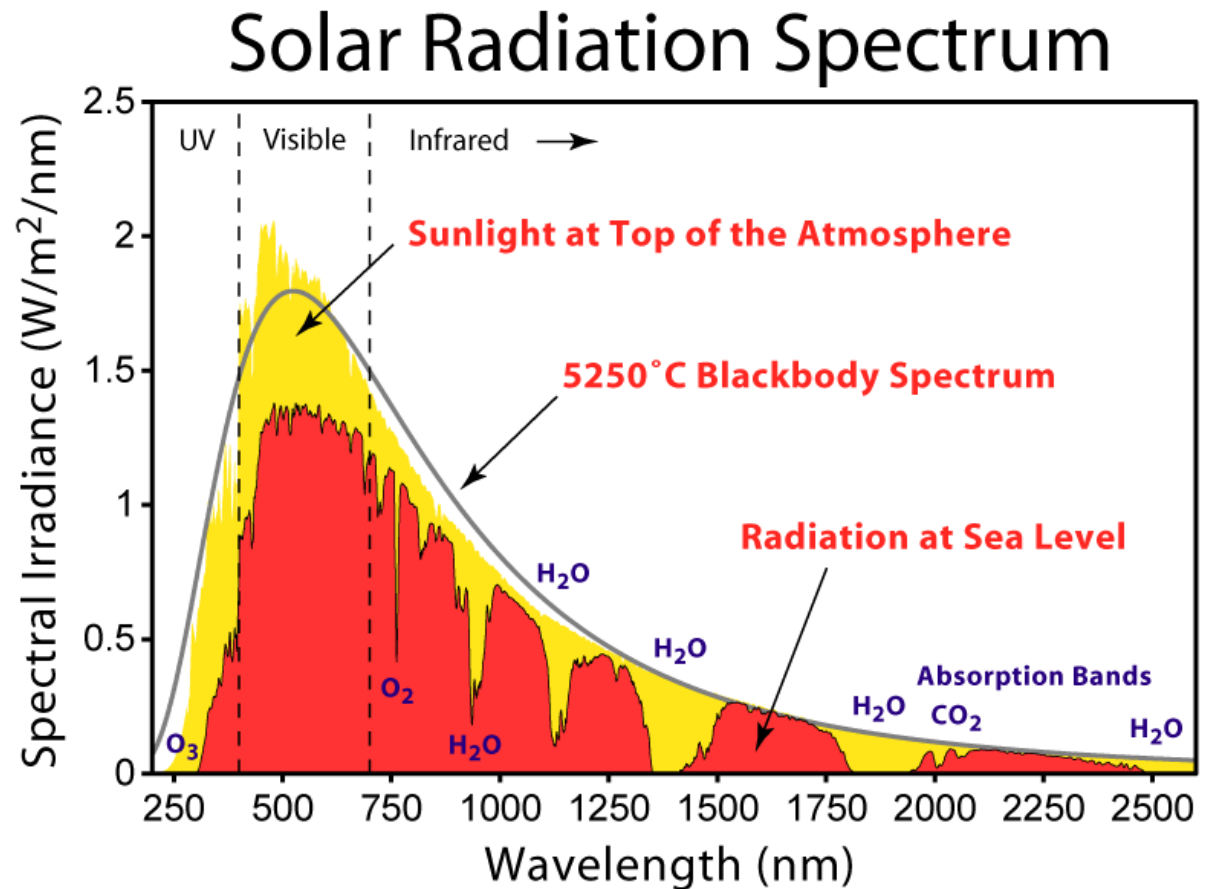


AST 453

- ❑ Some more basics on determining physical parameters of stars
- ❑ Photometry
- ❑ Luminosities
- ❑ Binaries, and relations between mass, radius, temperature for normal stars



Astrophysically Useful Numbers

For fundamental physical constants, see NIST CODATA page:

<http://physics.nist.gov/cuu/Constants/index.html>

Constants for Sun:

<http://www.pas.rochester.edu/~emamajek/sun.txt>

Solar radius $R_{\odot} = 695,660 \text{ km}$ Solar Mass $M_{\odot} = 1.989 \times 10^{30} \text{ kg}$

Solar Luminosity $L_{\odot} = 3.827 \times 10^{33} \text{ erg/s} = 3.827 \times 10^{26} \text{ W}$

Solar Effective Temperature $T_{\text{eff}\odot} = 5,772 \text{ K}$

Astronomical Unit = 149,597,870,700 m

Jupiter radius $R_J = 71,492 \text{ km}$ Jupiter mass $M_J = 1.8986 \times 10^{27} \text{ kg}$

Earth radius $R_E = 6,378 \text{ km}$ Earth mass $M_E = 5.9736 \times 10^{24} \text{ kg}$

A perusal of the astrophysics literature shows a mix of units:

SI, cgs, solar units, eV, Angstroms. Get used to converting units!

Apparent magnitudes: stellar photometry

$$m - m_0 = -2.5 \log_{10}(f/f_0)$$
$$m = -2.5 \log_{10}(f) + \text{zeropoint}$$

f = observed flux, m = magnitude

Usually this measured at some effective wavelength λ and measured with respect to some reference object (the star Vega has often been used).

Units of flux “ f ” are usually either quoted in either

erg/s/cm²/Angstrom

erg/s/cm²/μm

erg/s/cm²/Hz

Jansky = 1 Jy = $10^{-26} \frac{\text{W}}{\text{m}^2 \cdot \text{Hz}}$ (usually used in radio and IR)

<http://www.astro.utoronto.ca/~patton/astro/mags.html>

Absolute Magnitudes

Absolute magnitude M is the star's brightness at 10 pc

$$\begin{aligned}\text{Distance modulus } (m - M) &= -2.5 \log((L/d^2)/(L/(10\text{pc})^2)) \\ &= 5\log(d_{\text{pc}}) - 5\end{aligned}$$

For Sun at “V” band ($\lambda=0.55\mu\text{m}$), $m_V = -26.75$, $M_V = 4.82$

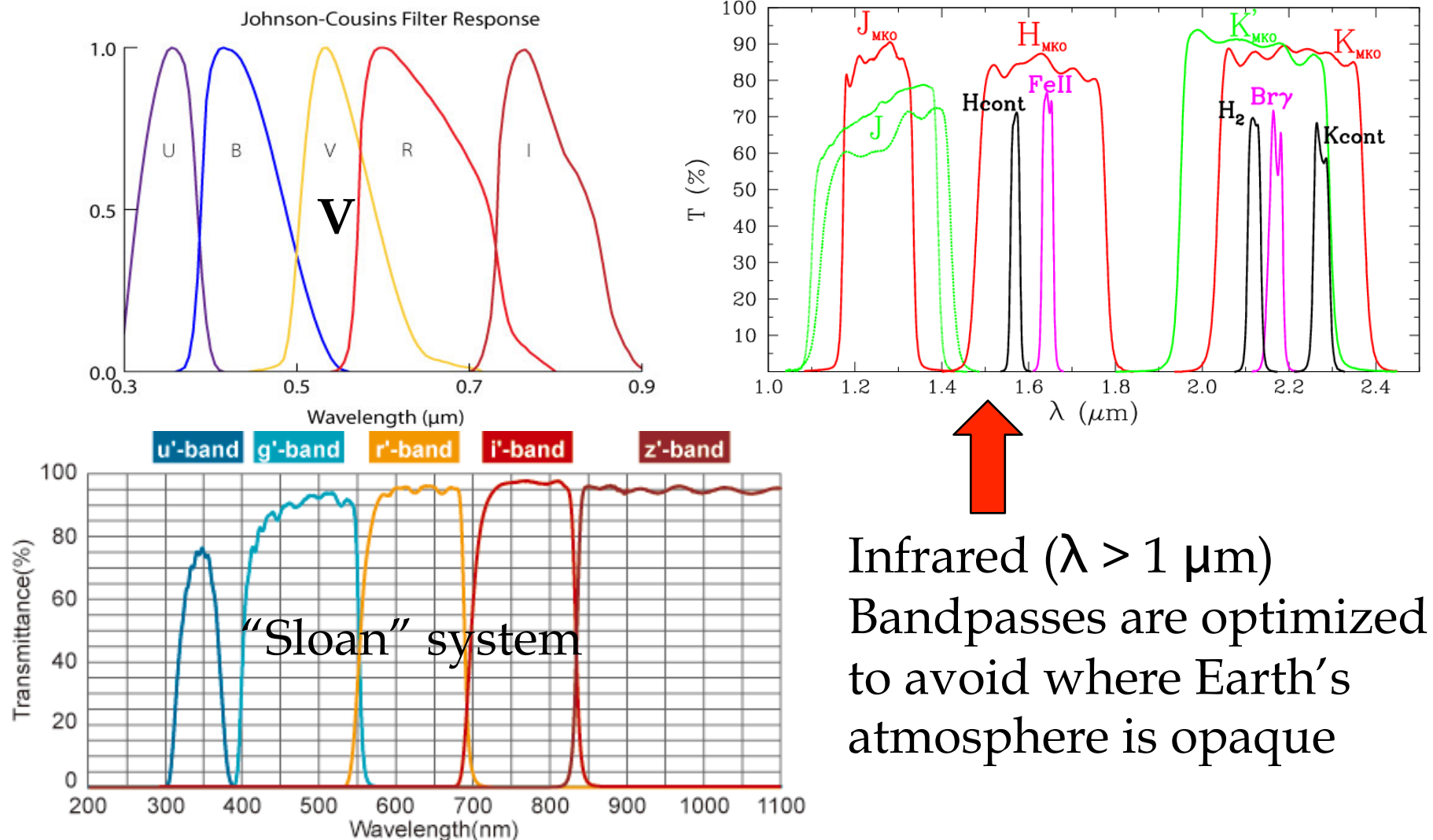
On this system $m_V(\text{Vega}) = 0.03$ (don't ask why not zero...)

Example: Vega has apparent V-band mag: $m_V(\text{Vega}) = 0.03$

And parallax $\varpi = 0.129$ arcsec \Rightarrow Distance = $1/\varpi = 7.75$ pc
so absolute magnitude of Vega in V-band is: $M_V = 0.58$

Databases on magnitudes, parallaxes (distances) for stars: [SIMBAD](#), [VizieR](#), [NStED](#)

Filters for measuring brightnesses at various λ



e.g. http://www.asahi-spectra.com/opticalfilters/astronomical_filter.html

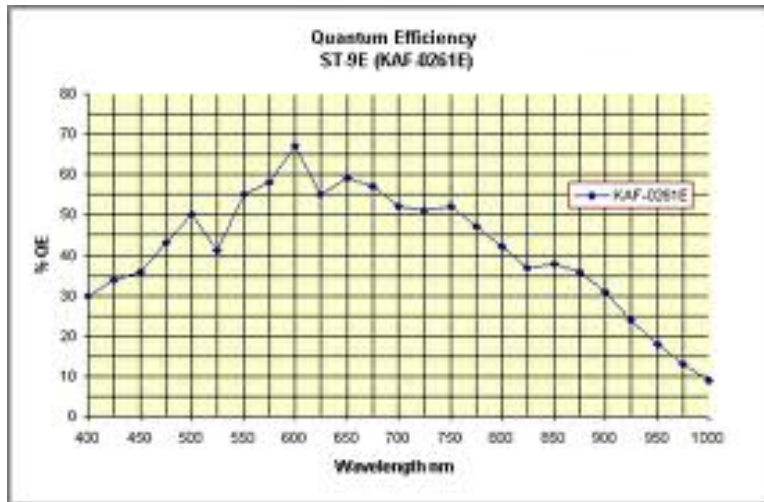
Ghinassi et al. 2002

6 Sept 2012

Astronomy 453, Fall 2012

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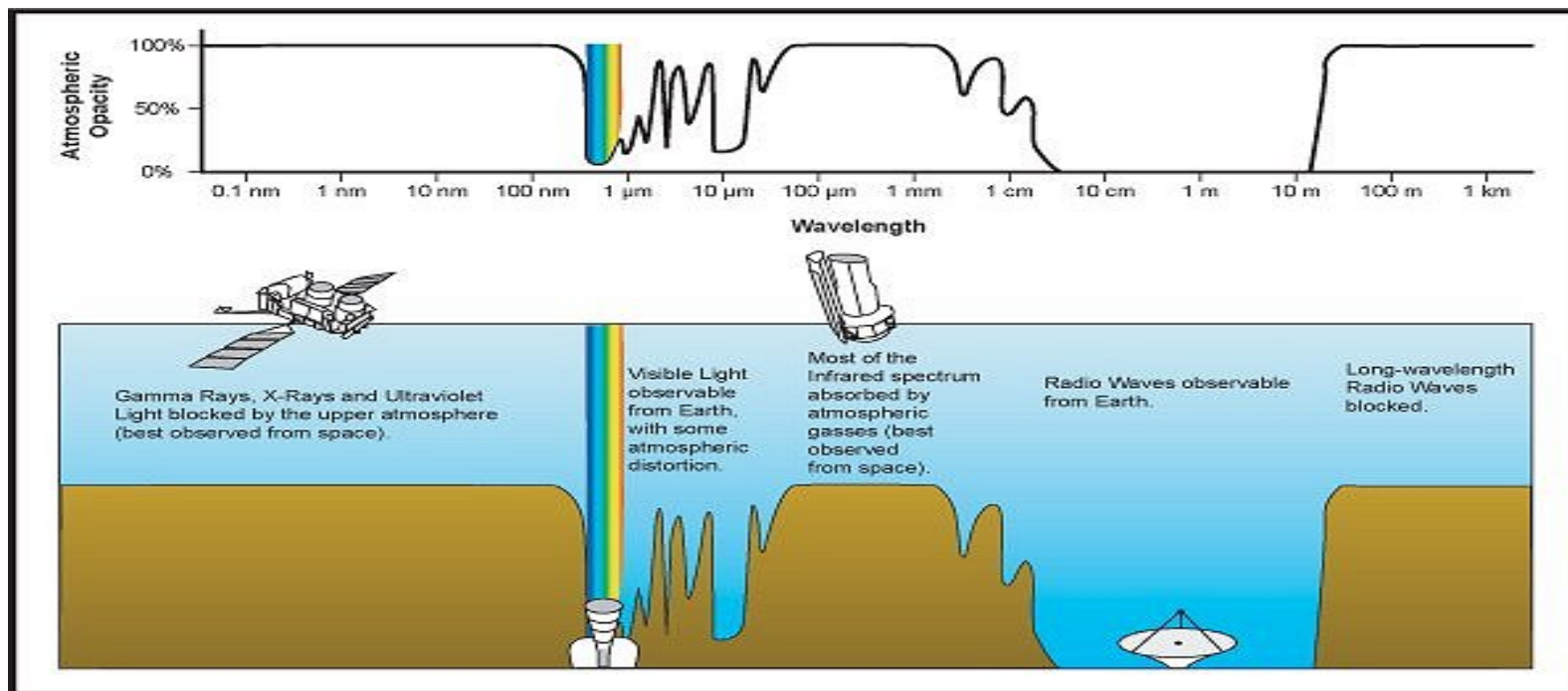
Other transmission profiles to keep in mind



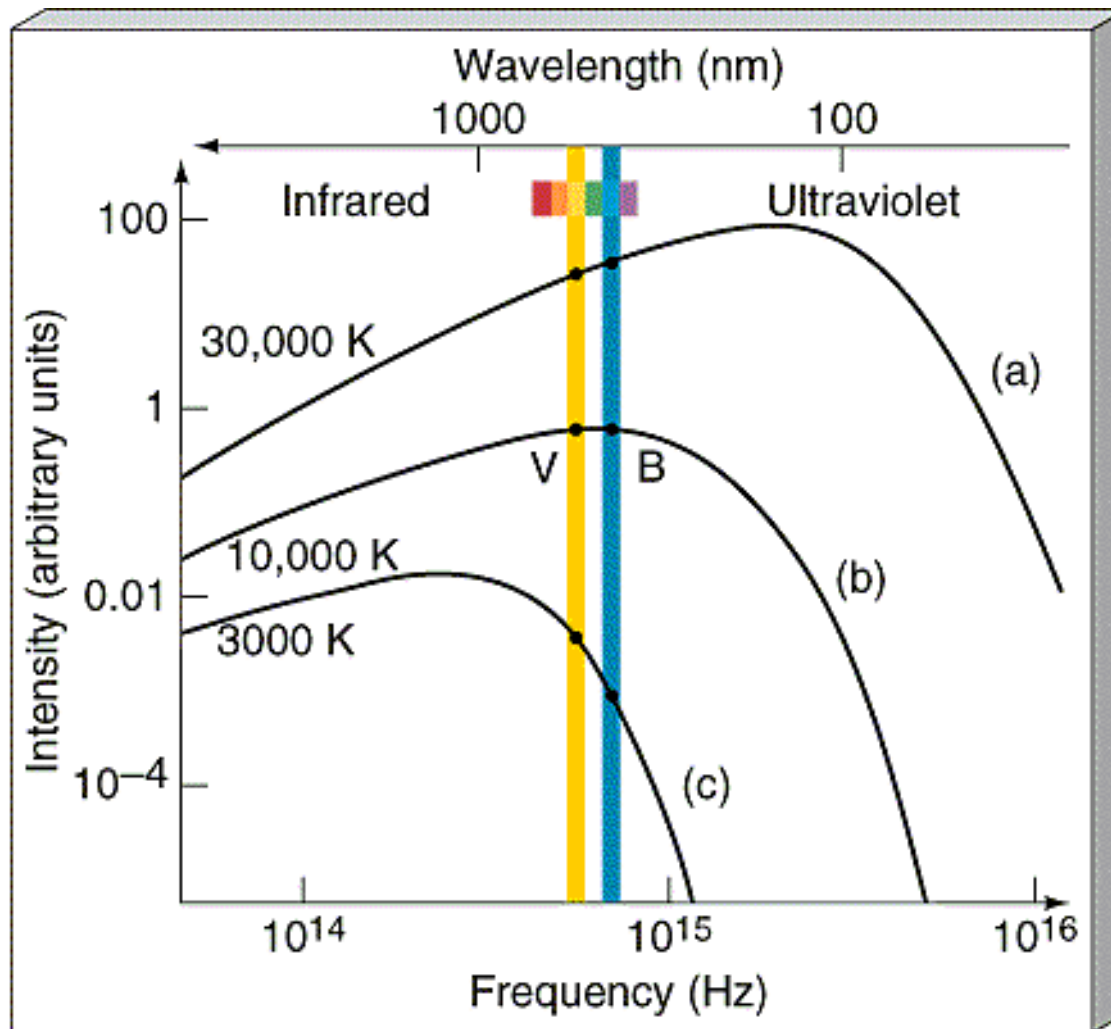
Typical CCD QE curve

[Santa Barbara Instrument Group](#)

Earth's atmospheric "windows"



Brightness/magnitude at two λ s => Colors



$$m_B - m_V = B - V \\ = -2.5 \log(f_B/f_V)$$

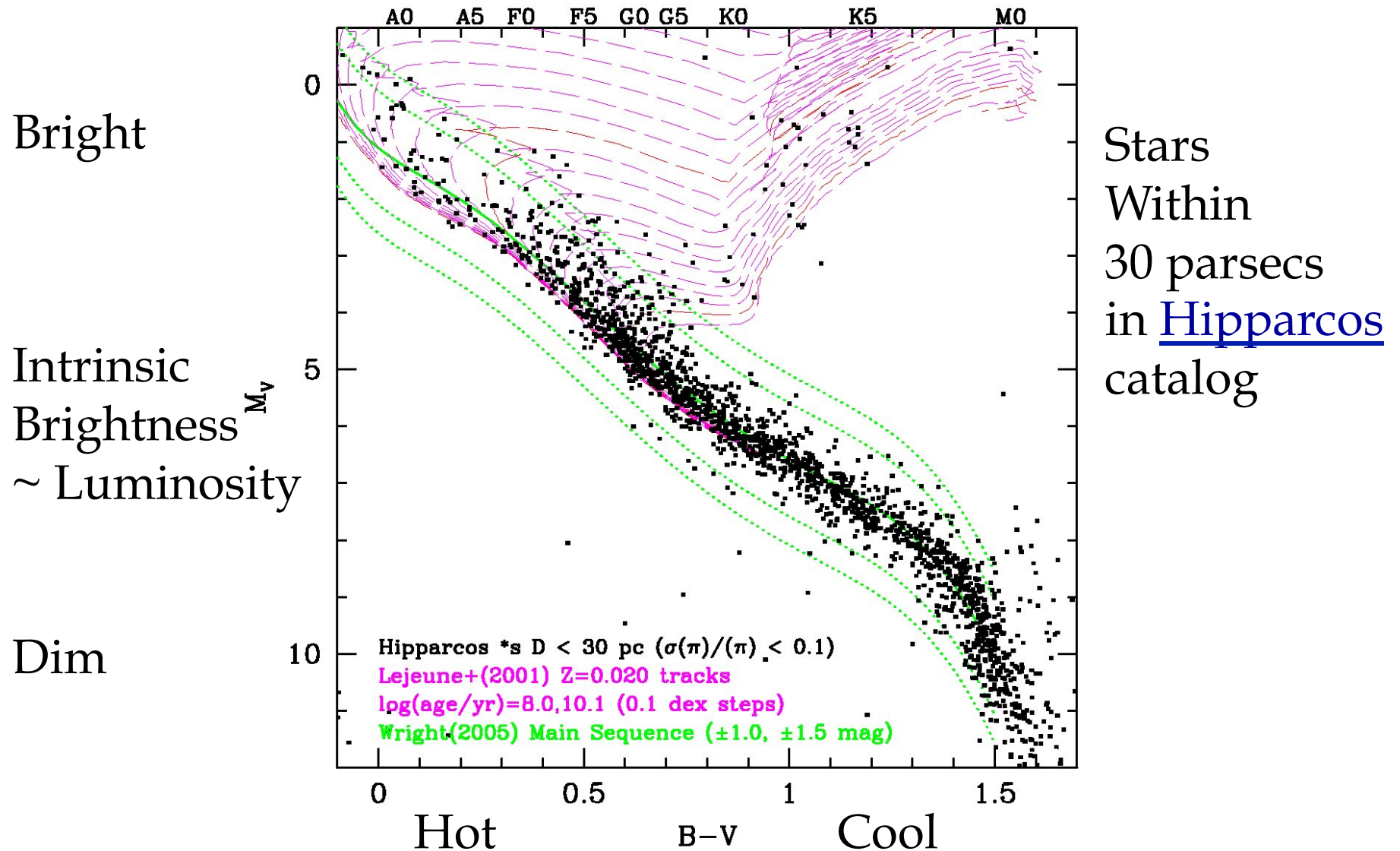
Where $B-V = 0.00$ mag
For Vega
($T_{\text{eff}} \sim 9500$ K)

$$B-V(\text{Sun}, T_{\text{eff}} = 5778\text{K}) \\ = 0.65 \text{ mag}$$

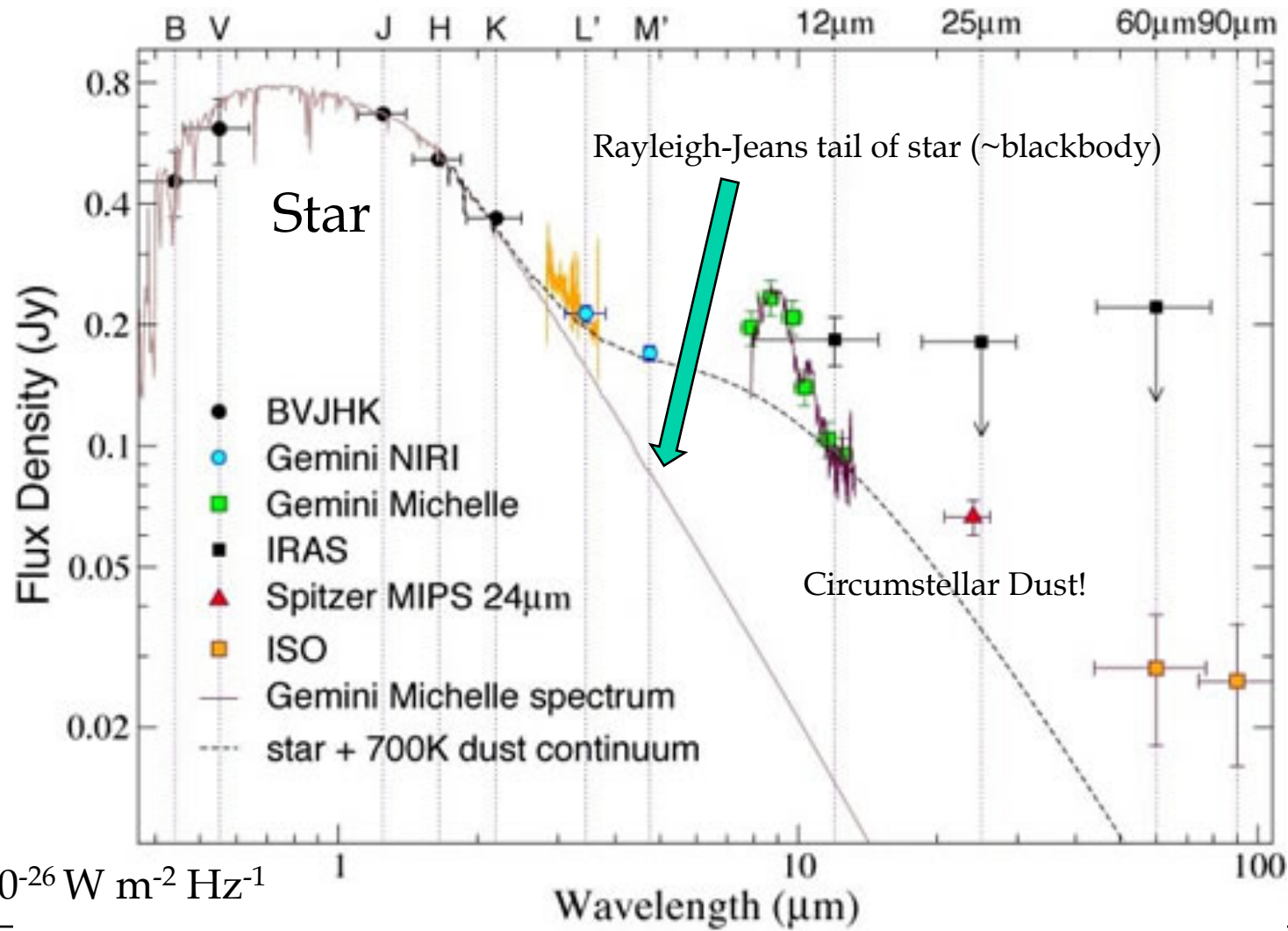
$$B-V(T_{\text{eff}} = 30000\text{K}) \\ = -0.32 \text{ mag}$$

$$B-V(T_{\text{eff}} = 3000\text{K}) \\ = 1.8 \text{ mag}$$

Combining colors and absolute magnitudes => HR diagram



How do we calculate luminosity with magnitudes (at some λ) and distances?



$$1 \text{ Jy} = 10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1}$$

How do we calculate luminosity with magnitudes (at some λ) and distances?

We can work with “bolometric magnitudes” (m_{bol}) – i.e. accounting for a star’s light at (ostensibly) all wavelengths.

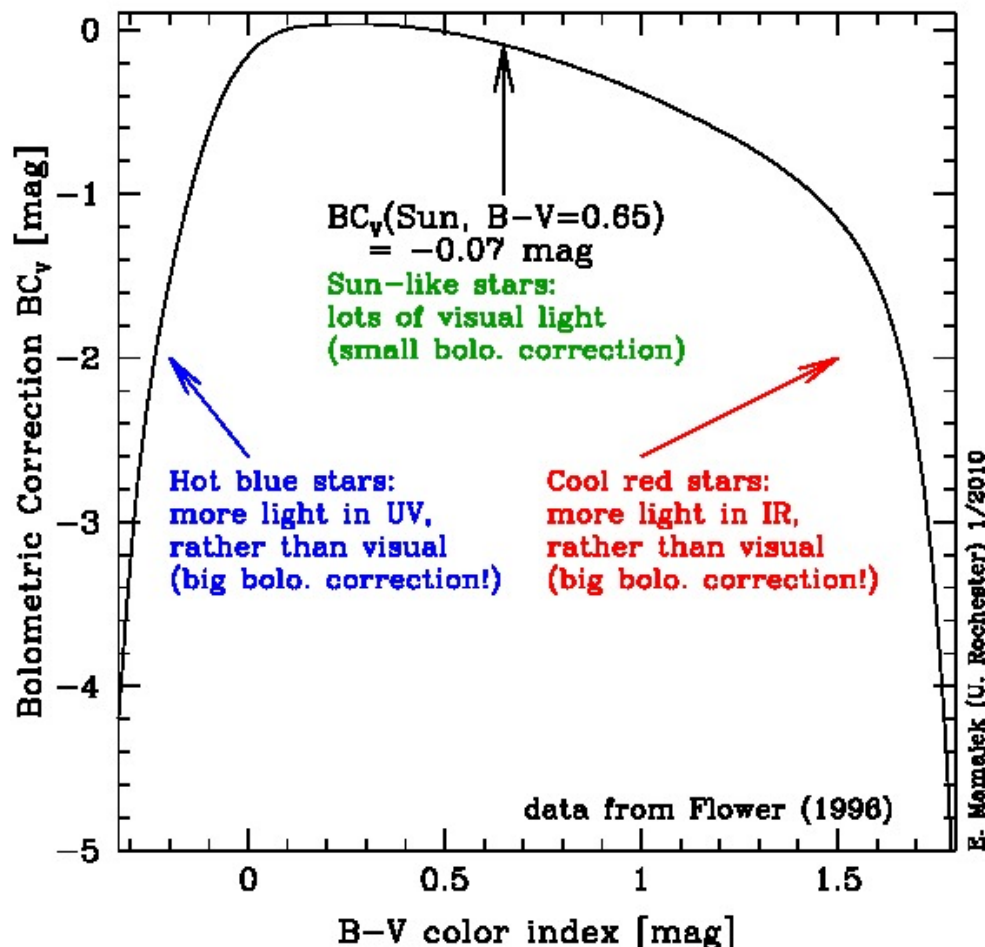
$$m_{\text{bol}} = m_V \text{ (V-band magnitude)} + \text{BC (‘‘bolometric correction’’)}$$

The “BC” accounts for the light not being emitted in the band whose magnitude you’ve measured (in this case all the light blueward and redward of the V-band $\sim 0.55\mu\text{m}$ filter).

Absolute bolometric magnitude: $M_{\text{bol}} = m_{\text{bol}} - 5\log(d_{\text{pc}}) + 5$
 $m_V(\text{Sun}) = -26.74$; $M_{\text{bol}}(\text{Sun}) = 4.75 \text{ mag (by convention)}$
 $M_V(\text{Sun}) = 4.83$; $\text{BC}_V(\text{Sun}) = -0.08 \text{ mag}$

Bolometric corrections (getting luminosities from magnitude at one band!)

Bolometric correction at V-band
for main sequence stars



Shape of spectrum, and this correction factor, can be determined if one knows the color of the star (difference between apparent magnitudes at two λ s).

A scaling factor for flux is the same as an offset in magnitude. This offset is called the bolometric correction, BC , and is determined **empirically** from observed stellar spectra.

$$m_{\text{bol}} = m_V + BC$$

$$B - V = m_B - m_V$$

Luminosity... finally!

$$\text{Log}_{10}(L/L_{\text{sun}}) = -0.4(M_{\text{bol}} - M_{\text{bol}}(\text{Sun})) = -0.4M_{\text{bol}} + 1.90$$

$$\text{Where } L_{\text{sun}} = 3.827 \times 10^{33} \text{ erg/s} = 3.827 \times 10^{26} \text{ W}$$

Origin: spacecraft measurements measure “solar constant”
to be 1361 W/m² at 1 AU distance from Sun: and $L = f * 4\pi R^2$

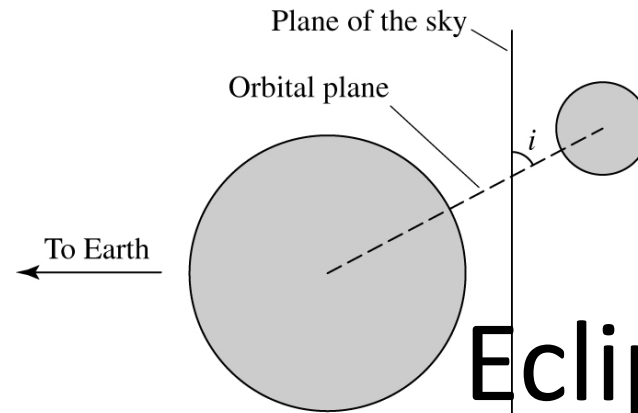
Stefan-Boltzmann law: surface flux = $f_{\text{bol}} = \sigma T_{\text{eff}}^4$

$$\sigma = \text{Stefan-Boltzmann Constant} = 5.67 \times 10^{-5} \text{ erg/cm}^2/\text{s/K}^4$$

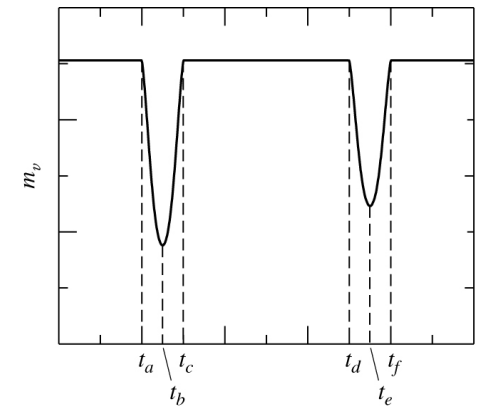
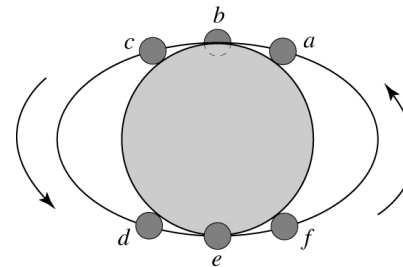
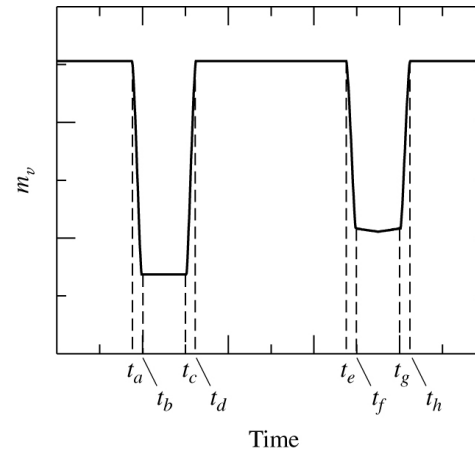
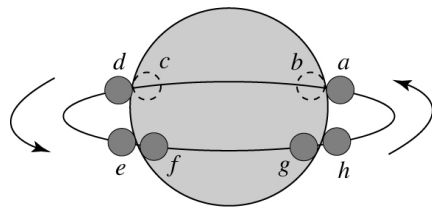
$$\text{Luminosity } L = 4\pi R^2 (\sigma T^4)$$

Sun's radius is opaque to radiation at $R_{\text{sun}} = 695,600 \text{ km}$

We define the star's effective temperature T_{eff} to be the
temperature of a blackbody that would emit the same
luminosity (power) as a star of a given radius



Eclipsing Binaries (primary source of accurate stellar masses)

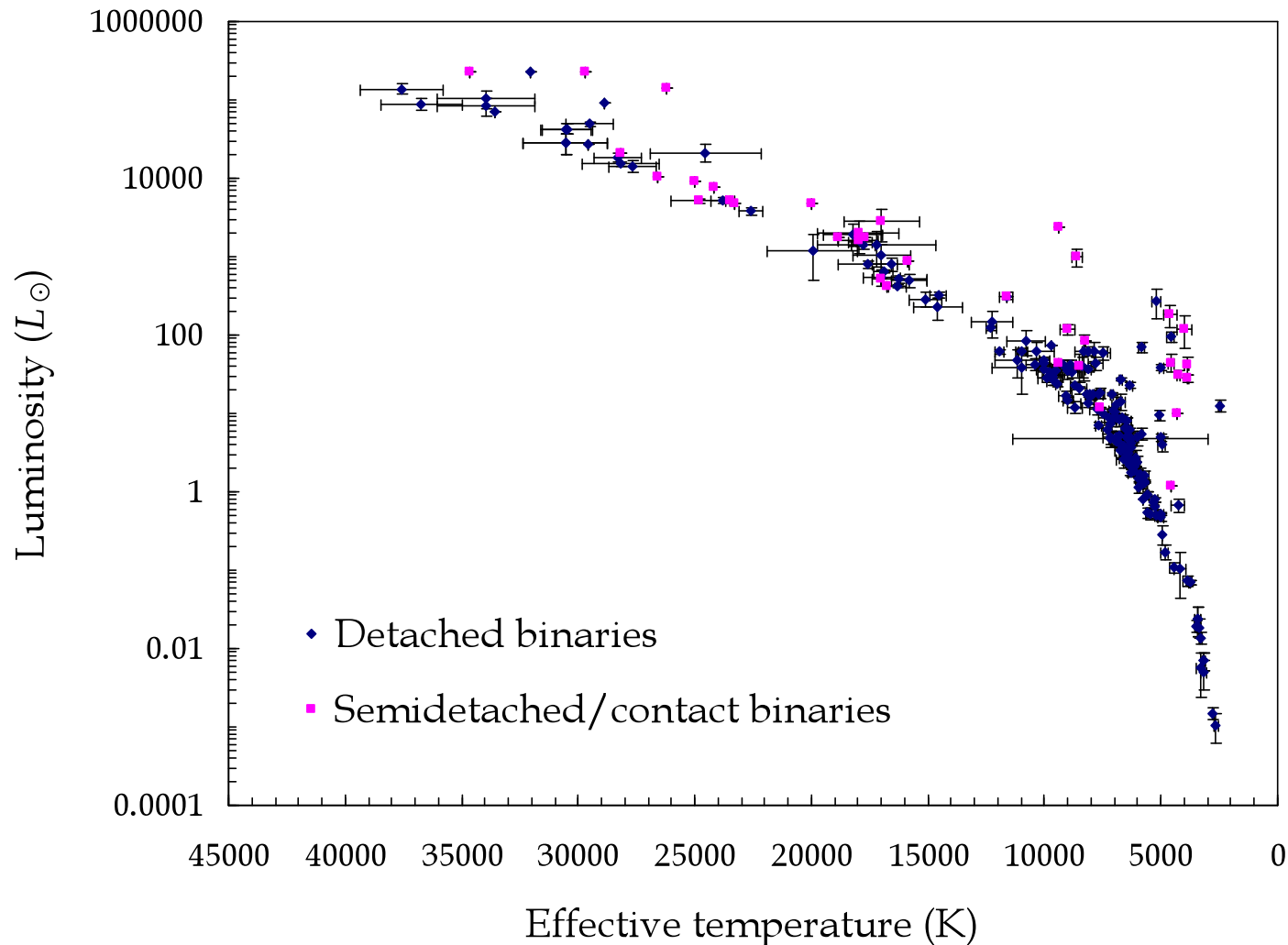


Binaries from which one gets useful mass measurements

Most normal stars turn out to be members of binary systems.

- ❑ Resolved **visual binaries**: see stars separately, measure orbital axes and speeds directly. Few demonstrate “useful” orbital motion (i.e. period $\sim \tau_{\text{human}}$)
- ❑ **Astrometric binaries**: only brighter member seen, with periodic wobble in the track of its proper motion.
- ❑ **Spectroscopic binaries**: unresolved (relatively close) binaries told apart by periodically oscillating Doppler shifts in spectral lines. Periods = days to years.
 - Spectrum binaries: orbital periods longer than period of known observations.
 - **Eclipsing binaries**: orbits seen nearly edge on, so that the stars actually eclipse one another. (Most useful.)

Luminosity-temperature relation for binary stars (mostly eclipsing) with well-determined orbits



Compiled
by Oleg
Malkov
(1993),
based
mostly on
work over
many
decades by
Dan
Popper.

Stellar masses determined for binary systems

- If orbital major axes (relative to center of mass) or radial velocity amplitudes are known, so is the ratio of masses:

$$\frac{m_1}{m_2} = \frac{a_2}{a_1} = \frac{v_{2r}}{v_{1r}}$$

- If the period, P , and the sum of major axis lengths, $a = a_1 + a_2$, are known, Kepler's third law can give masses separately:

$$P = \frac{4\pi^2}{G(m_1 + m_2)} a^3$$

Stellar masses determined for binary systems

- If only radial velocities are known, the sum of masses (from Kepler's third law) is:

$$m_1 + m_2 = \frac{P}{2\pi G} \left(\frac{v_{1r} + v_{2r}}{\sin i} \right)^3$$

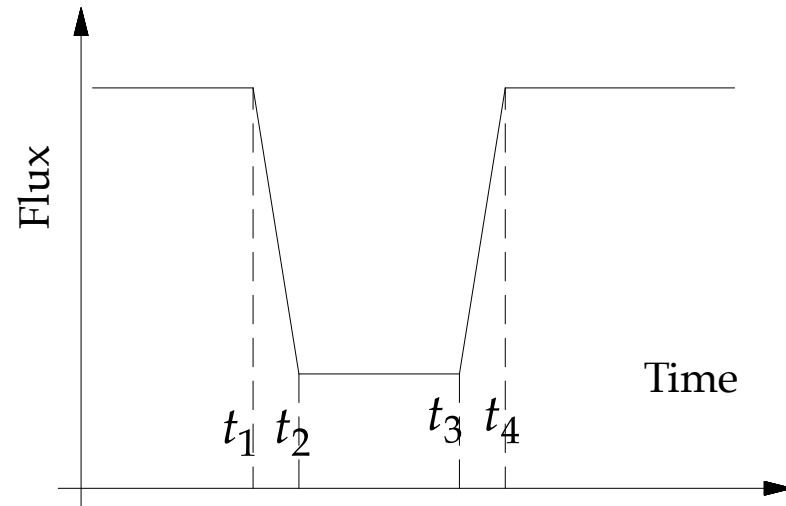
- If orientation angle of orbit, i , is known, this allows separate determination of the masses; that's why eclipsing binaries are so important ($\sin i$ must be close to unity in such cases).

Other uses for totally-eclipsing binary systems

Duration of eclipses and shape of light curve can be used to determine **radii** of stars:

$$R_s = \frac{v_1 + v_2}{2} (t_2 - t_1)$$

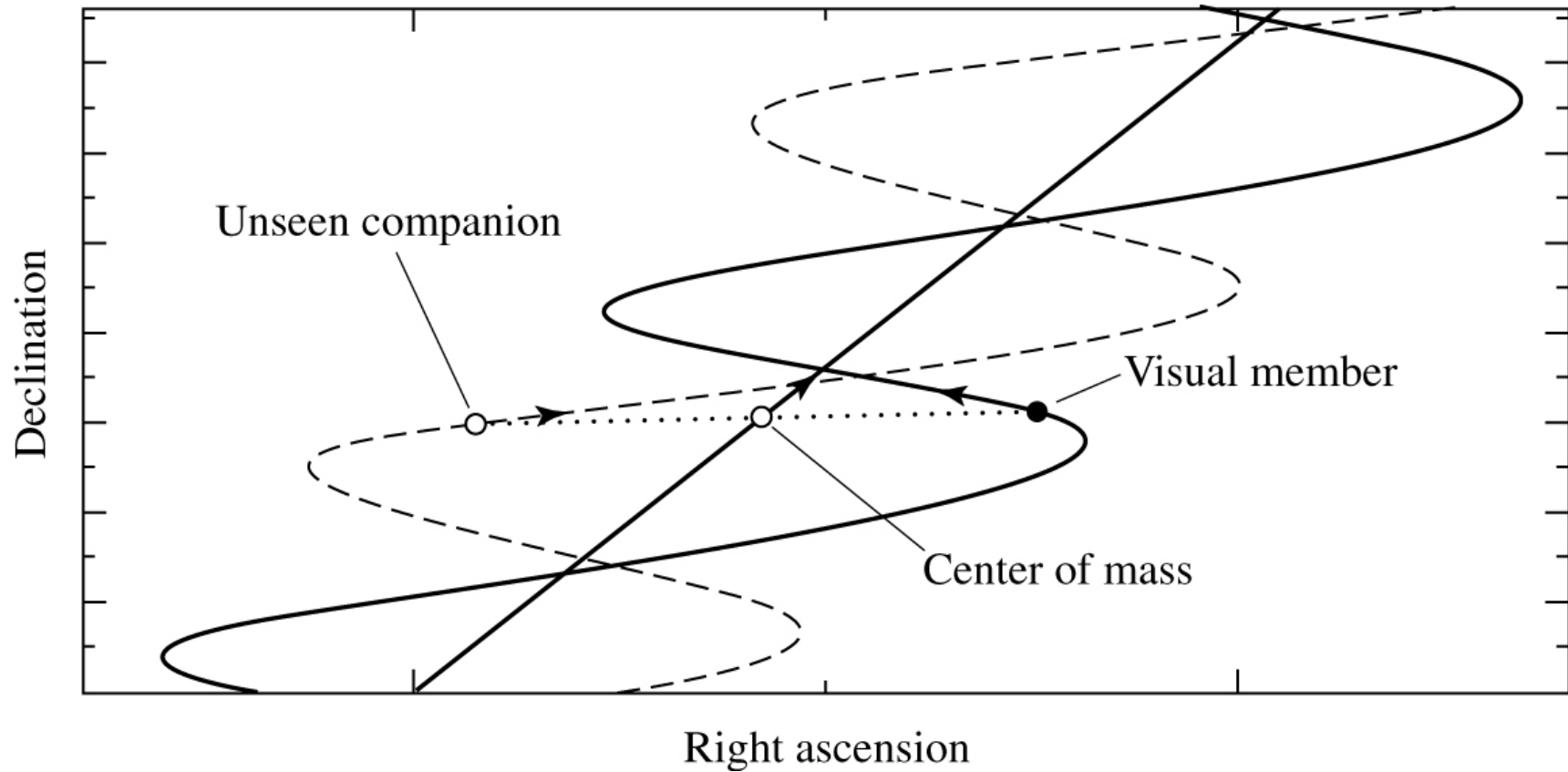
$$R_\ell = \frac{v_1 + v_2}{2} (t_3 - t_1)$$



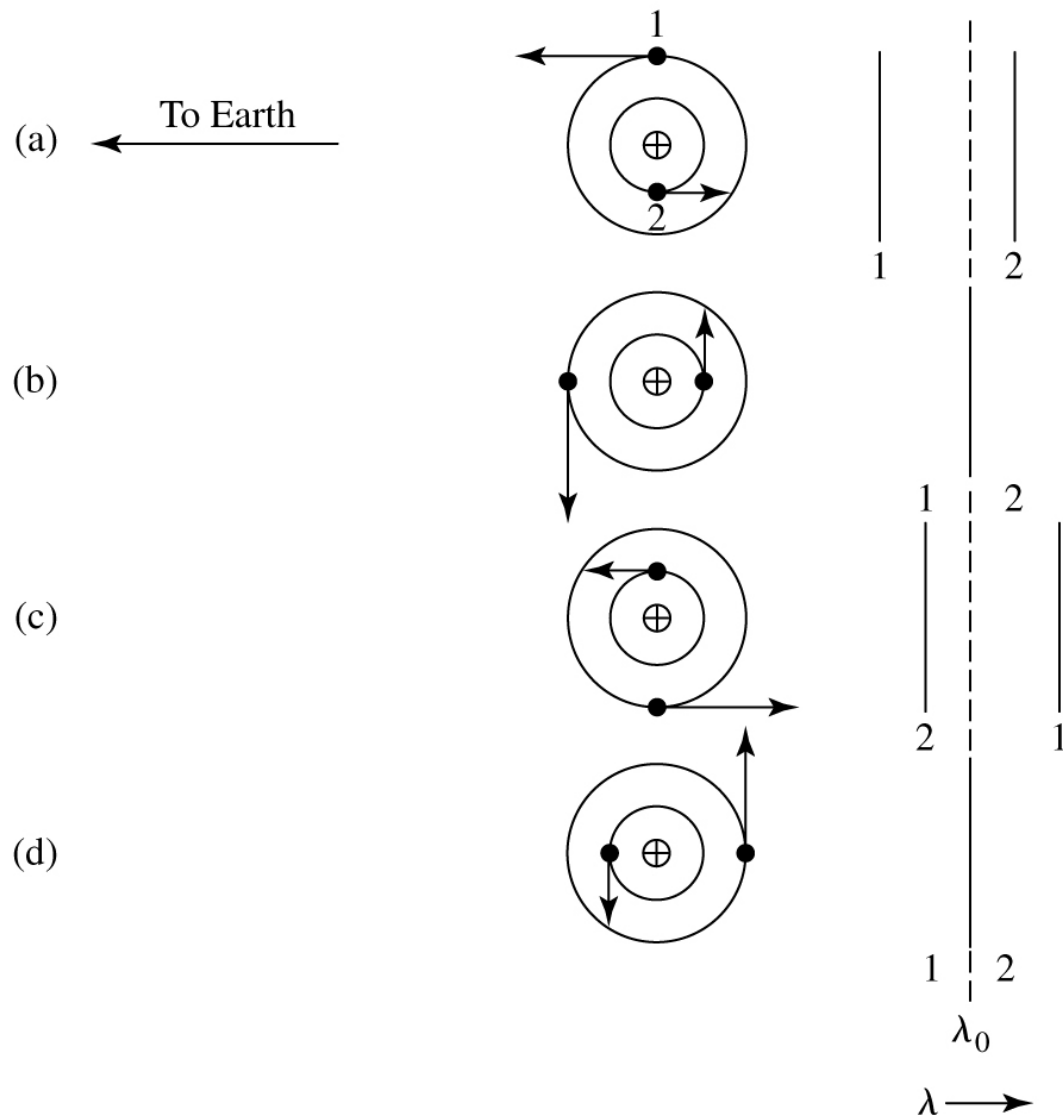
Relative depth of primary (deepest) and secondary brightness minima of eclipses can be used to determine the **ratio of effective temperatures of the stars**:

$$\frac{F_0 - F_{\text{primary}}}{F_0 - F_{\text{secondary}}} = \left(\frac{T_{e,s}}{T_{e,\ell}} \right)^4 .$$

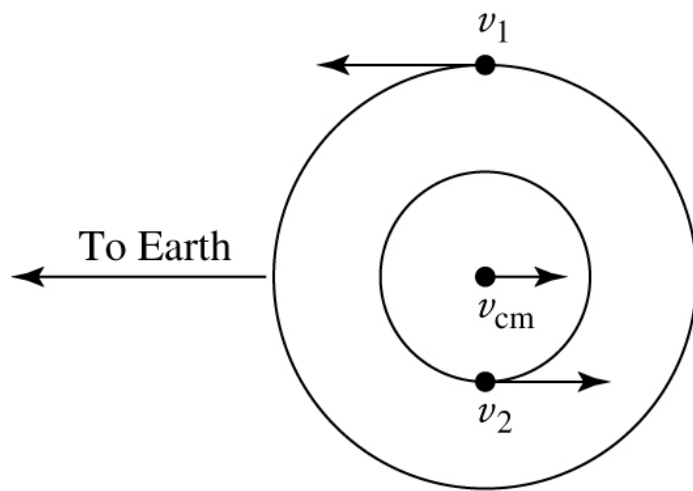
Astrometric Binary



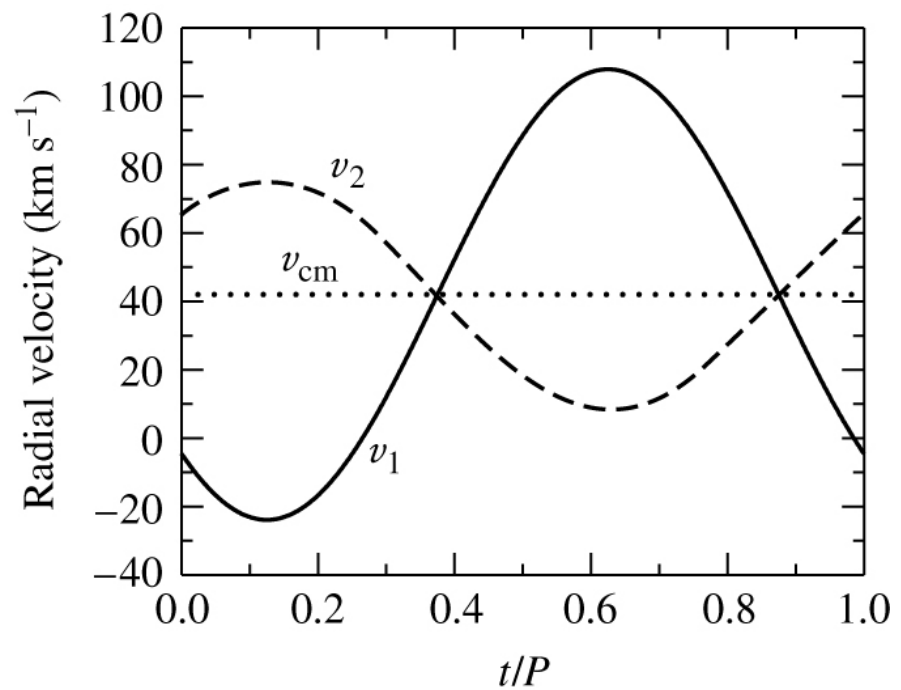
Spectroscopic Binary



Radial Velocity vs. Time for Double-lined SB in Circular Orbit

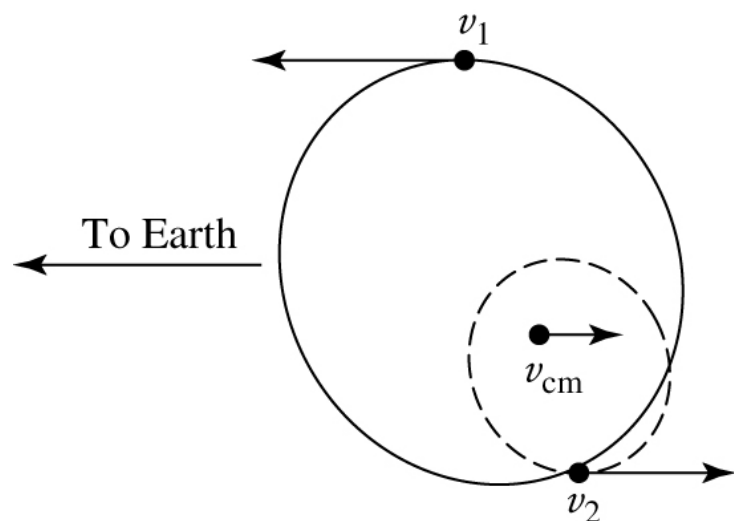


(a)

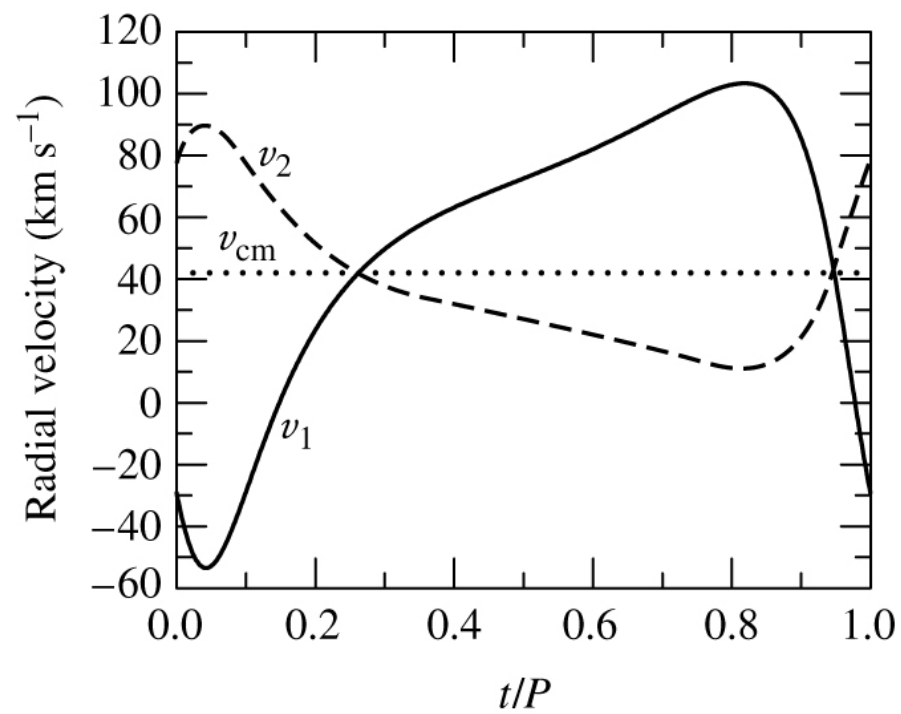


(b)

Radial Velocity vs. Time for Double-lined SB in Elliptical orbit ($e=0.4$)

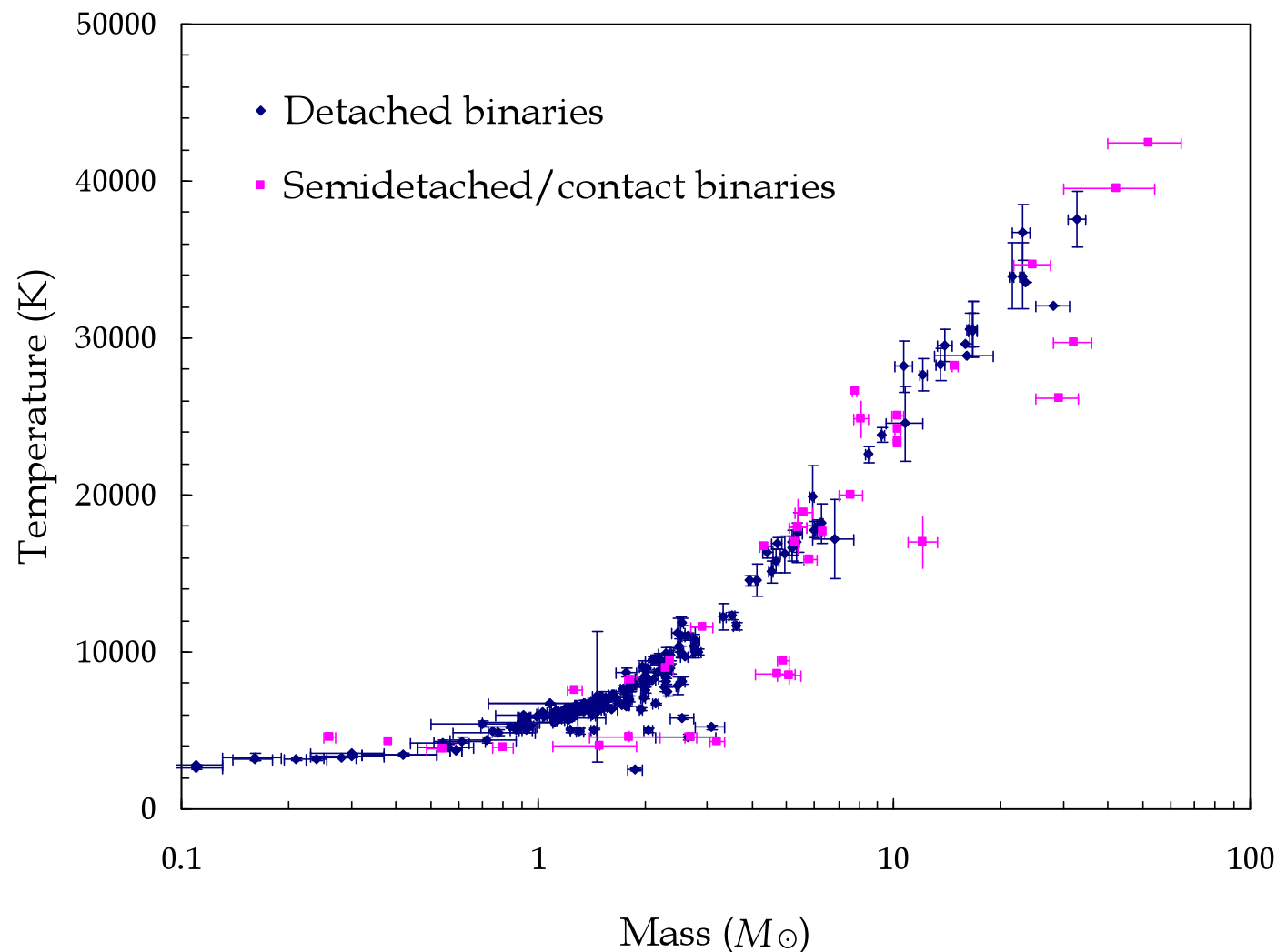


(a)



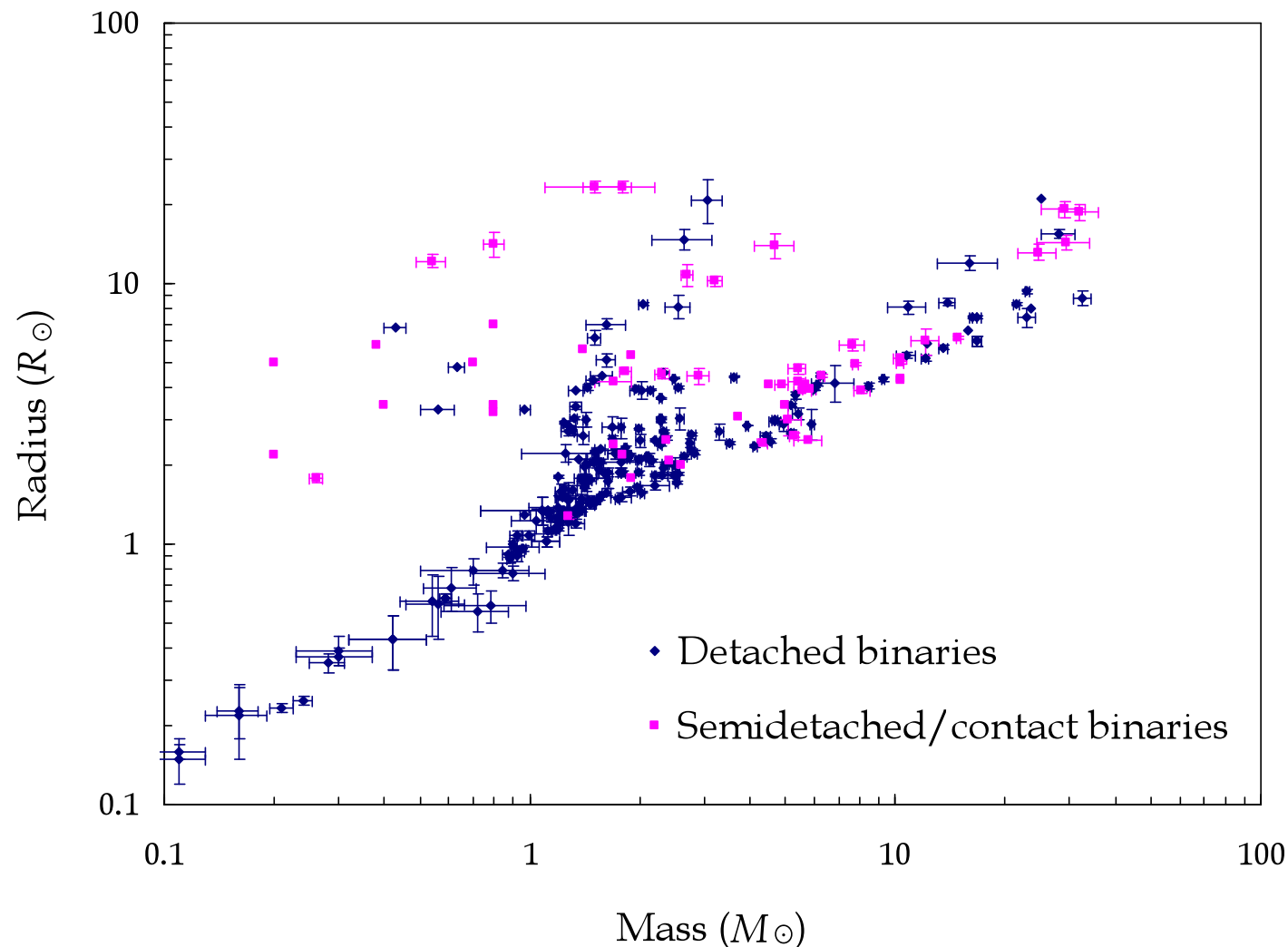
(b)

Temperature-mass relation for binary stars with well-determined orbits



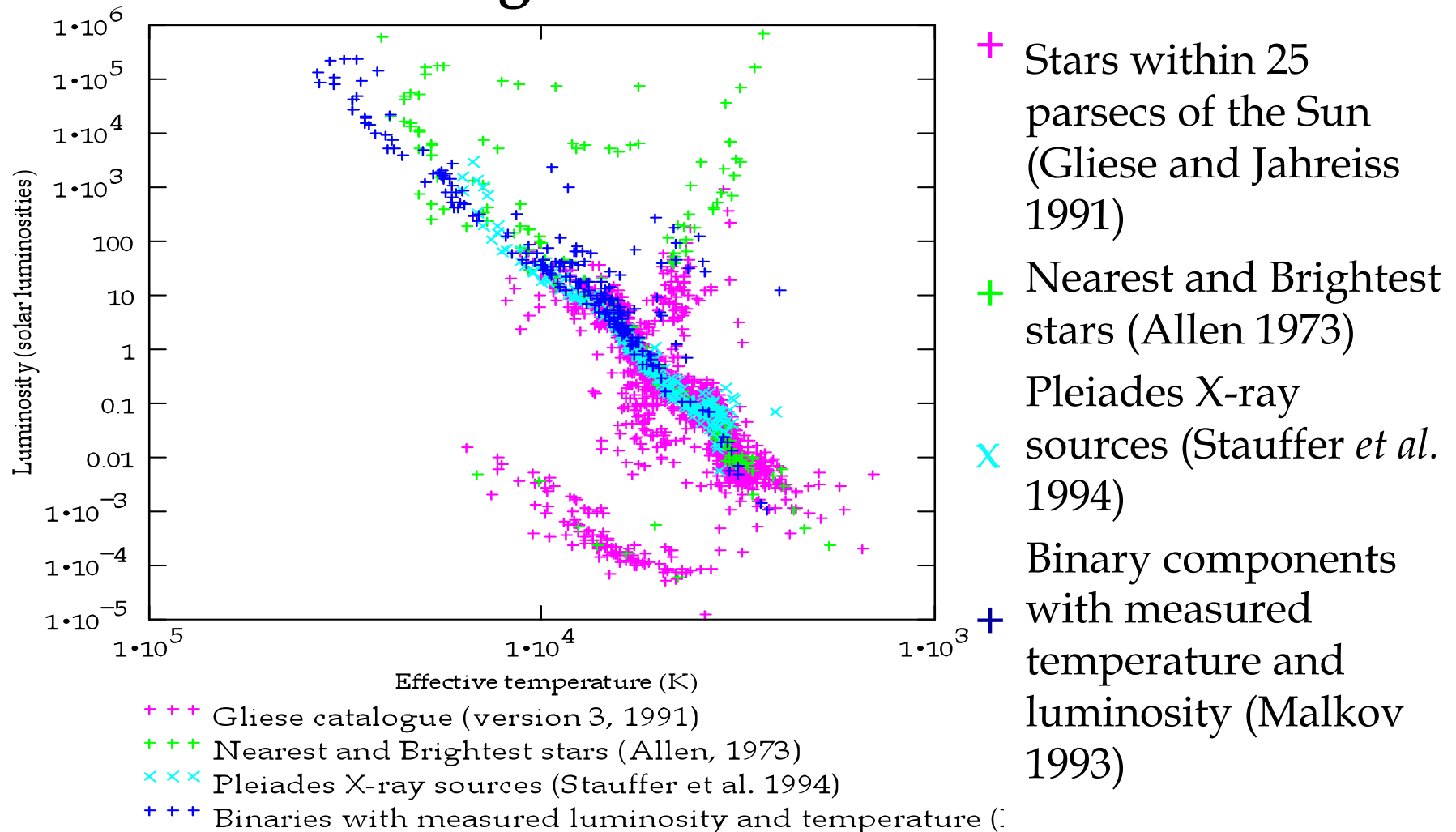
Compiled by Malkov (1993), based mostly on work over many decades by Popper.

Radius-mass relation for binary stars with well-determined orbits



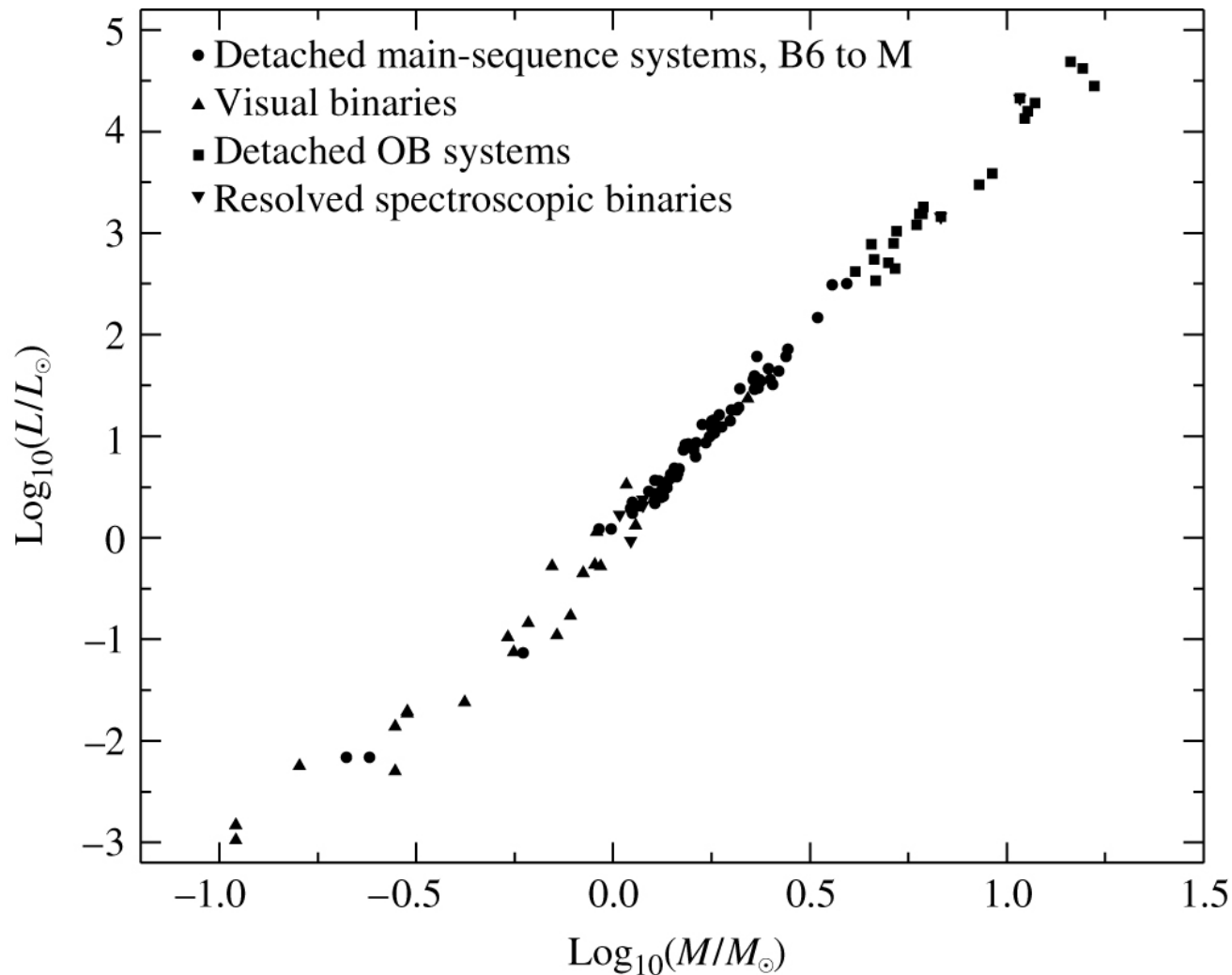
Compiled by Malkov (1993), based mostly on work over many decades by Popper.

Why do we think these results apply to stars in general? Well ...



Mass-Luminosity Relation

(our stellar models need to reproduce this!)



Data from
Popper
(1980;
ARA&A 18,
115)