Physics 218 Practice Midterm Exam

Spring 2004

If this were a real exam, you would be reminded here of the **exam rules**: "You may consult *only* one page of formulas and constants and a calculator while taking this test. You may *not* consult any books, nor each other. All of your work must be written on the attached pages, using the reverse sides if necessary. The final answers, and any formulas you use or derive, must be indicated clearly. Exams are due an hour and fifteen minutes after we start, and will be returned to you during at the next lecture. Good luck."

This practice test is meant mostly to illustrate the style, length, variety, and degree of difficulty of the problems on the real exam; not all the topics we have covered are represented here. This is not to say that these topics won't appear on the real exam, or that there will be problems just like the ones here on the real exam! You should expect problems on any topic we have covered in lecture, recitation and on the homework.

For best results please try hard to work out the problems before you look at the solutions.

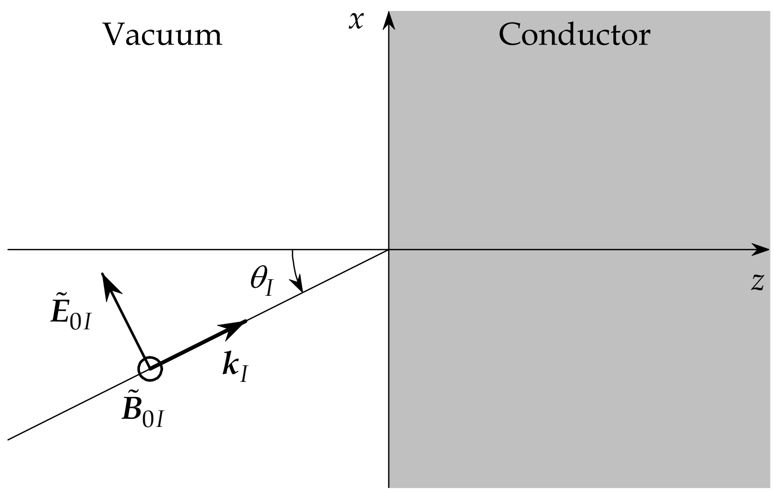
Name: ________________________

Problem 1 (20 points)

In a certain material, the group velocity of electromagnetic waves is twice the phase velocity. Derive an expression for these waves's wavenumber, k, as a function of their angular frequency, ω.

Problem 2 (40 points)

Light, propagating in vacuum, is incident obliquely (incidence angle θ_I) on the planar surface of a good conductor $\left(\sigma \gg \varepsilon\omega/4\pi\right.$, or $\sigma \gg \varepsilon_r \varepsilon_0 \omega$ in MKS units$\left.\right)$ for which $\varepsilon = \mu = 1\left(\varepsilon = \varepsilon_0, \mu = \mu_0 \text{ in MKS}\right)$. The light is polarized, with the electric field in the plane of incidence.



a. What is the angle of the transmitted wavevector with respect to the normal? (Hint: it's not equal to θ_I.)

Problem 2 (continued)

b. Using the electromagnetic boundary conditions, write down enough equations to determine the reflected and transmitted electric field amplitudes. (Don't solve them yet.)

Problem 2 (continued)

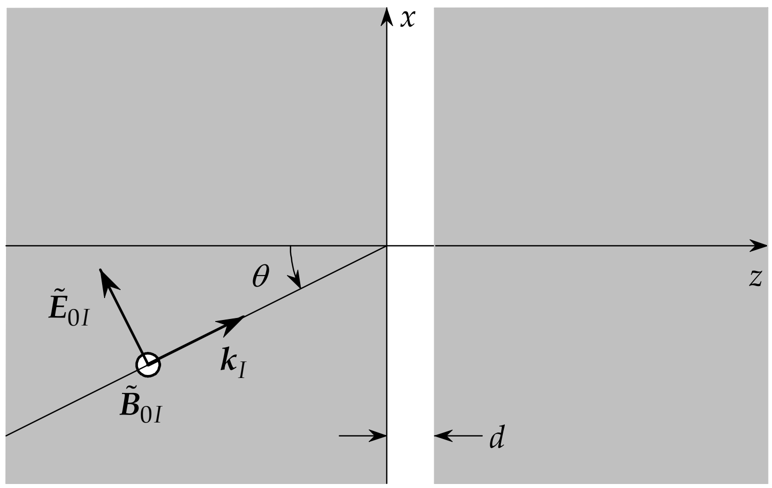
c. Solve for the reflected electric field amplitude, in terms of the incident amplitude, and from this derive the time-averaged intensity reflection coefficient $\rho = I_R / I_I$.

Problem 2 (continued)

d. Is there a “Brewster’s angle” for reflection from conductors? Explain why or why not.

Problem 3 (40 points)

Frustrated total reflection. Light with angular frequency ω, propagating in glass (n = 1.5) and polarized in the plane of incidence, encounters a plane-parallel, vacuum-filled gap at an angle θ greater than $\arcsin(1/n)$ -- that is, an angle at which total reflection would be expected for a single surface. Glass fills the whole Universe, apart from the gap.



Calculate the intensity of light transmitted through the gap. Is it possible to make the gap transparent, at this frequency?

Problem 3 (continued)

Problem 4 (20 points)

Consider using, as a gauge condition,

$$\begin{aligned} V &= 0 \quad \text{everywhere,} \\ \boldsymbol{A} &\neq 0 \quad . \end{aligned}$$

Show explicitly that this can describe electromagnetic wavs in vacuum; that is, find the appropriate $\boldsymbol{A}$, and show that it gives electric and magnetic fields with the correct properties of waves in a vacuum.