Fermi Project Update

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1 Critical Technology

$$T_* = T_{\eta=\eta_C} = Critical\ Technology$$

This critical technology is defined to be the minimum value of technology so that if systems have tech below this, they will not be able to reach any of their neighbors. In order to find this, we make the following assumptions.

1.1 Assumptions

1. Low η Limit (low density limit) means that
$$\eta = \eta_c \Longleftrightarrow T_S = T_C$$
2. Non-Relatavistic Limit ($v << c$) means that
$$d_p \approx r_0 e^T$$
3. Calculating in Periodic box means that both v_s and ρ are constants

1.2 Calculation

Now, our condition for critical technology can be written like (assuming low η)

$$T_S = T_C = \frac{1}{\pi f \rho d_p^2 v_s}$$

In the periodic box, the density and root mean squared velocities are constant. In the non-relativistic limit, our expression for probe range is given by

$$d_p \approx r_0 e^T = (\pi f \rho T_s v_s)^{-1/2}$$

Solving for T gives us the answer...

$$\boxed{T_* = -1/2 \ln\left(\pi f \rho T_s v_s r_0^2\right)}$$

With our typical units, this becomes

$$T_* = -\frac{1}{2} \ln\left(\frac{f T_s}{2.6 \times 10^{11}\, yr}\right)$$

2 Periodic Box Paramater Sweep

2.1 Calculating Inputs

- Variables Visualized
  - $T = Technology$
  - $X = \dfrac{\#\ settled\ systems}{\#\ total\ systems} = Settled\ Fraction$
- Given
  - $\rho = 0.08\ pc^{-3} = stellar\ density$
  - $d_p = 10^{-2.51}\,kpc \approx 10\ lyr = Probe\ Range$
  - $v_p = 10^{-4}\,c \approx 30\ km/s = Probe\ Range$
  - $v_s = 50\ km/s = Stellar\ Velocity$
  - $\Delta T = \dfrac{\sigma}{\sqrt{2\theta}} = 1 = Technology\ Standard\ Deviation$
    * $\theta = Rate\ of\ Convergence\ (Myr^{-1})$
      · $T_\theta = 1/\theta = Tech\ Convergence\ to\ Mean\ Time\ (Myr)$
    * $\sigma = Degree\ of\ Randomness\ (Myr^{-1/2})$
      · $T_\sigma = 1/\sqrt{\sigma} = Timescale\ for\ Technological\ Spread\ (Myr)$
  - $N = 10^4 = Number\ of\ systems/stars$
  - $X_i = \dfrac{N_{abio}}{N} = 0.1 = Initial\ Settled\ Fraction$
  - $N_{abio} = NX_i = 100 = Number\ of\ Abiogenesis\ Seeds$
- Specify f
  - $\eta = f\rho d_p^3 = 2.36f$
  - $T_C = (\pi f\rho d_p^2 v_s)^{-1} = \dfrac{8,148\,yrs}{f} = Collision\ Timescale$
  - $T_R = \dfrac{(f\rho)^{-1/3}}{v_s} = \dfrac{45,386\,yrs}{f^{1/3}} = Reconfiguration\ Timescale$
    * $\Delta t = T_R/10 = \dfrac{4,539\,yrs}{f^{1/3}} = Timestep$
  - $L = \left(\dfrac{N}{f\rho}\right)^{1/3} = \dfrac{50\,pc}{f^{1/3}} = Box\ Length/Width/Height$
  - Misc...
    * $D = 1 - e^{-\frac{4\pi}{3}\eta} = Odds\ of\ System\ Having\ At\ Least\ One\ Neighbor\ in\ Range$
    * $T_p = \Delta t = Probe\ Launch\ Period$
    * $T_\ell = DT_p + (1-D)T_c = Launch\ Time$
- Specify $T_\theta/T_C \equiv k$
  - $T_\theta = kT_c = 8,148\left(\dfrac{k}{f}\right)\ yrs$
  - $\theta = \dfrac{1}{T_\theta} = 125\left(\dfrac{f}{k}\right)\ Myr^{-1}$
  - $\sigma = \Delta T\sqrt{2\theta} = \left(\dfrac{250f}{k}\right)\ Myr^{-1/2}$

- Specify $X_{eq} \equiv 1 - \frac{T_\ell}{T_S}$ = *Equilibrium Settled Fraction*
  - $T_{runtime} = \max\left(5T_\theta,\ \dfrac{T_C}{X_{eq}}\right)$ = *Total Simulation Runtime*
  - $T_S = \dfrac{T_\ell}{1 - X_{eq}}$ = *Civilization Average Lifetime*

2.2 Output Plots

2.2.1 Line Plots Showing Temporal Evolution

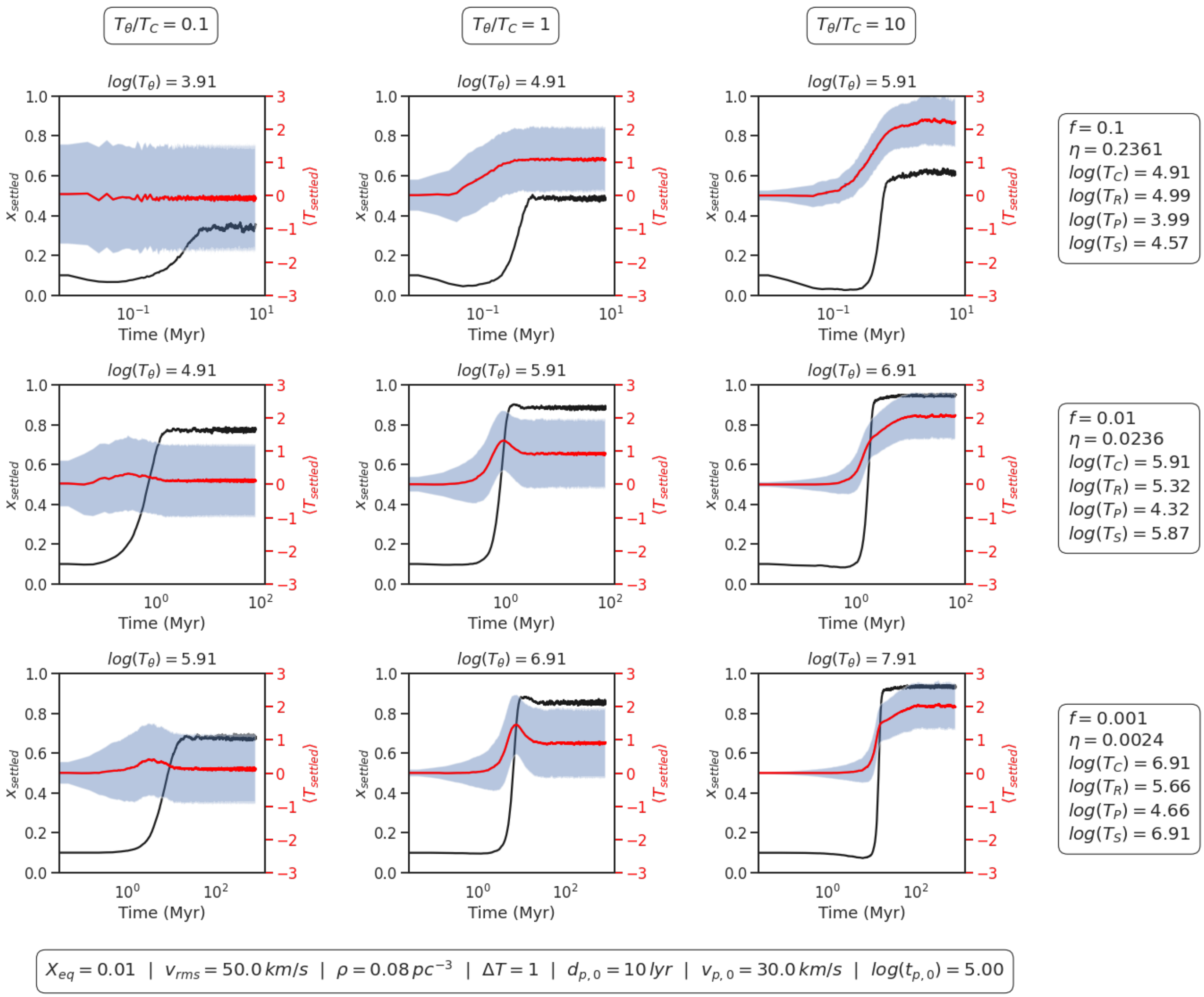


Figure 1: These plots show the behavior of all settled systems, as time evolves. So $x_{settled}$ is the fraction of settled systems over the total number of systems, and $\langle T_{settled} \rangle$ is the mean technology of the settled systems. Also, the blue shaded region around the technology line shows the standard deviation of technology (shown in the bottom as $\Delta T = 1$). Note also that the x axis has a log scale.

2.2.2 Histogram Showing Final Distribution of Technology

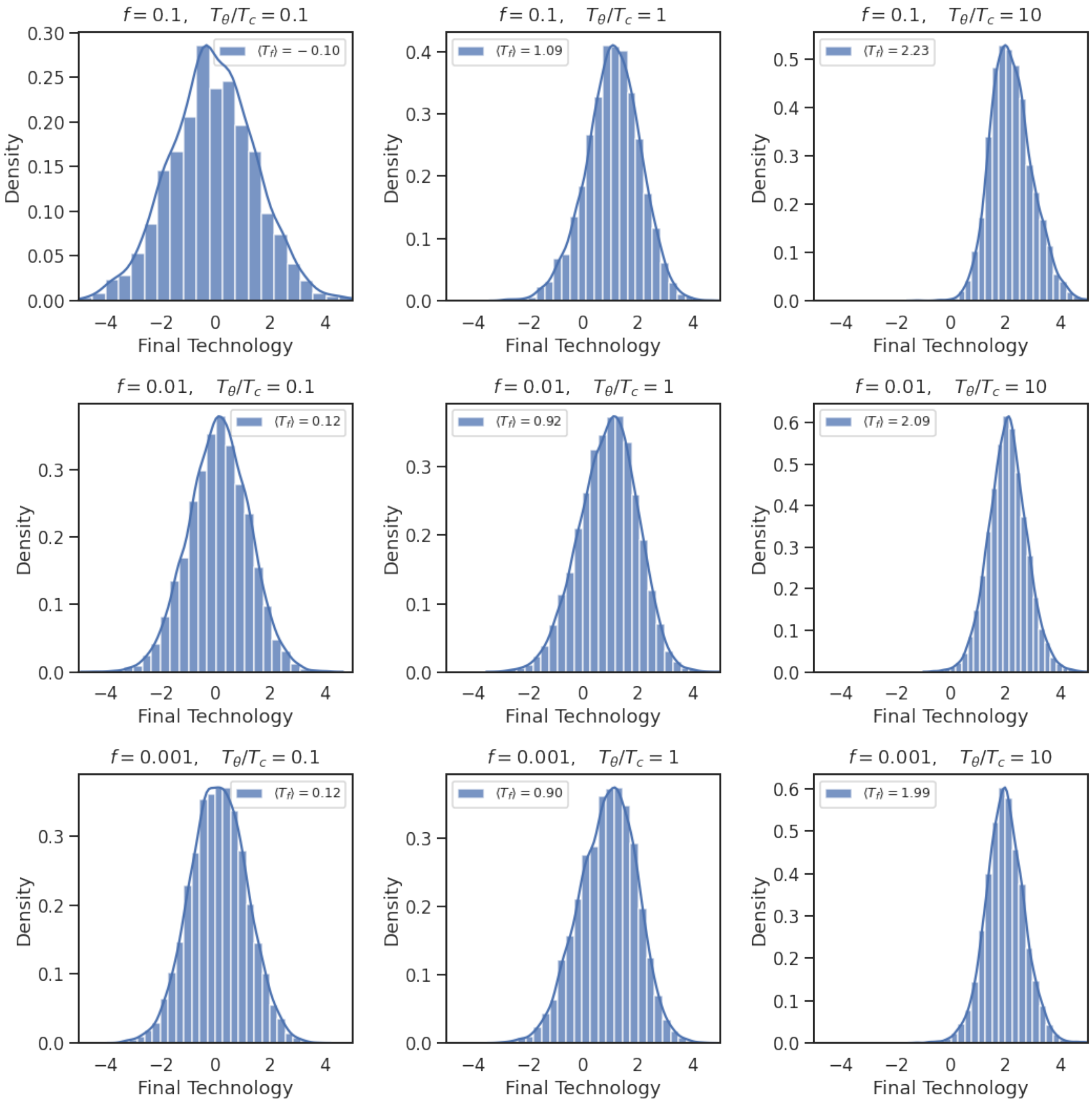
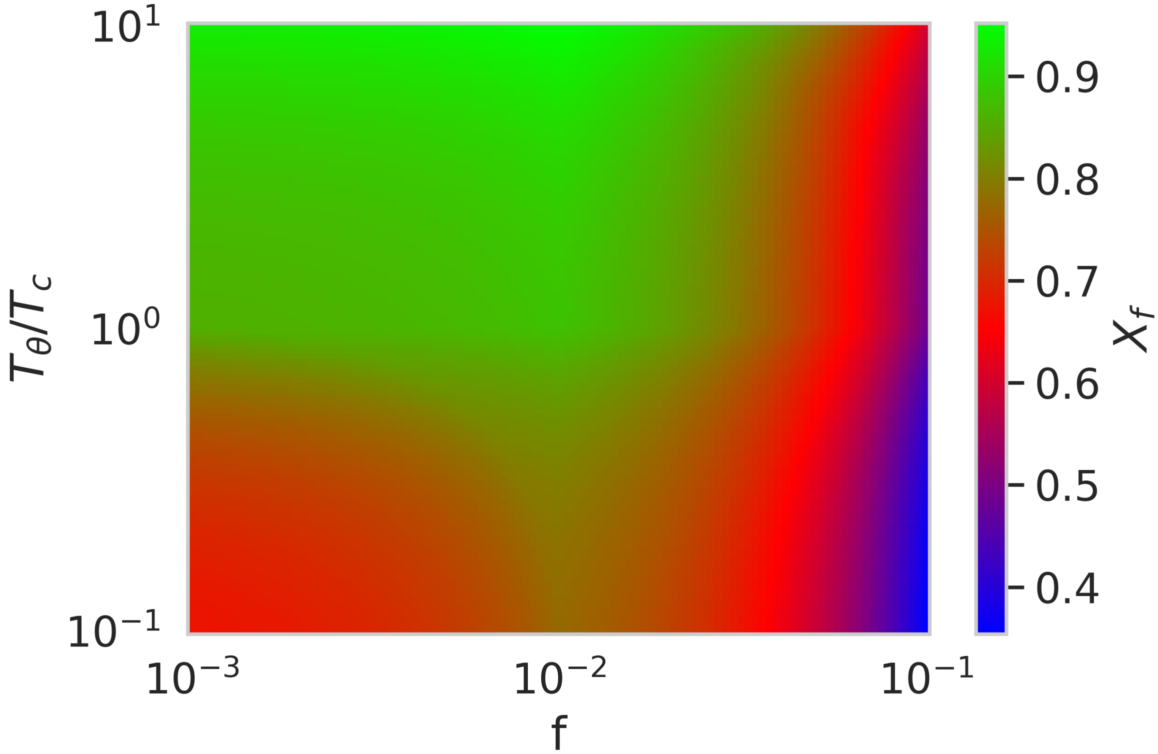


Figure 2: These plots show the final distribution of technology for all settled systems. As T_θ/T_c increases, as does the value of final mean technology ($\langle T_f \rangle$)

2.2.3 Phase Diagram Summarizing Parameter Sweep



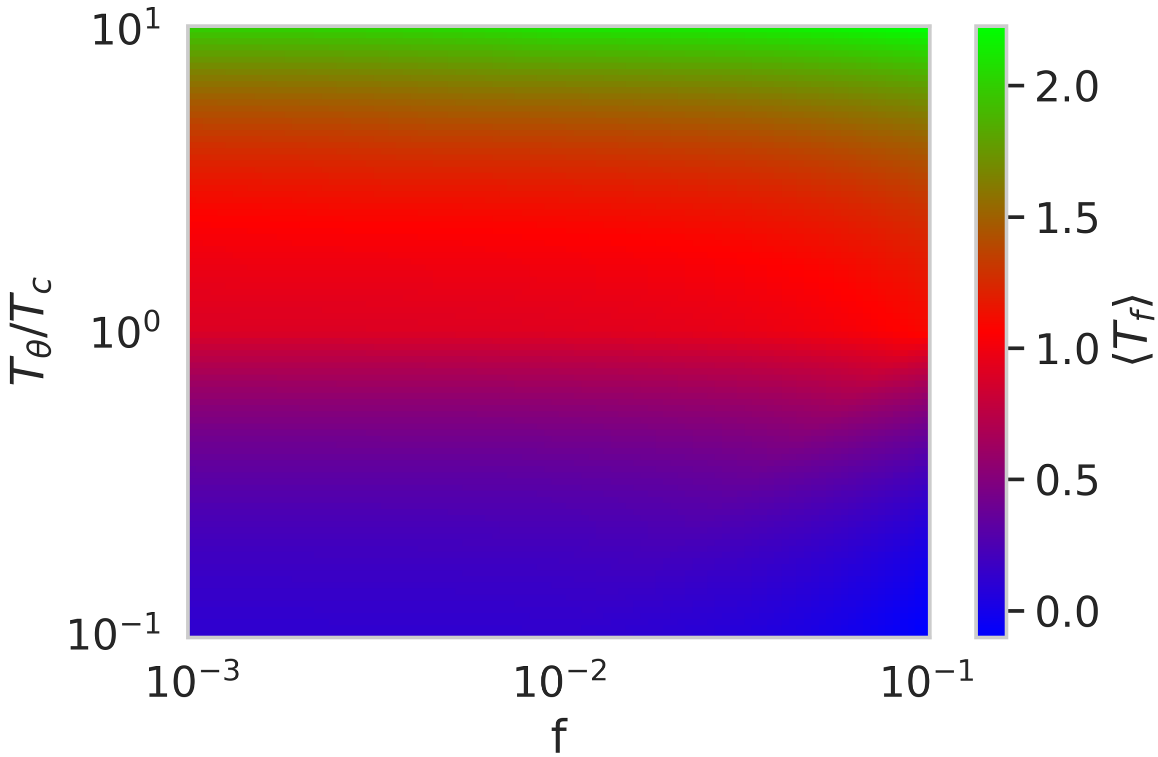


Figure 3: X_f is the final settled fraction, while $\langle T_f \rangle$ is the final mean technology of settled systems