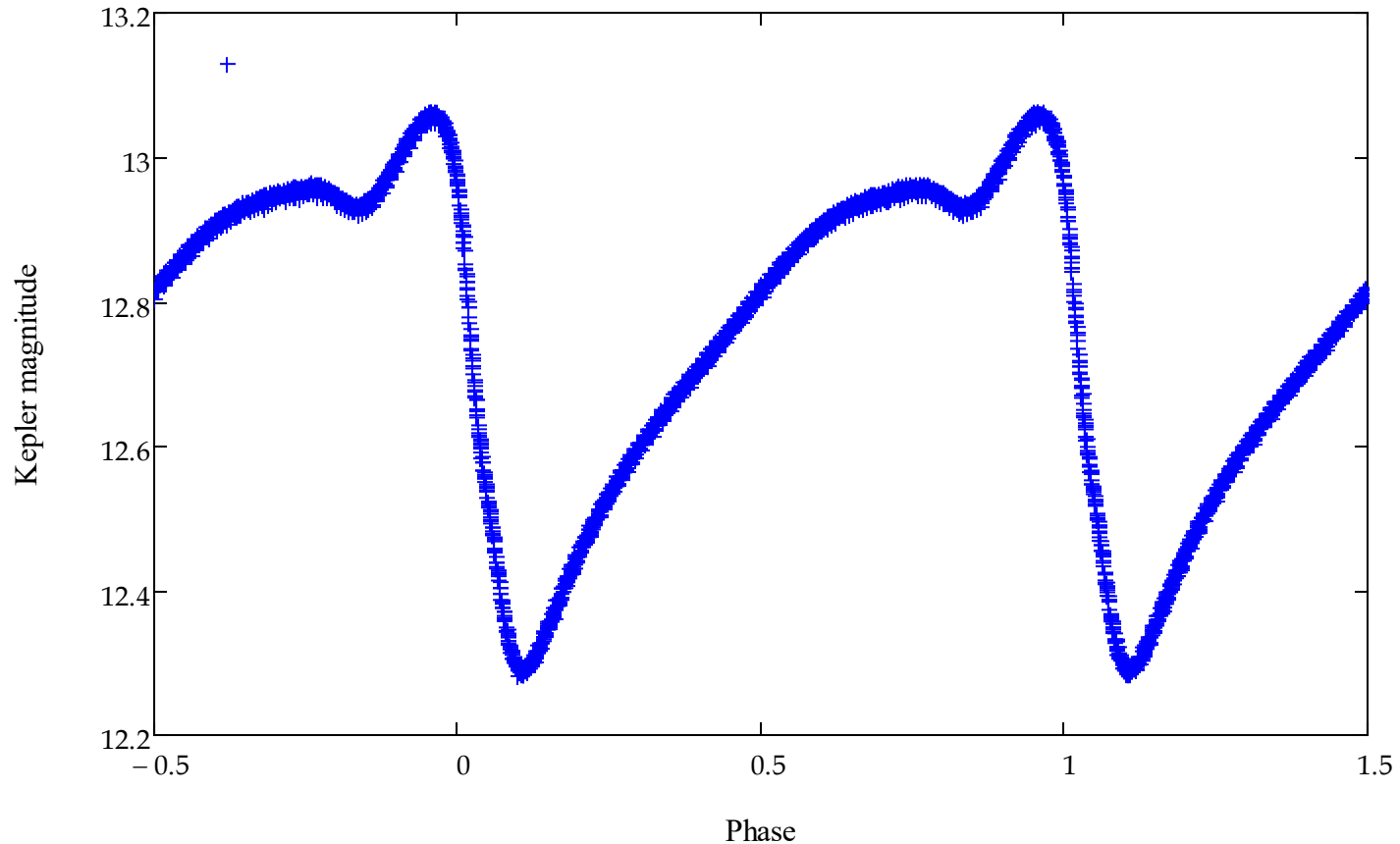
Example 6: phase-folded light curve for RR Lyrae

No errorbars plotted, as the large Blazhko effect would make it too confusing.