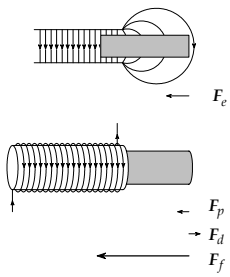
Today in Physics 217: magnetism in matter

- Magnetism
- Magnetization and bound currents
- Ampère's Law and magnets
- Linear magnetic media
- Cautions about H
- Example: H and Ampère's Law



Magnetism

There are three forms of magnetism in matter, all having to do with **spins of atomic electrons**.

- Electrons have spin, and spinning charges have magnetic dipole moments that can be aligned by external magnetic fields.
- However, the spin of electrons, and the pairings of electrons within atoms, are manifestations of **quantum-mechanical** effects, so magnetism is not really understandable simply from our present **classical** viewpoint.

Anyway, here are the three kinds of magnetism:

Magnetism (continued)

- **Ferromagnetism**
  The layman's magnetism, this is the only *strong* form of magnetism.
  - It comes from aligned spins of unpaired d electrons in transition metals, especially nickel, iron and cobalt (hence the name), and rare earths like samarium.
- **Paramagnetism**
  Analogous to electric polarization, and much weaker than ferromagnetism; can almost understand classically.
  - Produced by interaction of $\boldsymbol{B}$ with dipole moments of unpaired s or p electrons. Aluminum, for example, has one p electron in its valence shell, and is paramagnetic. So is molecular oxygen, which has two unpaired spins among its bonding electrons.

Magnetism (continued)

- **Diamagnetism**
  Comes from the interaction of $\boldsymbol{B}$ with induced magnetic dipoles in atoms; also much weaker than ferromagnetism.
  - Contrary to the way electric polarization works, diamagnetism is characterized by magnetization in the direction *opposite* that of the applied field.
  - Seen best in materials in which all the electrons are paired off (Ne, N_2, ...).

Whatever the type, we can define a **magnetization**, in analogy with the electric polarization $\boldsymbol{P}$:

$$\boldsymbol{M} = \frac{\text{magnetic dipole moment}}{\text{unit volume}}$$

Magnetization and bound currents

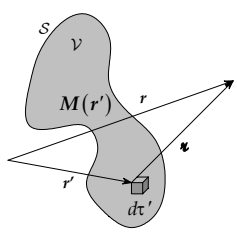
Like $\boldsymbol{P}$, which we interpret in terms of bound charge, $\boldsymbol{M}$ can be interpreted in terms of bound currents, which we can characterize by consideration of the magnetic potential for a dipole.

$$\boldsymbol{A} = \frac{\boldsymbol{m}\times\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2} = \int_{\mathcal{V}} \frac{\boldsymbol{M}\times\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}\, d\tau'$$

$$= \int_{\mathcal{V}} \boldsymbol{M}\times\nabla'\left(\frac{1}{\mathscr{r}}\right) d\tau'$$

Use Product Rule #7:

$$= \int_{\mathcal{V}} \left[\frac{1}{\mathscr{r}}\nabla'\times\boldsymbol{M} + \nabla'\times\left(\frac{\boldsymbol{M}}{\mathscr{r}}\right)\right] d\tau'$$



Magnetization and bound currents (continued)

Now, for any vector function $\boldsymbol{v} = \boldsymbol{v}(\boldsymbol{r})$ and a constant vector $\boldsymbol{C}$, we can write the divergence theorem as

$$\int \nabla\cdot(\boldsymbol{v}\times\boldsymbol{C})\, d\tau = \oint (\boldsymbol{v}\times\boldsymbol{C})\cdot d\boldsymbol{a}$$

$$\int \left[\boldsymbol{C}\cdot(\nabla\times\boldsymbol{v}) + \boldsymbol{v}\cdot\cancelto{0}{(\nabla\times\boldsymbol{C})}\right] d\tau = \oint (\boldsymbol{v}\times\boldsymbol{C})\cdot d\boldsymbol{a} \qquad \text{Product Rule \#6}$$

$$\boldsymbol{C}\cdot\int \nabla\times\boldsymbol{v}\, d\tau = \oint (\boldsymbol{v}\times\boldsymbol{C})\cdot d\boldsymbol{a} \qquad \boldsymbol{C} \text{ is constant}$$

$$= \boldsymbol{C}\cdot\oint d\boldsymbol{a}\times\boldsymbol{v} \qquad \text{triple product rule \#1}$$

$$\int \nabla\times\boldsymbol{v}\, d\tau = \oint d\boldsymbol{a}\times\boldsymbol{v} \quad .$$

Thus $\boldsymbol{A} = \int_{\mathcal{V}} \frac{1}{\mathscr{r}}\nabla'\times\boldsymbol{M}\, d\tau' + \oint_{\mathcal{S}} \frac{\boldsymbol{M}\times d\boldsymbol{a}'}{\mathscr{r}}$.

Magnetization and bound currents (continued)

Thus
$$\boldsymbol{A} = \int_{\mathcal{V}} \frac{1}{\mathscr{r}} \nabla' \times \boldsymbol{M} d\tau' + \oint_{\mathcal{S}} \frac{\boldsymbol{M} \times d\boldsymbol{a}'}{\mathscr{r}}$$
$$= \frac{1}{c} \int_{\mathcal{V}} \frac{1}{\mathscr{r}} \boldsymbol{J}_b d\tau' + \frac{1}{c} \oint_{\mathcal{S}} \frac{1}{\mathscr{r}} \boldsymbol{K}_b d\boldsymbol{a}' \quad ,$$

where we have defined the **bound current densities**:

$$\boldsymbol{J}_b(\boldsymbol{r}') = c\nabla' \times \boldsymbol{M}(\boldsymbol{r}') \quad , \qquad (\text{cf. } \rho_b = -\nabla \cdot \boldsymbol{P},$$
$$\boldsymbol{K}_b(\boldsymbol{r}') = c\boldsymbol{M} \times \hat{\boldsymbol{n}}\big|_{\mathcal{S}} \quad . \qquad \sigma_b = \boldsymbol{P} \cdot \hat{\boldsymbol{n}}\big|_{\mathcal{S}})$$

[In MKS, $\boldsymbol{J}_b(\boldsymbol{r}') = \nabla' \times \boldsymbol{M}(\boldsymbol{r}')$, $\boldsymbol{K}_b(\boldsymbol{r}') = \boldsymbol{M} \times \hat{\boldsymbol{n}}\big|_{\mathcal{S}}$,

$$\boldsymbol{A} = \frac{\mu_0}{4\pi} \int_{\mathcal{V}} \frac{1}{\mathscr{r}} \boldsymbol{J}_b d\tau' + \frac{\mu_0}{4\pi} \oint_{\mathcal{S}} \frac{1}{\mathscr{r}} \boldsymbol{K}_b d\boldsymbol{a}' \quad .]$$

Ampère's Law revisited

With bound currents we can do to Ampère's Law about what we did to Gauss's law before:

$$\nabla \times \boldsymbol{B} = \frac{4\pi}{c} \boldsymbol{J} = \frac{4\pi}{c}\left(\boldsymbol{J}_{\text{free}} + \boldsymbol{J}_b\right) = \frac{4\pi}{c}\left(\boldsymbol{J}_f + c\nabla \times \boldsymbol{M}\right)$$
$$\nabla \times \left(\boldsymbol{B} - 4\pi \boldsymbol{M}\right) = \frac{4\pi}{c} \boldsymbol{J}_f$$
$$\Rightarrow \quad \nabla \times \boldsymbol{H} = \frac{4\pi}{c} \boldsymbol{J}_f \quad , \quad \boldsymbol{H} \equiv \boldsymbol{B} - 4\pi \boldsymbol{M} \quad (\text{cf. } \boldsymbol{D} = \boldsymbol{E} + 4\pi \boldsymbol{P})$$

Or, in MKS, $\nabla \times \boldsymbol{H} = \boldsymbol{J}_f$, $\boldsymbol{H} \equiv \frac{1}{\mu_0} \boldsymbol{B} - \boldsymbol{M}$.

Linear magnetic media

As before, when the definition of $\boldsymbol{D}$ led to that of the electric susceptibility χ_e and the dielectric constant ε, we can define

$\cancel{\boldsymbol{M} = \chi_m \boldsymbol{B} \quad (\text{cf. } \boldsymbol{P} = \chi_e \boldsymbol{E})}$ That would be too sensible.

$$\boldsymbol{M} = \chi_m \boldsymbol{H}$$

In these terms,

$$\boldsymbol{B} = \boldsymbol{H} + 4\pi \boldsymbol{M} = \boldsymbol{H}\left(1 + 4\pi\chi_m\right) \equiv \mu \boldsymbol{H} \qquad (\text{cf. } \boldsymbol{D} = \varepsilon \boldsymbol{E}).$$

or, in MKS,

$$\boldsymbol{B} = \mu_0 \left(\boldsymbol{H} + \boldsymbol{M}\right) = \mu_0 \left(1 + \chi_m\right) \boldsymbol{H} \equiv \mu \boldsymbol{H} \quad .$$

μ is called the **relative permeability.** Contrary to ε, μ is the same in cgs and MKS, though χ_m , equally dimensionless, is not.

Linear magnetic media (continued)

The run of χ_m for the three forms of magnetism:

- **Ferromagnetism**: $\chi_m > 0$ and *very* large. For instance, pure, annealed iron has $\mu \approx 10000$.
- **Paramagnetism**: $\chi_m > 0$ but $\ll 1$. Typically, $\chi_m(\text{cgs}) \approx 10^{-6}$.
- **Diamagnetism**: $\chi_m < 0$, $|\chi_m| \ll 1$. Typically somewhat smaller than is typical in paramagnetism.

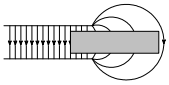
$\mu \cong 1$.

The consequence is that μ is so close to unity for anything that isn't ferromagnetic, that one could easily avoid noticing magnetism in everyday life.

Linear magnetic media (continued)

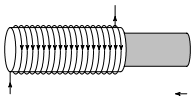
Practical realization of the three different kinds of linear magnetic media, compared to electric polarization:

Capacitor and dielectric



← Force on dielectric

Solenoid and magnet



Force:
← paramagnetic
→ diamagnetic
← ferromagnetic

Cautions (?) about *H*

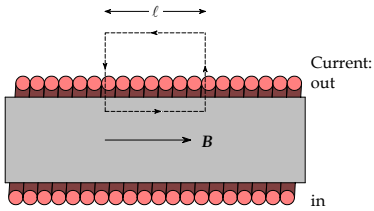
One must be careful about the use of ***H***, for many of the same reasons that we are cautious about ***D***. But ***H*** does turn out to be somewhat more generally useful than ***D***.

- Reason: measurability. We can easily measure potential differences and free currents (with voltmeters and ammeters), so these relate best to the fields ***E*** and ***H***, not ***D*** and ***B***.
- Rule of thumb: ***H*** is useful whenever the situation is symmetrical enough to use Ampère's Law: infinite cylinders, planes and solenoids; toroids.

Remember, though, ***B*** is the fundamental field. There isn't a Biot-Savart law or Lorentz force law for ***H***.

Example: *H* and Ampère's Law

Example 6.3: An infinite solenoid carries a current I, has n turns per unit length, and is filled with a magnetic material with relative permeability μ. What is the magnetic field $\boldsymbol{B}$ inside, and what are the bound currents associated with the material?



Because there are bound currents we can't compute $\boldsymbol{B}$ directly, but we can compute $\boldsymbol{H}$.

Example: *H* and Ampère's Law (continued)

Clearly the field should be zero outside, axial inside, and perpendicular to the two vertical sides of the Ampèrean loop.

$\nabla \times \boldsymbol{H} = \frac{4\pi}{c}\boldsymbol{J}_f$ Integrate over area, use Stokes' theorem:

$$\oint \boldsymbol{H}\cdot d\ell = \frac{4\pi}{c} I_{f,\,\text{enclosed}}$$

$$H\ell = \frac{4\pi}{c} In\ell \quad \Rightarrow \quad \boldsymbol{H} = \frac{4\pi}{c} nI\hat{\boldsymbol{z}} \quad , \quad \boldsymbol{B} = \mu\boldsymbol{H} = \frac{4\pi}{c}\mu nI\hat{\boldsymbol{z}}.$$

$$\boldsymbol{K}_b = c\boldsymbol{M}\times\hat{\boldsymbol{n}} = c\chi_m\boldsymbol{H}\times\hat{\boldsymbol{s}} = 4\pi\chi_m nI\hat{\boldsymbol{\varphi}} \quad ,$$

$$\boldsymbol{J}_b = c\nabla\times\boldsymbol{M} = c\nabla\times\chi_m\boldsymbol{H} = 0 \quad .$$

In general, the bound current will reside on the surface (unless free current flows in the medium), and will follow the free current.

Example: *H* and Ampère's Law (continued)

The same thing, in MKS units:

$\nabla \times \boldsymbol{H} = \boldsymbol{J}_f$ Integrate over area, use Stokes' theorem:

$$\oint \boldsymbol{H}\cdot d\ell = I_{f,\,\text{enclosed}}$$

$$H\ell = In\ell \quad \Rightarrow \quad \boldsymbol{H} = nI\hat{\boldsymbol{z}} \quad , \quad \boldsymbol{B} = \mu\boldsymbol{H} = \mu nI\hat{\boldsymbol{z}}.$$

Bound charges:

$$\boldsymbol{K}_b = \boldsymbol{M}\times\hat{\boldsymbol{n}} = \chi_m\boldsymbol{H}\times\hat{\boldsymbol{s}} = \chi_m nI\hat{\boldsymbol{\varphi}} \quad ,$$

$$\boldsymbol{J}_b = \nabla\times\boldsymbol{M} = \nabla\times\chi_m\boldsymbol{H} = 0 \quad .$$