# Fermi Project Update

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## 1 Calculating Inputs

- Variables Visualized
  - $T = Technology$
  - $X = \frac{\#\ settled\ systems}{\#\ total\ systems} = Settled\ Fraction$
- Given
  - $\rho = 0.08\ pc^{-3} = stellar\ density$
  - $d_p = 10^{-2.51}\, kpc \approx 10\ lyr = Probe\ Range$
  - $v_p = 10^{-4}\, c \approx 30\ km/s = Probe\ Range$
  - $v_s = 50\ km/s = Stellar\ Velocity$
  - $\Delta T = \frac{\sigma}{\sqrt{2\theta}} = 1 = Technology\ Standard\ Deviation$
    - $\theta = Rate\ of\ Convergence\ (Myr^{-1})$
      - $T_\theta = 1/\theta = Tech\ Convergence\ to\ Mean\ Time\ (Myr)$
    - $\sigma = Degree\ of\ Randomness\ (Myr^{-1/2})$
      - $T_\sigma = 1/\sqrt{\sigma} = Timescale\ for\ Technological\ Spread\ (Myr)$
  - $N = 10^4 = Number\ of\ systems/stars$
  - $X_i = \frac{N_{abio}}{N} = 0.1 = Initial\ Settled\ Fraction$
  - $N_{abio} = NX_i = 100 = Number\ of\ Abiogenesis\ Seeds$
- Specify $f$
  - $\eta = f\rho d_p^3 = 2.36f$
  - $T_C = (\pi f \rho d_p^2 v_s)^{-1} = \frac{8,148\, yrs}{f} = Collision\ Timescale$
  - $T_R = \frac{(f\rho)^{-1/3}}{v_s} = \frac{45,386\, yrs}{f^{1/3}} = Reconfiguration\ Timescale$
    - $\Delta t = T_R/10 = \frac{4,539\, yrs}{f^{1/3}} = Timestep$
  - $L = \left(\frac{N}{f\rho}\right)^{1/3} = \frac{50\, pc}{f^{1/3}} = Box\ Length/Width/Height$
  - Misc...
    - $D = 1 - e^{-\frac{4\pi}{3}\eta} = Odds\ of\ System\ Having\ At\ Least\ One\ Neighbor\ in\ Range$
    - $T_p = \Delta t = Probe\ Launch\ Period$
    - $T_\ell = DT_p + (1-D)T_c = Launch\ Time$

- Specify $T_\theta/T_C \equiv k$
  - $T_\theta = kT_c = 8,148 \left(\frac{k}{f}\right) \; yrs$
  - $\theta = \frac{1}{T_\theta} = 125 \left(\frac{f}{k}\right) \; Myr^{-1}$
  - $\sigma = \Delta T\sqrt{2\theta} = \left(\frac{250f}{k}\right) \; Myr^{-1/2}$
- Specify $X_{eq} \equiv 1 - \frac{T_\ell}{T_S} = Equilibrium\ Settled\ Fraction$
  - $T_{runtime} = \max\left(5T_\theta,\ \frac{T_C}{X_{eq}}\right) = Total\ Simulation\ Runtime$
  - $T_S = \frac{T_\ell}{1 - X_{eq}} = Civilization\ Average\ Lifetime$

# 2 Analytic Model (Non-Relativistic)

$$\frac{dT}{dt} = \frac{X(1-X)T}{T_c d_p^{-2} e^{-2T}} - \frac{\Delta T}{T_\theta}$$
$$\frac{dX}{dt} = \frac{X(1-X)}{T_c d_p^{-2} e^{-2T}} - \frac{X}{T_s}$$

**Note**

- $d_p = d_{p,0} = Initial\ Value$
- $T_s = T_{s,0} = Initial\ Value$
- $T_c = T_{c,0} = Intial\ Value$

$$T_c \propto \frac{T_{c,0}}{\left(d_{p,0} e^T\right)^2}$$