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# Today in Physics 218: the blue sky

- ❑ The color and polarization of the sky; reddening in sunsets and interstellar clouds
- ❑ Demonstration of the wavelength and polarization dependence of Rayleigh scattering
- ❑ Magnetic dipole radiation



*The Trifid Nebula, M20, is a good example of many features of scattering by dielectric particles: it shows extinction (the dust lanes), strong wavelength dependence (the blue color of the reflection nebula), and reddening (in the colors of the background stars). Photo by Greg Bothun, U. Oregon.*

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# Total scattering cross section of a dielectric sphere

Last time, we found that

$$\langle P \rangle = \left( \frac{4\pi}{3} \right)^2 \frac{a^6 \chi_e^2 \omega^4}{3c^3} E_0^2 = \sigma_{sc} I_I = \sigma_{sc} \frac{c}{8\pi} E_0^2 \quad , \text{ or}$$

$$\sigma_{sc} = 2 \left( \frac{4\pi}{3} \right)^3 \frac{a^6 \chi_e^2 \omega^4}{c^4} .$$

for radiation by electric dipoles induced in dielectric spheres by incident light.

- Note that it's not  $\pi a^2$ .
- Note especially the strong dependence on angular frequency.

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## Rayleigh's explanation of the blue sky

This result was first obtained by Lord Rayleigh, and far-field dipole-radiative scattering of light is called Rayleigh scattering in his honor. Rayleigh immediately used the result in the ultimate explanation of the blue color, and polarization, of the sky. To wit:

- Molecules in the air are polarizable, small compared to the wavelength of visible light, and most of the atmosphere is many wavelengths away. Thus air fits the approximations we have made so far.
- Sunlight subjects molecules to sinusoidally-varying electric fields; the dipoles thus induced radiate (scatter) some of the incident light.

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## Rayleigh's explanation of the blue sky (continued)

- But because  $\sigma_{sc} \propto \omega^4$ , a larger fraction of the power in high frequencies (short wavelengths) is scattered than at low frequencies (long wavelengths).
- Consider blue light ( $\lambda \sim 430$  nm) and red light ( $\lambda \sim 630$  nm); the ratio of total scattering cross sections is

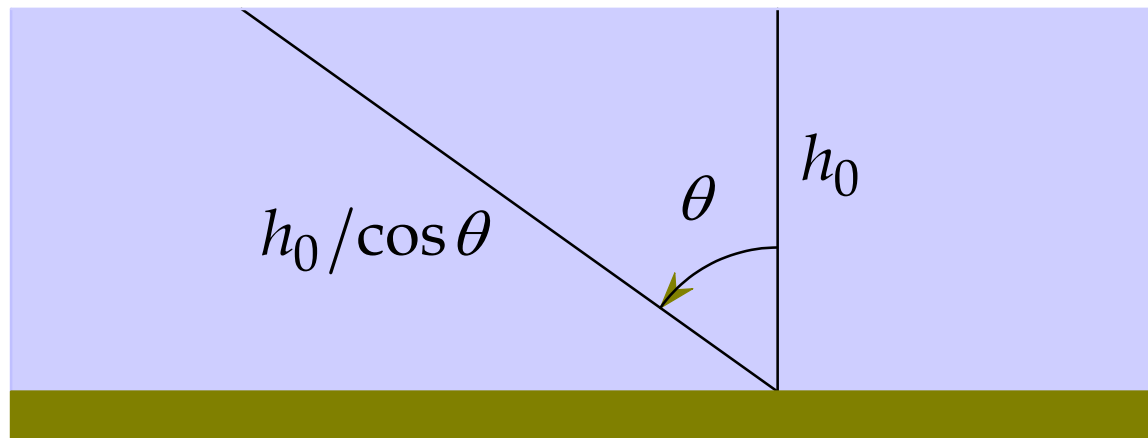
$$\frac{\sigma_{sc}(\text{blue})}{\sigma_{sc}(\text{red})} = \left(\frac{630}{430}\right)^4 = 6.1.$$

- Light from the sky during the day is all scattered sunlight. (Otherwise the sky wouldn't be dark at night.) The natural color of the sun isn't very red, so the scattered light is very heavily blue. That's why the sky is blue.

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## Rayleigh's explanation of the blue sky (continued)

- By the same token, light propagating through the atmosphere gets redder as it goes along, because more and more of the blue scatters away.
- This is why sunsets and sunrises are red. The path through the atmosphere is much longer toward the horizon than straight overhead, roughly by the secant of the angle from the zenith.



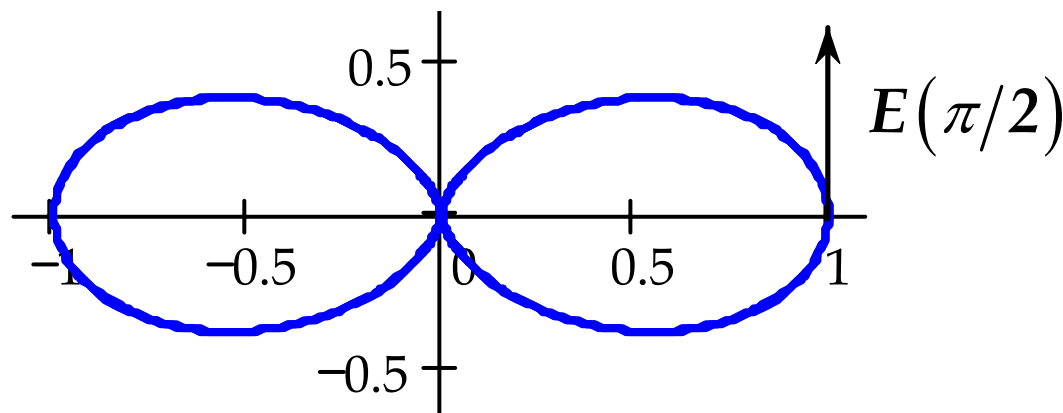
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## Polarization of the blue sky

- What about polarization? Recall that because the electric field of radiation by induced dipoles has the form

$$\mathbf{E} = -\frac{p_0 \omega^2 \sin \theta}{rc^2} \cos \omega \left( t - \frac{r}{c} \right) \hat{\theta} \quad ,$$

and is thus zero along the direction of the dipole,  $\theta = 0, \pi$ , and points parallel to the dipole along the direction in which the amplitude is largest,  $\theta = \pi/2$ .



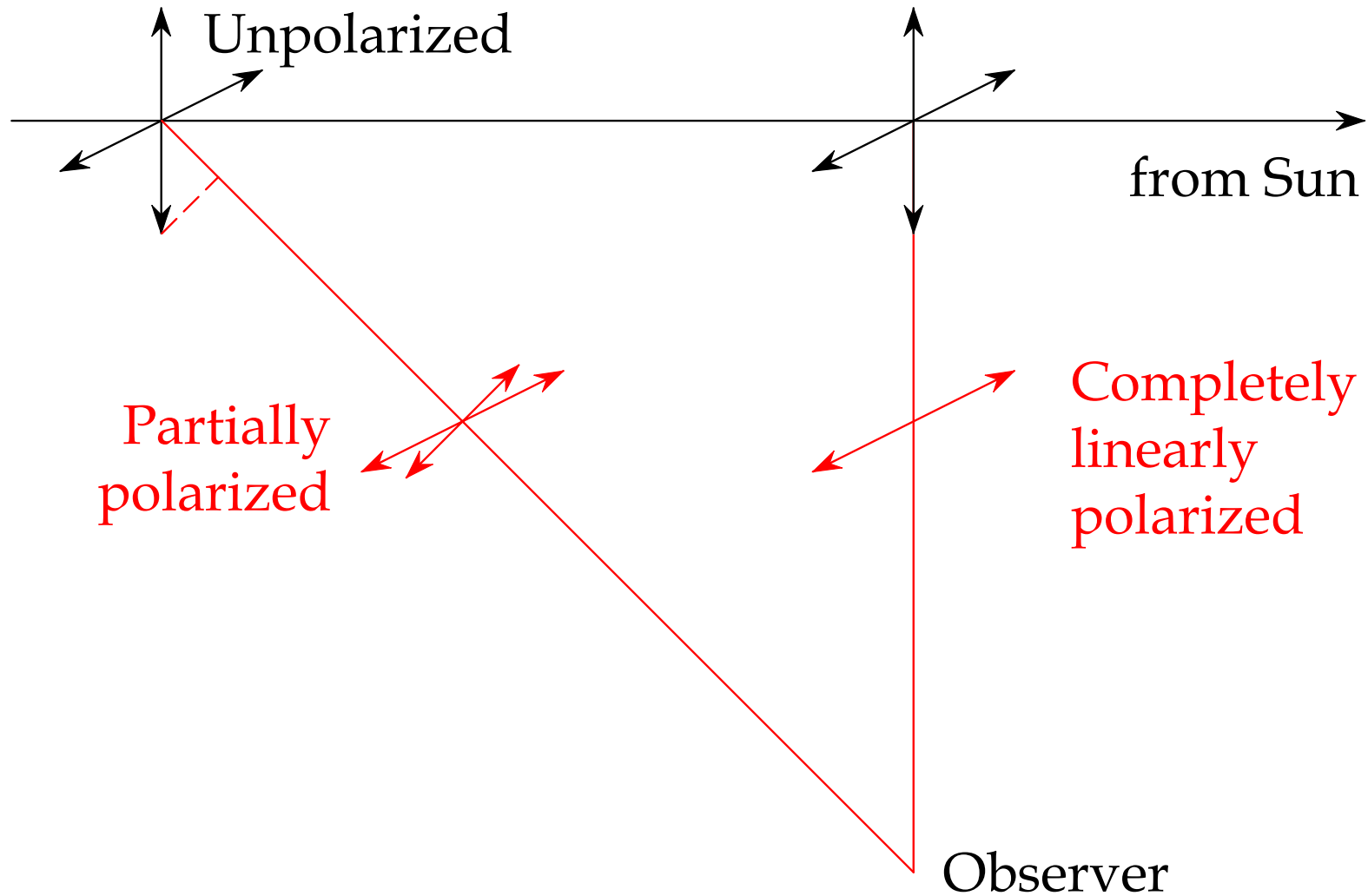
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## Polarization of the blue sky (continued)

- ❑ The induced dipoles in the air merely point in the same direction as  $E$  in the sunlight.
- ❑ Sunlight is unpolarized; it has a component of  $E$  in every direction other than  $k$ .
- ❑ But the induced dipole moment pointing in *your* direction will not radiate to you – only those with components perpendicular to this direction. So the sky is polarized.
- ❑ The direction of maximum polarization is the direction perpendicular to the direction toward the sun, where in principle the scattered light is completely polarized perpendicular to the plane of incidence.
  - In practice, multiple scattering modifies this a bit, but the effect is still strong.

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## Polarization of the blue sky (continued)





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## Hands-on demonstrations

We will demonstrate the wavelength dependence and polarization of scattering with some hand-held lasers and flashlights and polarizers.

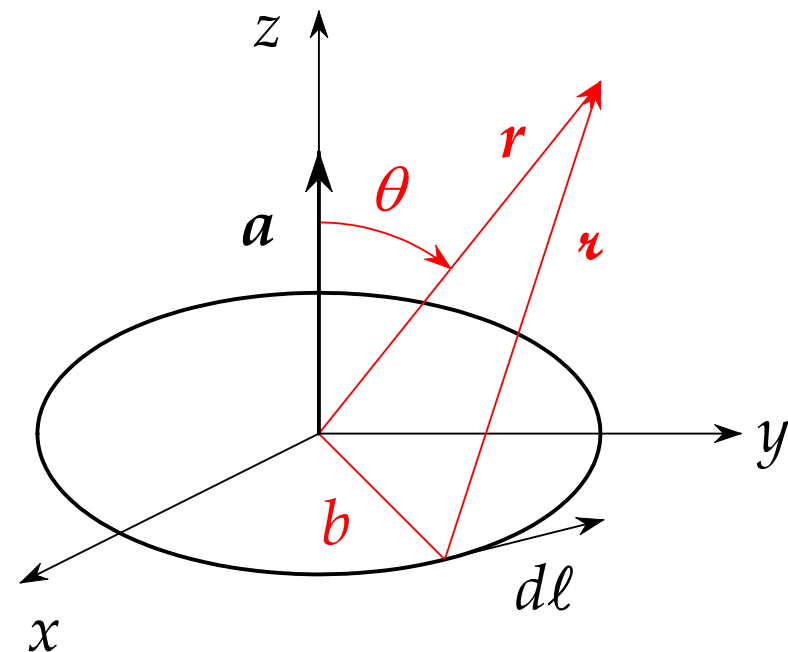
If it's not cloudy we will also step outside to observe the polarization of the sky.

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# Magnetic dipole radiation

Consider now an oscillating magnetic dipole, consisting of a circular loop of oscillating current:

$$\begin{aligned} I &= I_0 \cos \omega t \\ \mathbf{m} &= \frac{1}{c} I a \\ &= \frac{1}{c} I_0 \pi b^2 \hat{\mathbf{z}} \cos \omega t \\ &= \mathbf{m}_0 \cos \omega t \quad . \end{aligned}$$



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## Magnetic dipole radiation (continued)

We know that  $V = 0$  here, because the wire is neutral, so we can work out the fields from the vector potential alone:

$$\mathbf{A}(\mathbf{r}, t) = \frac{1}{c} \int \frac{I(t_r)}{r} d\ell = \frac{bI_0}{c} \hat{\phi} \int_0^{2\pi} \frac{\cos \omega(t - r/c)}{r} d\phi \quad .$$

This integral works out in much the same way as it did in the case of electric dipoles (it's also done in detail in the book), so we'll cut to the answers for the far field ( $r \gg c/\omega \gg b$ ):

$$\mathbf{A}_{\text{rad}}(\mathbf{r}, t) = \frac{m_0 \omega \sin \theta}{c r} \sin \omega \left( t - \frac{r}{c} \right) \hat{\phi} \quad ,$$

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## Magnetic dipole radiation (continued)

$$\mathbf{E}(\mathbf{r}, t) = -\frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} = \frac{m_0 \omega^2}{c^2} \frac{\sin \theta}{r} \cos \omega \left( t - \frac{r}{c} \right) \hat{\phi} \quad ,$$

$$\mathbf{B}(\mathbf{r}, t) = \nabla \times \mathbf{A} = -\frac{m_0 \omega^2}{c^2} \frac{\sin \theta}{r} \cos \omega \left( t - \frac{r}{c} \right) \hat{\theta} \quad .$$

The formulas for the field look just like those for electric dipole radiation, if we simply replace

$$p_0 \leftrightarrow m_0$$

$$\mathbf{E} \leftrightarrow \mathbf{B}$$

$$\mathbf{B} \leftrightarrow -\mathbf{E} \quad .$$

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## Magnetic dipole radiation (continued)

Because of this similarity, the Poynting vector comes out looking pretty familiar too:

$$\langle \mathbf{S} \rangle = \frac{c}{4\pi} \langle \mathbf{E} \times \mathbf{B} \rangle = \frac{c}{8\pi} \left( \frac{m_0 \omega^2}{c^2} \right)^2 \left( \frac{\sin \theta}{r} \right)^2 \hat{\mathbf{r}} = I \hat{\mathbf{r}} \quad ,$$

the same as the electric dipole case, simply switching  $p$  for  $m$ .

- Radiation from oscillating electric and magnetic dipoles are thus very similar. They do differ, however, in size. Suppose we compare dipoles of the same size  $b$  of each, with the same current,  $I = dq/dt$  :

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## Magnetic dipole radiation (continued)

$$\frac{\langle \mathbf{S}_{\text{mag}} \rangle}{\langle \mathbf{S}_{\text{elec}} \rangle} = \frac{\frac{c}{8\pi} \left( \frac{m_0 \omega^2}{c^2} \right)^2 \left( \frac{\sin \theta}{r} \right)^2}{\frac{c}{8\pi} \left( \frac{p_0 \omega^2}{c^2} \right)^2 \left( \frac{\sin \theta}{r} \right)^2} = \frac{m_0^2}{p_0^2} = \left( \frac{I_0 \pi b^2}{c q_0 \pi b} \right)^2$$
$$= \left( \frac{\omega q_0 \pi b^2}{c q_0 \pi b} \right)^2 = \left( \frac{\omega b}{c} \right)^2 \ll 1 \quad ,$$

since we're in the far field. This is why broadcast antennas look like electric dipoles instead of magnetic dipoles.

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## Magnetic dipole radiation (continued)

This is also related to the reason that electric dipole transitions in atoms and molecules are so much stronger than magnetic dipole transitions:

- they involve electrons in orbitals of a given *size*, to the associated magnetic dipoles are smaller than the electric dipoles by a factor of  $(\omega a/c)^2$ , where  $a$  is the atomic radius.