Physics 218 Practice Final Exam

Spring 2004

If this were a real exam, you would be reminded here of the **exam rules**: "You may consult *only* two pages of formulas and constants and a calculator while taking this test. You may *not* consult any books, nor each other. All of your work must be written on the attached pages, using the reverse sides if necessary. The final answers, and any formulas you use or derive, must be indicated clearly. Exams are due an hour and fifteen minutes after we start, and will be returned to you during at the next lecture. Good luck."

This practice test is meant mostly to illustrate the style, length, variety, and degree of difficulty of the problems on the real exam; not all the topics we have covered are represented here. This is not to say that these topics won't appear on the real exam, or that there will be problems just like the ones here on the real exam! You should expect problems on any topic we have covered in lecture, recitation and on the homework.

For best results please try hard to work out the problems before you look at the solutions.

Name: ________________________

Problem 1 (50 points)

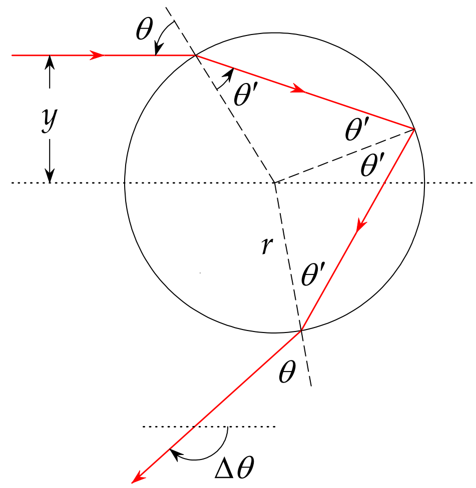
a. A spherical shell (inside radius a, outside radius b) has charge $Q\cos\omega t$ spread on its outer surface, and charge $-Q\cos\omega t$ spread uniformly on its inner surface. What is the electric field in the far-field zone, a distance $r \gg \lambda \gg b$ from the center of the sphere?

Problem 1 (continued)

b. Another shell is the same in all respect to the first one, except that the *centers* of the inner and outer surfaces are displaced with respect to one another by a distance d. What is the electric field in the far-field zone, a distance $r \gg \lambda \gg b$ from the center of the sphere?

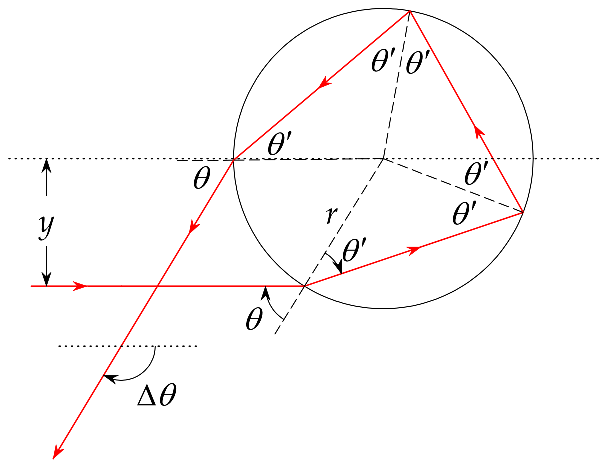
Problem 2 (50 points)

a. Calculate the distance y_0 from the center of a rain drop, at which the "rainbow ray" is incident: light that corresponds exactly to the scattering-angle extremum that in turn corresponds to the primary rainbow. Refer to the diagram at right for the geometry.



Problem 2 (continued)

b. Repeat the calculation of part a, but for the *secondary* rainbow. Again, refer to the diagram at right for the geometry.



Problem 2 (continued)

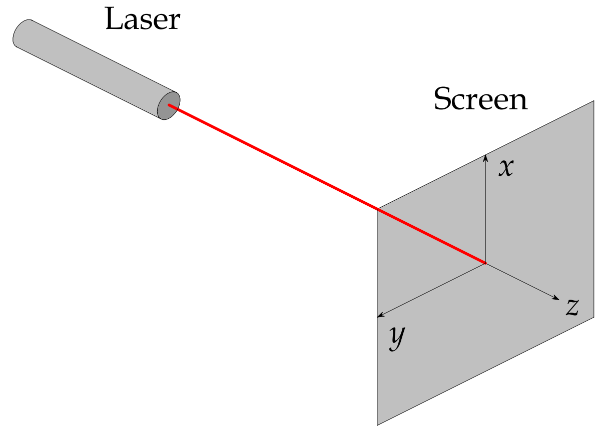
c. In the Bible (Genesis 9:13-15), it is written that shortly after the Flood subsided, God said to Noah and his family,

> I have set my bow in the clouds, and it shall be a sign of the covenant between me and the earth. Henceforth when I bring clouds over the Earth and the bow is seen in the clouds, I will remember my covenant that is between me and you and every living creature of all flesh; and the waters shall never again become a flood to destroy all flesh.

So before the Flood, raindrops did not produce rainbows, but afterward they did.

Describe how the refractive index of water would have had to change during this conversation, in order for the optical properties of raindrops to change like this.

Problem 3 (50 points)



Diffraction of a Gaussian beam. The electric field in a laser beam, as the beam leaves the end of the laser, is linearly polarized vertically, is axially symmetric, and has a magnitude which depends upon distance from the laser's axis as follows:

$$\boldsymbol{E} = \boldsymbol{E}_0 e^{-s^2/s_0^2} = \boldsymbol{E}_0 e^{-\left(x'^2+y'^2\right)/s_0^2} \quad .$$

The laser beam is pointed perpendicular to a screen which lies a very long distance r $(\gg s_0)$ away from the laser. What is the electric field on this screen, as a function of distance $q = \sqrt{x^2+y^2}$ from the point on the screen at which the laser is aimed?

Make a rough plot of the electric field amplitude as a function of q.

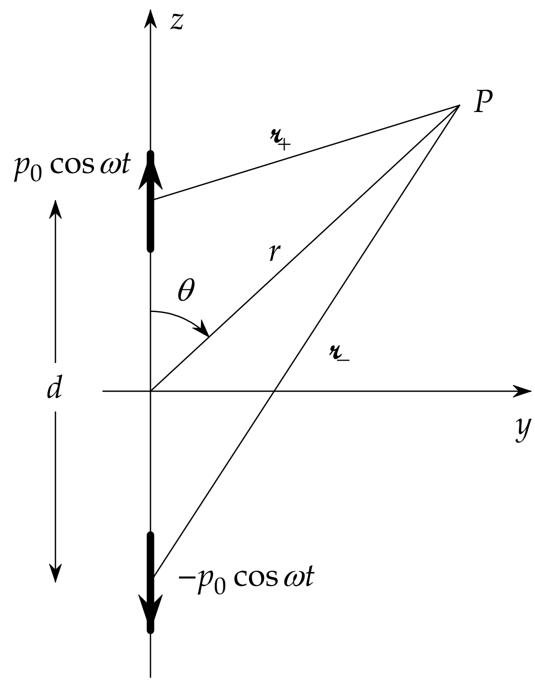
Hint: work in Cartesian coordinates initially, and complete the square in the exponent of the integrand, to carry out the integral.

Problem 3 (continued)

Problem 4 (50 points)

Electric quadrupole radiation. Two oscillating electric dipoles, separated by a distance d, are oriented as shown in the figure at right. Using what you know about the potentials for individual dipoles, calculate the scalar potential V in the far field $(r \gg \lambda \gg d)$.

Hints: Keep only terms that are first order in d. Note that neither dipole lies at the origin.



Problem 4 (continued)

Problem 5 (50 points)

a. Using the electromagnetic field tensor $F^{\mu\nu}$ and the dual tensor $G^{\mu\nu}$, show that

$$E^2 - B^2 \quad \text{(cgs units) or} \quad E^2 - c^2B^2 \quad \text{(MKS units)}$$

and

$$\boldsymbol{E} \cdot \boldsymbol{B}$$

are invariant under Lorentz transformations.

Problem 5 (continued)

b. Suppose that, in a certain inertial frame S, the electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{B}$ are neither parallel nor perpendicular. Show that in a different inertial frame $\overline{S}$, moving relative to S at velocity $\boldsymbol{v}$ given by

$$\boldsymbol{v} = \frac{c}{\gamma^2} \frac{\boldsymbol{E} \times \boldsymbol{B}}{B^2 + E^2} \quad \text{(cgs units)} \quad \text{or} \quad \frac{1}{\gamma^2} \frac{\boldsymbol{E} \times \boldsymbol{B}}{B^2 + E^2/c^2} \quad ,$$

the fields $\overline{\boldsymbol{E}}$ and $\overline{\boldsymbol{B}}$ are parallel.

Problem 5 (continued)

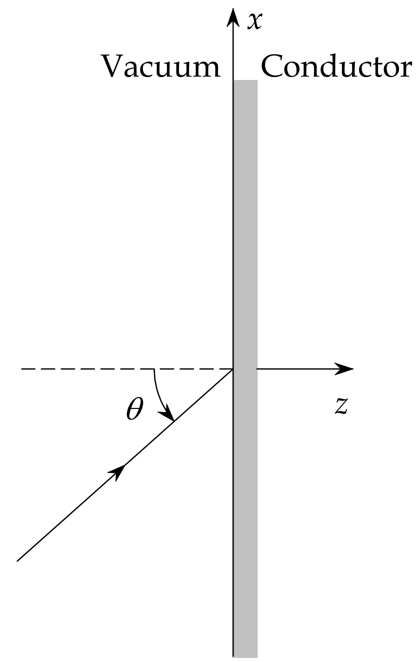
c. Is there a frame in which the electric and magnetic fields are perpendicular?

Problem 6 (50 points)

Unpolarized light with angular frequency ω is incident from vacuum, at angle θ, on a planar conducting surface. Calculate the degree of polarization,

$$\delta = \frac{I_\perp - I_\parallel}{I_\perp + I_\parallel} \quad ,$$

of reflected light, and describe in a few words the state of the light's polarization: nature, magnitude and direction. (Here $\perp$ and $\parallel$ mean perpendicular and parallel to the plane of incidence.)



Problem 6 (continued)