GEOPHYSICAL RESEARCH LETTERS, VOL. 33, L19711, doi:10.1029/2006GL026355, 2006

Thermocline flux exchange during the Pinatubo event

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Received 23 March 2006; revised 28 July 2006; accepted 11 September 2006; published 13 October 2006.

[1] We analyze the temperature anomaly of the Pinatubo eruption using an exact mathematical solution of a standard energy balance model that includes coupling between the mixed layer and the thermocline. Our solution yields a short response time τ = 4.4 months and a small climate sensitivity λ = 0.22 C/(W/m^2), implying short-term negative feedback. Also, our analysis determines a value of the effective eddy diffusion constant $\kappa = 2 \times 10^{-6}$ m^2/s that is much smaller than that assumed in many climate models. We find for this model that the heat flux to the thermocline reverses sign and integrates to zero for any forcing of finite duration. This effect should be observable in any future Pinatubo-type event. **Citation:** Douglass, D. H., R. S. Knox, B. D. Pearson, and A. Clark Jr. (2006), Thermocline flux exchange during the Pinatubo event, *Geophys. Res. Lett.*, *33*, L19711, doi:10.1029/2006GL026355.

1. Introduction

[2] Because of its large magnitude compared to other major climatic disturbances, the Pinatubo eruption of June 15, 1991, is of particular interest. At the peak of its effect, the temperature of the earth decreased by about 0.5 C and the outgoing long wave flux decreased by about 4 W/m^2. The eruption occurred during a period relatively free of other climate forcings such as solar variations and El Niño effects. Pinatubo has thus been considered ideal for theoretical analysis [*Hansen et al.*, 1992] and general circulation models (GCMs) have been applied to the problem [*Wigley et al.*, 2005a]. There are two critical unknowns whose values have had to be assumed in these applications, namely the intrinsic relaxation time for surface temperature anomalies and the effective eddy diffusion constant at the top of the thermocline. In a series of earlier publications [*Douglass and Knox*, 2005a, 2005b, 2005c] (a revised version of *Douglass and Knox* [2005a] incorporating the two subsequent papers is available at http://arXiv.org/abs/physics/0509166), to be called DKa, DKb, and DKc, we applied a model of the type considered by *Wigley and Schlesinger* [1985] and *Lindzen* [1994] and determined, by analysis of the temperature data, a relaxation time much shorter (months instead of years) than those assumed in the complex models.

[3] In our papers cited above the mixed-layer–thermocline coupling was either neglected or included in an approximate way. Here we reanalyze the data with an exact solution of the mathematical problem that includes the coupling. The present analysis enables a determination of the effective eddy diffusion coefficient ($\kappa = 2 \times 10^{-6}$ m^2/s) and a prediction of the peak thermocline-to-mixed-layer heat transfer ($-\Delta Q_{\max}$ = 0.5 W/m^2). We also find that $\Delta Q(t)$ changes sign during the event and is such that the exchange integrated over time is zero. For the model under consideration, this result holds for any forcing of finite duration. There is no net heating of the thermocline by transient events.

[4] From the analysis we obtain a relaxation time τ = 4.4 mo and a climate sensitivity 0.22 °C/(W/m^2), which then implies negative feedback. This is the same result obtained in our earlier papers, with slightly different values. The sensitivity appears to be at variance with 0.46 °C/(W/m^2), corresponding to a T_{2x} of 1.7 °C associated with a CO_2 doubling, expected by *Wigley et al.* [2005b]. There is no discrepancy if one acknowledges that the sensitivity for CO_2 doubling is not necessarily the same as that for aerosol forcing. The reason for the difference in this case is that the models applied to CO_2 doubling are designed for an approach to a long-term steady state (called "equilibrium") and that the assumed processes involved include positive delayed feedback. The CO_2-doubling sensitivity is inappropriate for the volcano forcing, which activates only rapidly occurring feedbacks.

[5] We emphasize that our intrinsic relaxation time, climate sensitivity, and effective eddy diffusion coefficient are determined by the model and the data. They do not represent assumed values.

[6] In Sections 2 and 3 we describe our exact solution and methods. Uncertainties in our estimates of parameter values are discussed in Section 3 and shown in Table 1. We conclude with a discussion (Section 4).

2. Temperature Anomaly and Heat Transfer

[7] We assume an earth covered by an ocean consisting of a mixed layer of effective depth h. There is a forcing ΔF as described above, and the mixed layer is coupled to the thermocline in which it is assumed that the flow is by eddy diffusion with an effective diffusion constant κ. There is a flux ΔQ at the interface between the mixed layer and the thermocline, defined positive downward, representing a "simple diffusive process which serves as a surrogate for all ocean processes acting to carry heat from [this] layer to the deeper layers of the ocean" [*Lindzen and Giannitsis*, 1998]. Since we deal solely with global averages, we do not consider the complexities introduced by lateral transport. In particular, we are assuming that the global average of the aerosol density produced by Pinatubo is independent of its spatial distribution.

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0094-8276/06/2006GL026355$05.00

Table 1. Fitting Parameters and Coordinates of Temperature Minima[a]

	Units	Lower Error Limit	Value at Best Fit	Upper Error Limit	Best Fit With λ and h Fixed
R^2			0.83		0.69
λ	°C/(W/m^2)	0.18	0.22	0.26	0.46
κ	m^2/s	10^{-8}	2×10^{-6}	1.3×10^{-5}	5×10^{-5}
h	m	5	13	27	30
τ	years	0.14	0.37	0.77	1.79
Surface cooling flux minimum	(time/t_V, flux in W/m^2)		(1.72, −1.75)		(2.51, −1.18)
Surface temperature minimum	(time/t_V, °C)		(1.72, −0.53)		(2.51, −0.35)

[a]Upper and lower error limits are defined in the text, section 3.

[8] Equations governing a model of the kind we consider here have been given by *Wigley and Schlesinger* [1985], *Lindzen* [1994], and others:

$$c_V h \frac{dU}{dt} + \frac{U}{\lambda} = \Delta F - \Delta Q, \quad (1a)$$

$$\Delta Q = -c_V \kappa \left(\frac{\partial W}{\partial x}\right)_{x=0}, \quad (1b)$$

$$\frac{\partial W}{\partial t} = \kappa \frac{\partial^2 W}{\partial x^2}, \quad (1c)$$

$$U(t) = W(t, 0). \quad (1d)$$

Here U and W are the mixed-layer and thermocline temperature anomalies, respectively, referring to variations in the temperatures from reference values. These variations are driven by the radiative forcing ΔF. The thermocline anomaly W varies with both time t and depth x (measured downward from the interface with the mixed layer). The quantity λ is the climate sensitivity in °C/(W/m^2), as discussed directly below. The full-earth ocean has specific heat $c_V = 4.1 \times 10^6$ J/m^3/°C and a mixed-layer depth h. In an earlier approximate treatment of a similar problem [*Wigley and Schlesinger*, 1985], the effective depth (equivalently, an effective heat capacity) was related to an actual mixed layer depth h_0 by $h = \gamma h_0$, where the fraction $\gamma = 0.71$ depended on land fraction, land-ocean and air-sea heat transfer coefficients, and climate sensitivity. In equation (1c), κ is the effective eddy diffusion constant. Using equations (1) alone is consistent with an assumption that the thermocline has infinite depth, with $W(t, \infty) = 0$. We have also treated the case of a finite-depth thermocline in which the flux vanishes at a depth H, $(\partial W/\partial x)_{x=H} = 0$.

[9] Equation (1a) is the equation of conservation of energy/flux. equation (1b) expresses the continuity of heat flux at the interface between the mixed layer and the thermocline. equation (1c) is the heat diffusion equation and equation (1d) is a boundary condition of temperature continuity at the mixed-layer/thermocline interface. When $dU/dt = 0$ and $\Delta Q = 0$ one obtains from equation (1a) the familiar steady-state result ΔT (surface) = $U = \lambda \Delta F$, where λ is the "equilibrium" climate sensitivity parameter under radiative forcing. We will generally write $\lambda = g\lambda_0$, where $\lambda_0 = 0.30$ °C/(W/m^2) is the sensitivity with no feedback [*Shine et al.*, 1995; *Knox*, 2004] and g is the system gain. This gain factor $g = 1/(1 - f)$, where f is an effective feedback, is determined by the combination of all feedback effects. The values of g, and therefore of f, depend on the particular climate forcing and must be determined in each case. Our analysis produces these values but does not say anything about the underlying mechanisms.

[10] In the case of the Pinatubo event, the time course of the forcing, assumed globally averaged, is taken as that of the aerosol optical density (AOD) as determined by *Ammann et al.* [2003]. As we showed in DKa, $\Delta F = -0.439A(t/t_V)\exp(-t/t_V)$ is an excellent fit to the assumed aerosol forcing. Here A = 21 W/m^2 is a theoretically derived forcing constant [*Hansen et al.*, 2002] and t_V (7.6 months) is the time of the peak aerosol density. Equations (1) may be simplified by introducing the response time $\tau = \lambda h c_V$, a dimensionless time $t^* = t/t_V$ and a dimensionless spatial variable $x^* = x/h$. The results for the combination of equations (1a) and (1b) and for equation (1c) are

$$\frac{dU}{dt^*} + \beta U = -0.439 A \lambda \beta t^* \exp(-t^*) + \alpha^2 \left(\frac{\partial W}{\partial x^*}\right)_{x^*=0} \quad (2)$$

and

$$\frac{\partial W}{\partial t^*} = \alpha^2 \frac{\partial^2 W}{\partial x^{*2}}, \quad (3)$$

where $\alpha = \sqrt{\kappa t_V / h^2}$ and $\beta = t_V/\tau$. U, W, and $A\lambda$ are still in temperature units.

[11] We obtain exact solutions of equations (2) and (3) under the boundary condition (1d) by the method of Laplace transforms [see, e.g., *Dickinson and Schaudt*, 1998]. For economy we show here only two intermediate steps, in the case $W(t, \infty) = 0$. The Laplace transform of the temperature anomaly is found to be

$$\overline{U}(s) = -\frac{0.439 A \lambda \beta}{(1+s)^2 (s + \beta + \alpha\sqrt{s})}, \quad (4)$$

where the transform has been taken with respect to the dimensionless time t^*, and s is the transform variable. This function is converted to $U(t^*)$ in the time domain by an inverse transform that employs an appropriate choice of contour in the complex plane. The imaginary axis is such a contour, and the result may be put into the form

$$U(t^*) = -\frac{0.439 A \lambda \beta}{\pi} \mathrm{Re} \int_0^\infty \frac{\exp(iqt^*)}{(1+iq)^2 \left[iq + \alpha(1+i)\sqrt{q/2} + \beta\right]} dq \quad (5)$$

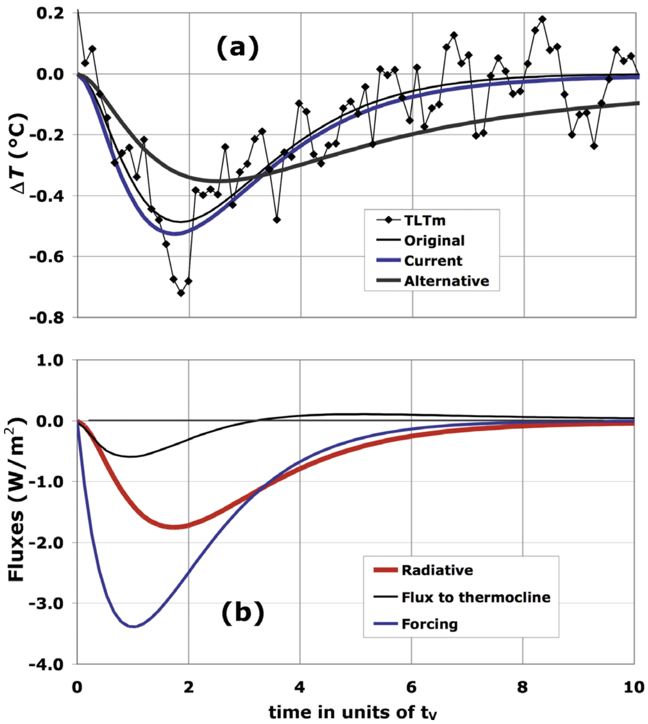

Figure 1. (a) Time course of the temperature anomaly induced by the Pinatubo eruption. Circles are the TLTm data set (see text, section 3). "Previous" is the function $U(t)$ obtained without coupling to the thermocline, and "Current" is $U(t)$ with coupling, both from the formalism of this paper and fit according to the method of section 3. "Alternative" is the best possible fit with the constraint λ = 0.46 °C/(W/m^2), as suggested by *Wigley et al.* [2005b], and for a choice of h = 30 m. (b) Time course of the fluxes associated with the exact solution. "Radiative" is $\Delta T(t)/\lambda$, "Flux to thermocline" is $\Delta Q(t)$, and "Forcing" is $\Delta F(t)$. Note the reversal of sign of $\Delta Q(t)$ at $t \sim 3.3t_V$.

where, on the contour chosen, $s = iq$. This is a form suitable for numerical integration. The adjustable parameters in the solution are λ, α, and β, or equivalently λ, κ, and τ.

3. Methods and Results

3.1. Fitting Procedure

[12] In the three papers DKa, DKb, and DKc, and in the present paper, the basic procedure is as follows: the proxy for the forcing due to Pinatubo given in the previous section is assumed in the computation of the surface temperature anomaly $\Delta T(t) = U(t)$. By adjusting parameters of the model, the result is fitted to TLTm, a temperature data set [*Christy et al.*, 2000] corrected to exclude the effects of El Niño as described in DKa and assumed to be representative of the surface temperature. In DKa, only equation (1a) was used, with ΔQ = 0, and two parameters (λ, τ) were fit to data. In DKb,c, we assumed that ΔQ was proportional to $U(t)$, which resulted in a simple correction to (λ, τ), as described in DKb. The proportionality constant was estimated from equation (1b) under the assumption of a time-averaged value of $(\partial W/\partial x)_{x=0}$ with the use of a recently measured value of $\kappa = 1.2 \times 10^{-5}$ m^2/s [*Ledwell et al.*, 1998]. In the present case, all three parameters (λ, τ, κ) were varied to obtain a best fit of U to TLTm by a standard least-squares analysis. Because of the relation $\tau = \lambda h c_V$, h therefore also varied.

[13] The exact solution that best fits the Pinatubo data is shown in Figure 1a as a heavy blue line. The solution found in DKa is shown as a thin black line. The new fit is characterized by a coefficient of determination R^2 = 0.83; for the old fit, R^2 = 0.79 (recomputed with the present algorithm). The inclusion of the thermocline coupling affects other fitting parameters; λ increases from 0.18 (DKa,b) to 0.22 °C/(W/m^2) and h decreases from 21 m to 13 m. Table 1 summarizes all relevant parameters. The gain g is 0.22/0.30 = 0.73, yielding a feedback f = −0.36. The effective depth h = 13 m corresponds to an actual depth h_0 = 20 m, as discussed above.

[14] Figure 1b shows the time dependence and relative magnitudes of the forcing $\Delta F(t)$, the radiative relaxation $\Delta T(t)/\lambda$, and the thermocline flux $\Delta Q(t)$, which is calculated from equation (1b) and the exact expression for $\partial W(t, 0)/\partial x$, which in turn is found by inverting its Laplace transform. Of particular interest is the reversal of sign of $\Delta Q(t)$, which is an essential feature in the case of an "impulse" forcing that leaves the system in its original state after relaxation. Physically, the reversal occurs because the induced temperature change just below the top of the thermocline overtakes the relaxing temperature change in the mixed layer. We find that the integral of $\Delta Q(t)$ over time is zero, as it must be.

[15] For the case of a finite thermocline with a zero-flux boundary condition at a depth H, an entirely analogous procedure may be followed. We have done this, determining that for a typical value H = 475 m, the solutions are identical to those found for infinite H. This is consistent with the fact that the diffusion time for a thermocline of that depth exceeds the duration of the transient event.

3.2. Fitting Uncertainties

[16] Our maximum of 0.83 in R^2 as a function of κ, λ, and h is assumed to be a global maximum, because of the rather simple form of the data and fitting function. To see how well defined the parameter values were at the peak, we varied each one separately, by examining solutions in the neighborhood of the maximum along the three axes of the parameter space. We took the points where a parameter axis crossed the R^2 = 0.69 surface as the limit of validity for solutions with that parameter. In the case of τ, which depends on both λ and h, the limits reflect only the limits of h, to which the result is more sensitive. The results are shown in Table 1 as upper and lower error limits.

[17] The value R^2 = 0.69 was chosen arbitrarily, but it is conservative in that it characterizes one solution that is clearly a poor fit: in the example discussed below ("Alternative"), where λ and h are fixed at 0.46 and 30, R^2 becomes 0.69 and is associated with $\kappa = 5 \times 10^{-5}$ m^2/s. The fit is then visibly bad (see Figure 1a).

[18] At the urging of a reviewer, we performed a different estimate of the uncertainties. The method proposed was to consider a fuzzy error band (0.15 to 0.20C) above and below the Pinatubo temperature signal, designed to account for the possibility of uncertainties in the analysis of the data resulting from variability of ENSO

and solar effects and from unknown forcings. Then the three parameters were to be varied in order to determine solutions that stayed within the band. Thus ranges of values of the three parameters were to be determined. We did this and found values consistent with those determined by our R^2 method. We feel that our method is better, since it deals with the real data and uses a well-defined criterion for assignment of error limits.

4. Discussion

4.1. Negative Feedback

[19] Our low values of climate sensitivity imply negative feedback. We found no values of ocean parameters that would lead to positive feedback. The implication is that the negative-feedback processes occur in the atmosphere. This is consistent with the ''infrared iris'' effect [*Lindzen et al.*, 2001].

4.2. Value of the Effective Eddy Diffusion Coefficient

[20] The value of κ found by our analysis is 2×10^{-6} m^2/s, smaller than the value chosen in the approximate DKb treatment and 50 times smaller than values assumed in many climate model calculations. Our value is only about three times smaller than that of *Polzin et al.* [1995], who give 7×10^{-6} m^2/s as a diapycnal background mixing rate for studies made at larger depths. This much smaller value of κ could make important changes in certain climate model simulations.

4.3. Flux Reversal

[21] As pointed out above, we predict a reversal of sign in the heat flux that occurs soon after the peak forcing (see Figure 1b). The magnitude of the effect is such that it should be observable with suitable instrumentation after a future Pinatubo-type event.

4.4. Failure of Approximate Treatments

[22] Our exact solution shows that the time-averaging or ''separability'' approximation made in DKb is a poor one because $\partial W(t, 0)/\partial x$ is very large near $t = 0$. We note that an approximation in which $\partial W(t, 0)/\partial x$ is monotonically proportional to $1/\sqrt{t}$ [*Wigley and Schlesinger*, 1985] is also inadequate for this problem because $\Delta Q(t)$ must change sign to return the system to its original steady state.

4.5. Relation of Fluxes to Sensitivity

[23] *Wigley et al.* [2005b] stated that our volcano-associated sensitivity λ_{vol} [DKa] did not yield the ''known'' value of λ_{2x}, the sensitivity to CO_2 doubling, of the DOE PCM model. We suggested earlier [DKb] that λ_{vol} need not be equal to λ_{2x}. Radiative forcing formalism was developed with the idea that the different climate forcings could have different feedbacks associated with different time scales and hence different values of λ. If the time scales are not different, or there is another similar explanation, this poses a problem for the modelers.

[24] We considered the properties of a solution (''Alternative'') with λ_{vol} fixed at 0.46, a value expected by *Wigley et al.* [2005a] to apply to this problem. Next, we chose $h = 30$ m, typical of GCMs, which determined $\tau = 21$ months. This left only κ to vary. Its value for a best data fit was 5×10^{-5} m^2/s. This solution is shown as the heavy gray line in Figure 1a. It is seen to be a poor fit. The minimum is too weak and occurs six months after the minimum in the data. The value $\lambda = 0.46$ combined with a ''typical'' relaxation time of 21 months is obviously inapplicable to the Pinatubo event. This becomes even more clear when τ is allowed to range freely. In that case the ''best'' solution with $\lambda = 0.46$ is unphysical ($\tau < t_V$, $h < 1$ m).

4.6. What About the DOE PCM Model of *Wigley et al.* [2005b]?

[25] These authors show, using the PCM model, that the temperature anomaly could be fit by the $\kappa = 0$ solution of our model with $\lambda_{vol} = 0.17$ and $\tau = 8.3$ months. They rejected this solution because λ_{vol}, either their own value or ours, was not equal to the ''known'' value of $\lambda_{2x} = 0.46$ from the PCM model. Their suggestion that the low value of λ is caused by neglecting ''thermal inertia'' associated with heat flow to the ocean is shown to be incorrect by the current analysis. We do not know the PCM-model parameters affecting the flow to the ocean, but in an earlier paper [*Wigley et al.*, 2005a] it was stated that the MAGICC model was equivalent and the parameters of the ocean part of this model are known. The poorly-fitting solution labeled ''Alternative'' in Figure 1a has ocean parameter values appropriate to the MAGICC model.

5. Summary

[26] Our exact solution of the mixed layer-thermocline problem for Pinatubo yields a short response time $\tau = 4.4$ months and a small sensitivity $\lambda = 0.22$ C/(W/m^2), implying a negative feedback factor $f = -0.36$. The sensitivity value is in disagreement with ''equilibrium'' sensitivities as calculated by the GCMs. In addition, we determine an effective eddy diffusion constant $\kappa = 2 \times 10^{-6}$ m^2/s, fifty times smaller than that used in many climate models.

[27] Our exact solution yields a flux flow between the mixed layer and the thermocline that is small in magnitude and shows a previously unrecognized reversal of sign shortly after the peak temperature anomaly. The integrated flux is found to be zero, which is a general consequence of finite-duration forcings on the present model.

[28] **Acknowledgments.** The authors are indebted to one of the reviewers of this manuscript for the important suggestion that we state explicitly error limits on parameter values.

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