Fermi Project Update

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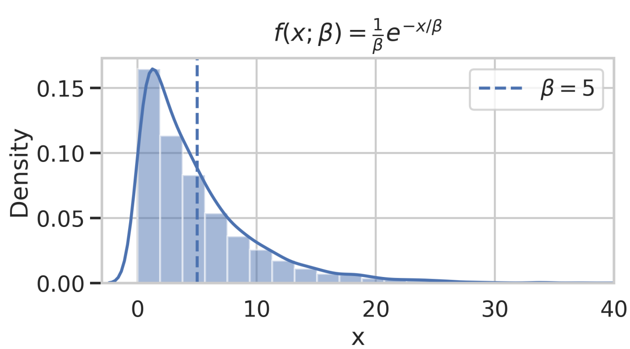
1 March Update

1.1 Introduction

Goal: Add as an input a fixed probability of destruction per timestep.
Method: Since we are considering events which occur continuously and independently at a constant average rate, this type of process is called a **Poisson point process**. Furthermore, the probability distribution describing the time between events in a Poisson point process is the **exponential distribution**, shown below.[1]



1.1.1 Collapse Time

Let

$$T_s = \textit{Average Civilization Lifetime}$$

Then

$$P(t) = \frac{1}{T_s}e^{-t/T_s} = \textit{probability of a civilzation having a lifetime equal to } t$$

[1]The mean (expectation) value is given by β. The median value is $\beta \ln 2$

And

$$\begin{aligned}
P(t \leq T_*) &= \int_0^{T_*} P(t)dt \\
&= \frac{1}{T_s}\int_0^{T_*} e^{-t/T_s}dt \\
&= 1 - e^{-T_*/T_s} = \textit{probability of a civilization having a lifetime less than or equal to } T_* \\
&= \textit{probability of a civilization experiencing spontaneous collapse within some time } T_*
\end{aligned}$$

Thus, if $T_* = 10^6\ yrs = 1\ Myr$, then...

$$\boxed{P(t \leq 1\,Myr) = 1 - \exp\left(-\frac{1\,Myr}{T_s}\right) = \textit{probability of civilization dying per Myr} \equiv P_{death}}$$

1.1.2 Abiogenesis Time

Similarly, let

$$T_a = \textit{Average Timescale for Abiogenesis Event}$$

Then

$$P(t) = \frac{1}{T_a}e^{-t/T_a} = \textit{probability of an abiogenesis event occuring at some time } t.$$

And

$$P(t \leq T_*) = \int_0^{T_*} P(t)dt = \frac{1}{T_a}\int_0^{T_*} e^{-t/T_a}dt = 1 - e^{-T_*/T_a} = \textit{probability of an abiogenesis event occuring within some time } T_*$$

Thus, if $T_* = 10^6\ yrs = 1\ Myr$, then...

$$\boxed{P(t \leq 1\,Myr) = 1 - \exp\left(-\frac{1\,Myr}{T_a}\right) = \textit{probability of abiogenesis event per Myr} \equiv P_{abio}}$$

1.2 Parameter Sweeps

- **Columns**: ($[\log T_s] = \log yrs$)
  - Top: $\log T_s = 8$ (low P_{death})
  - Middle: $\log T_s = 7$
  - Bottom: $\log T_s = 6$ (high P_{death})
- **Rows**: ($[\log T_a] = \log yrs$)
  - Left: $\log T_a = 20$ (low P_{abio})
  - Right: $\log T_a = 16$ (high P_{abio})
- **Misc:** The width of the technology gaussian curve is dicated by σ and θ
  - $T_\sigma = 1/\sigma^2 = \textit{Timescale for Technology to Increase (Myr)}$
  - $T_\theta = 1/\theta = \textit{Timescale for Technology to Drift back to Zero (Myr)}$
  - $\Delta T = \frac{\sigma}{\sqrt{2\theta}} = \textit{Width of the Gaussian}$

Each model was run for 100 Myr. The first page shows runs with a $\sigma = 5$, while the next page shows the same runs with a $\sigma = 0.5$. Since the timescale for technology to advance is essentially $1/\sigma^2$, that means that the timescale for the first set of runs to increase technologically is $1/5^2 = 0.04\,Myr$, while the timescale for the second set of runs to increase technologically is $1/0.5^2 = 4\,Myr$. These runs are denoted "High Sigma" and "Low Sigma" respectively.

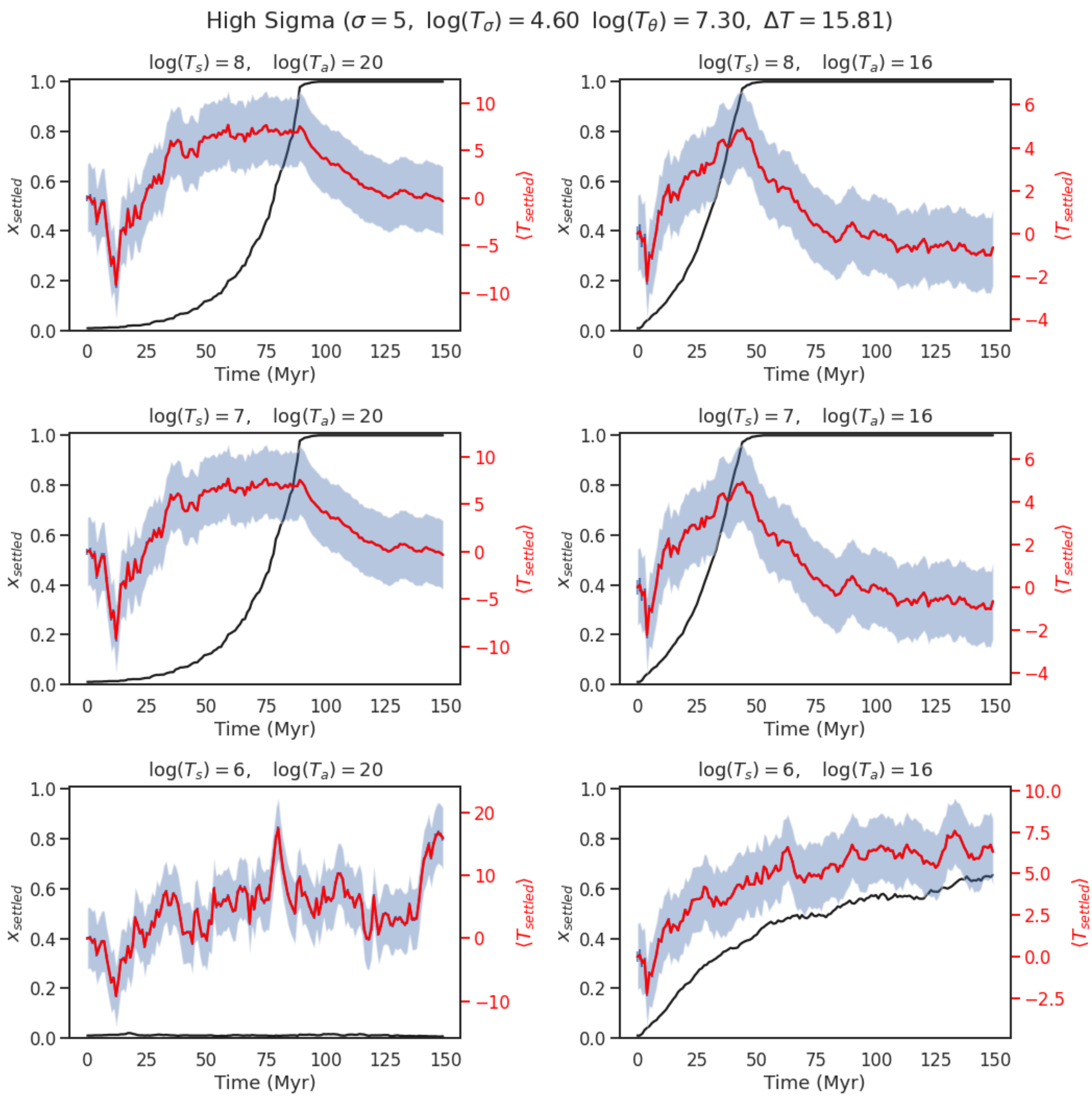


Figure 1: The black line shows x, the settled fraction, defined as the number of settled systems over the total number of systems. The black line correlates with the black tick marks on the left y-axis. The red line shows the average technological value of the settled systems, and correlates with the red tick marks on the right y-axis. Poisson error bars ($\sqrt{N}/N$) have also been included in the line plots of average technology.

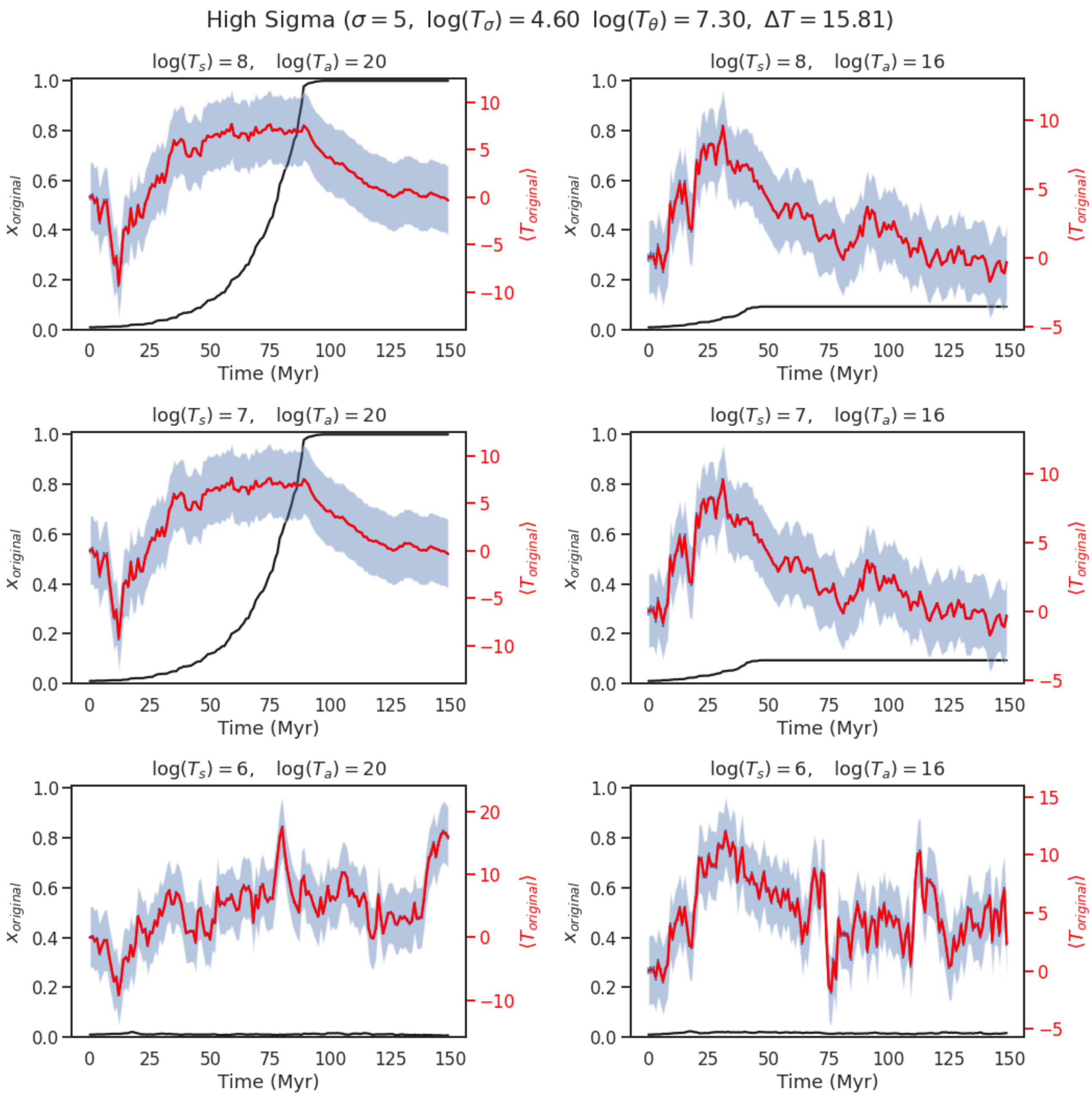


Figure 2: This figure is different from the previous in that we are only considering systems that have been settled by the original 10 systems. So x is the number of settled systems that have the same seed as the original 10 seeds divided by the total number of systems. While $\langle T \rangle$ now calculates the average value of technology among those systems that have the same seed as the original 10 systems (including the original 10). Poisson error bars ($\sqrt{N}/N$) have also been included in the line plots of average technology.

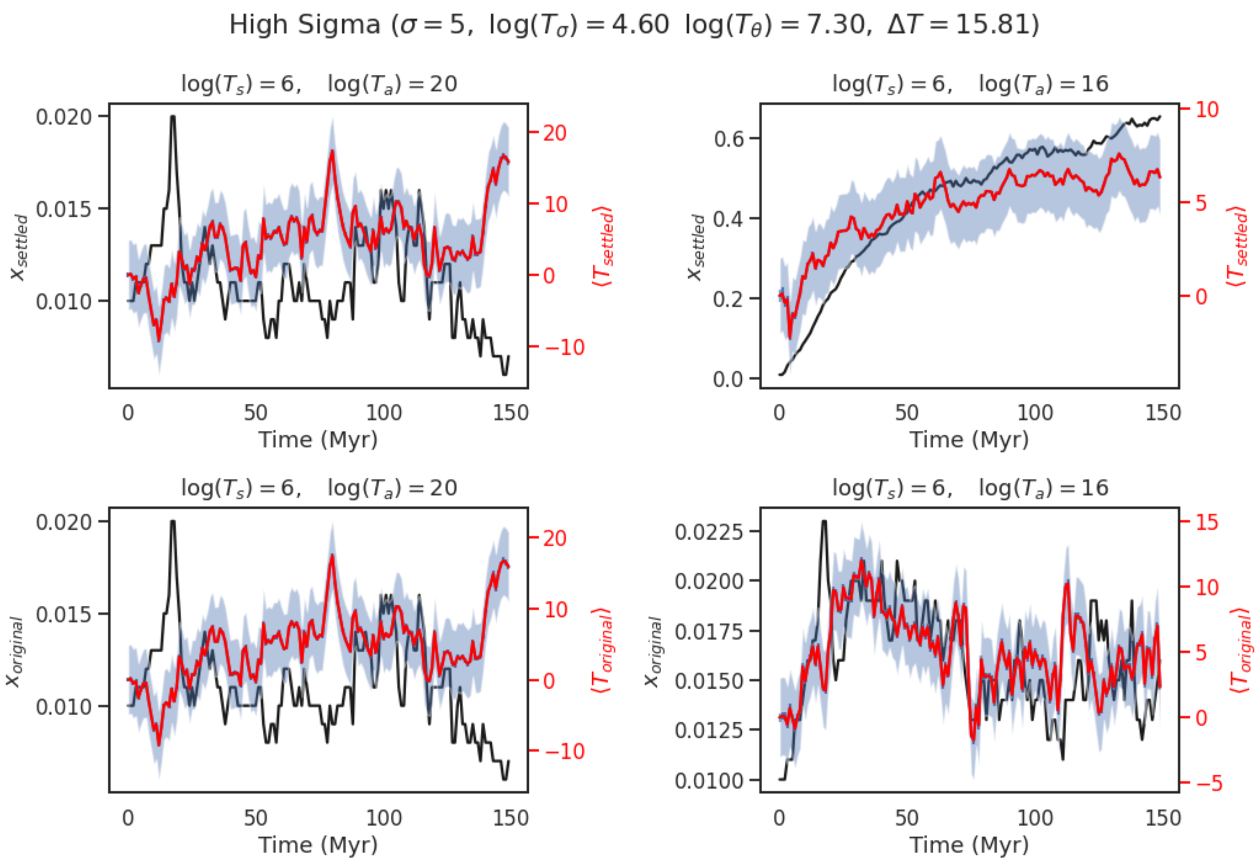


Figure 3: This figure shows the last two rows from the past two figures, zoomed in to show the behavior of x. Thus, these models constitute high P_{death}, so civilizations have a high probability of dying out every timestep.

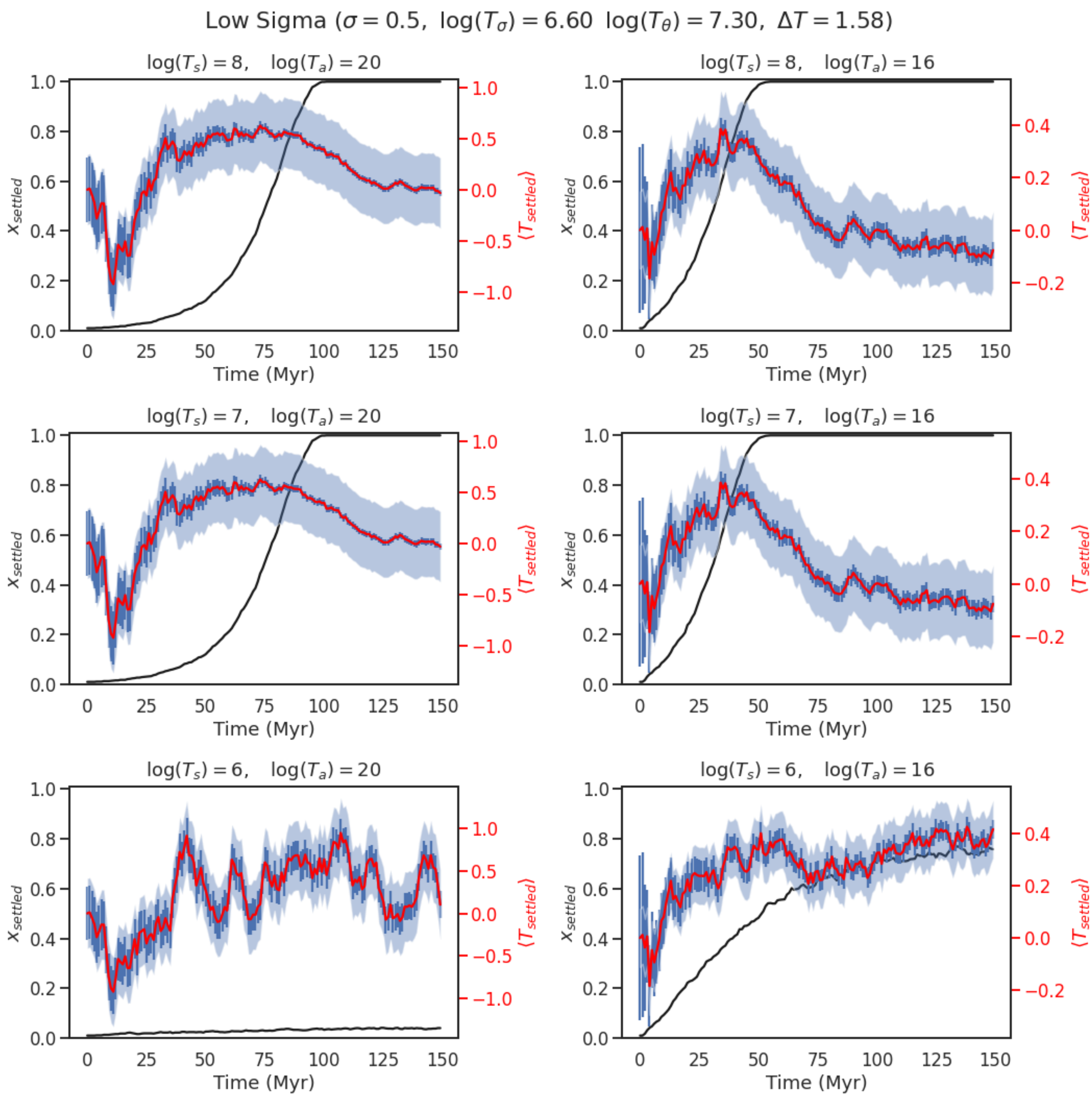


Figure 4: The black line shows x, the settled fraction, defined as the number of settled systems over the total number of systems. The black line correlates with the black tick marks on the left y-axis. The red line shows the average technological value of the settled systems, and correlates with the red tick marks on the right y-axis. Poisson error bars ($\sqrt{N}/N$) have also been included in the line plots of average technology.

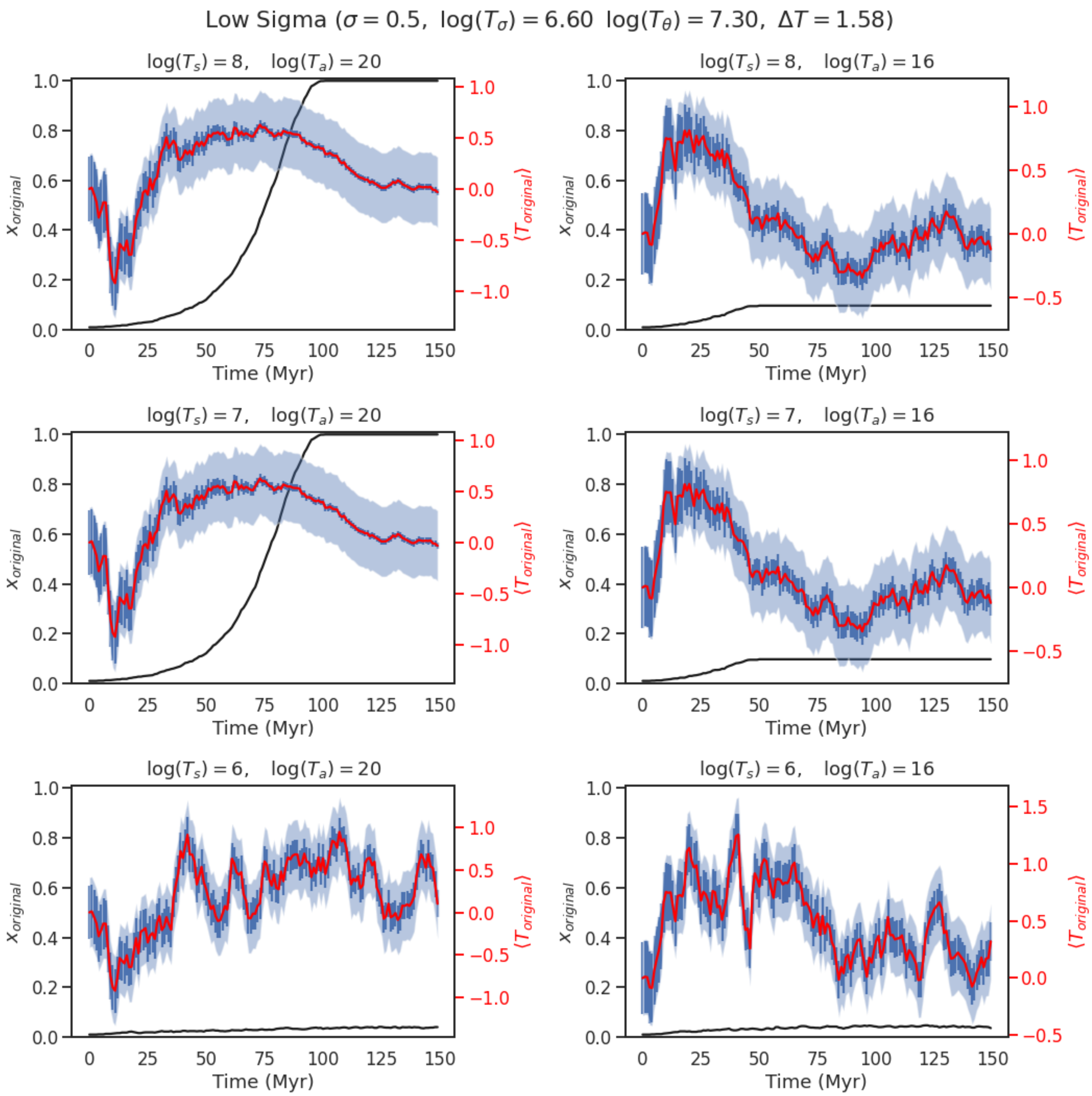


Figure 5: This figure is different from the previous in that we are only considering systems that have been settled by the original 10 systems. So x is the number of settled systems that have the same seed as the original 10 seeds divided by the total number of systems. While $\langle T \rangle$ now calculates the average value of technology among those systems that have the same seed as the original 10 systems (including the original 10). Poisson error bars ($\sqrt{N}/N$) have also been included in the line plots of average technology.

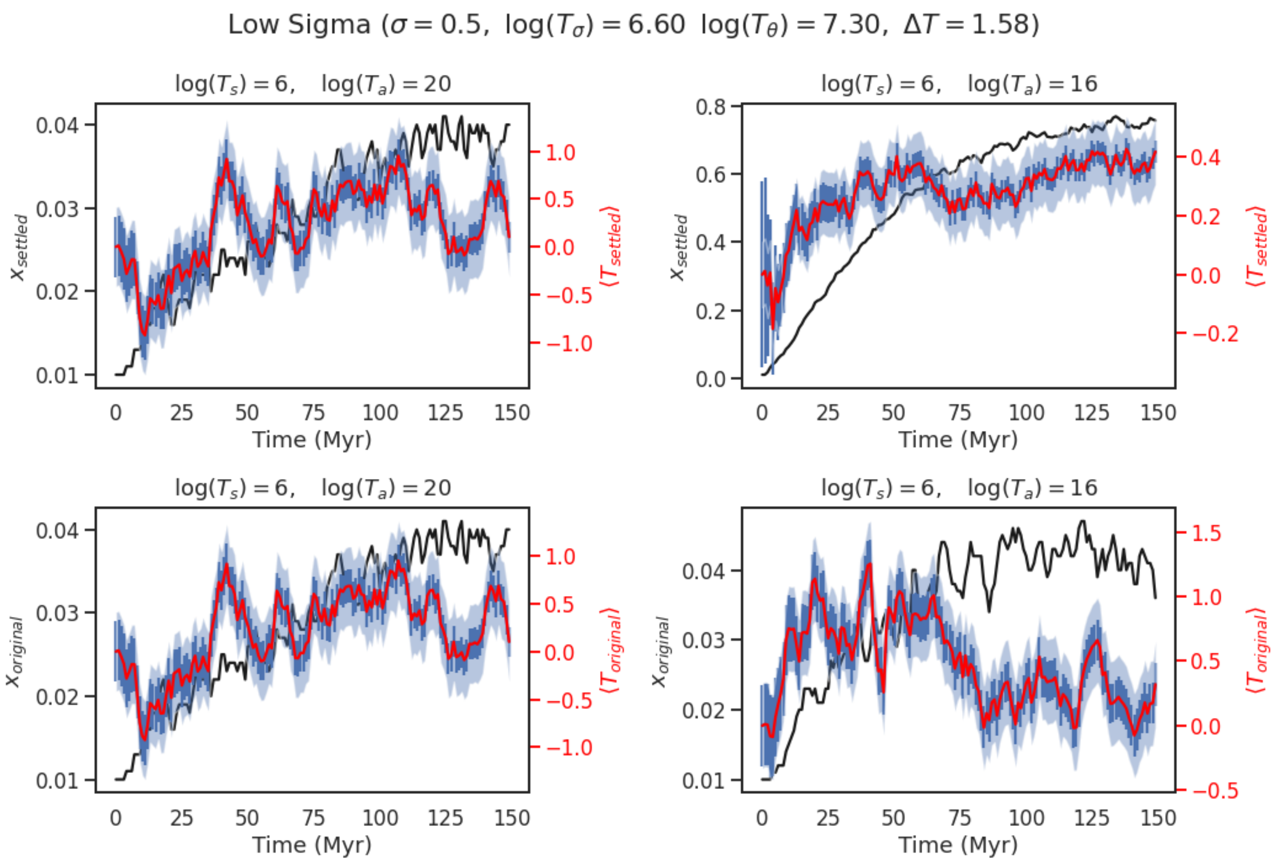


Figure 6: This figure shows the last two rows from the past two figures, zoomed in to show the behavior of x. Thus, these models constitute high P_{death}, so civilizations have a high probability of dying out every timestep.

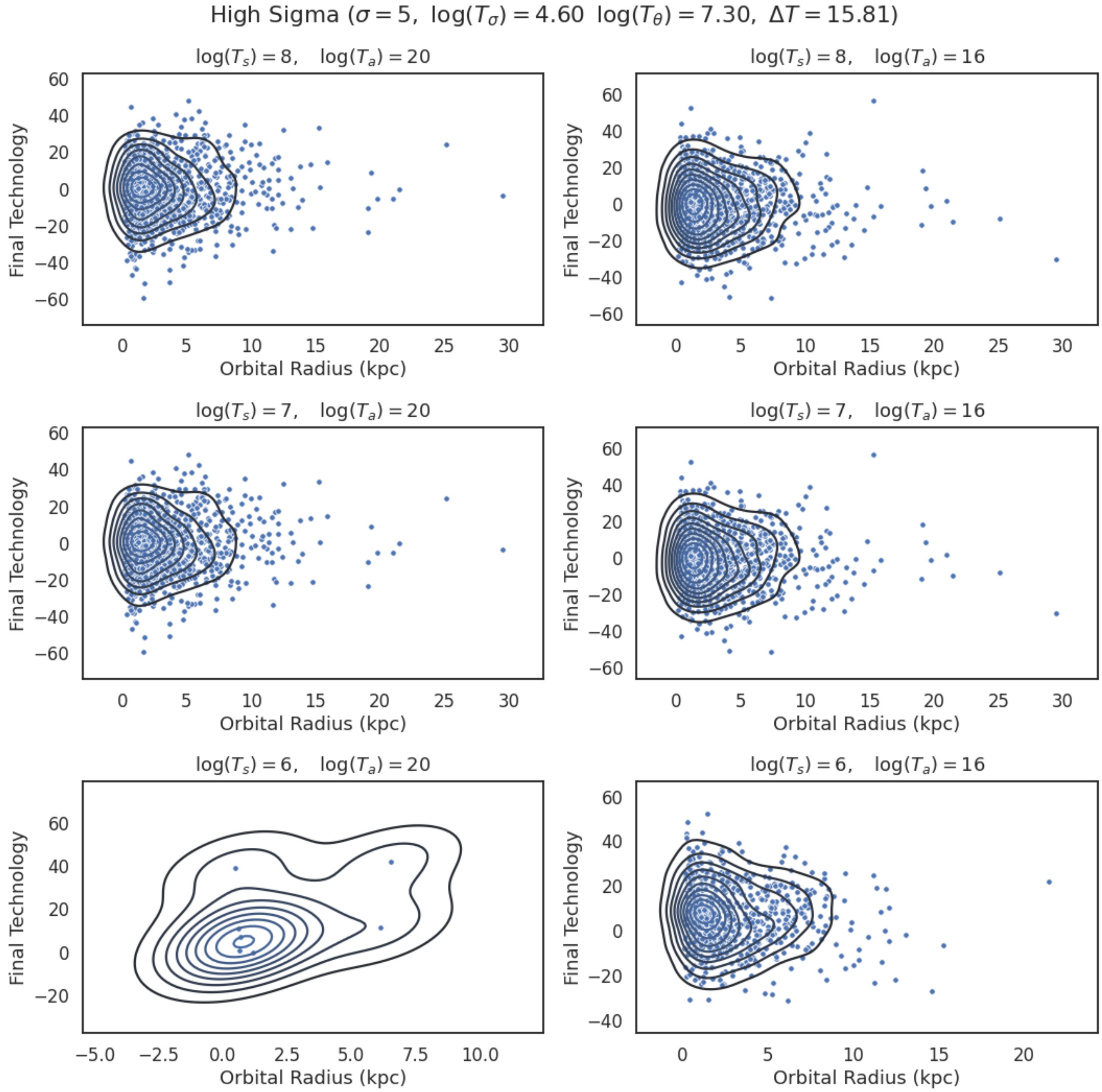


Figure 7: This shows a scatter plot capturing the final configuration of our models (taken at time $t = 100\, Myr$). All settled systems are shown, with their orbital radii on the x-axis and their technological abilities on the $y-axis$. The civilizations of the left column tend to have higher technological abilities as orbitall radii increases.

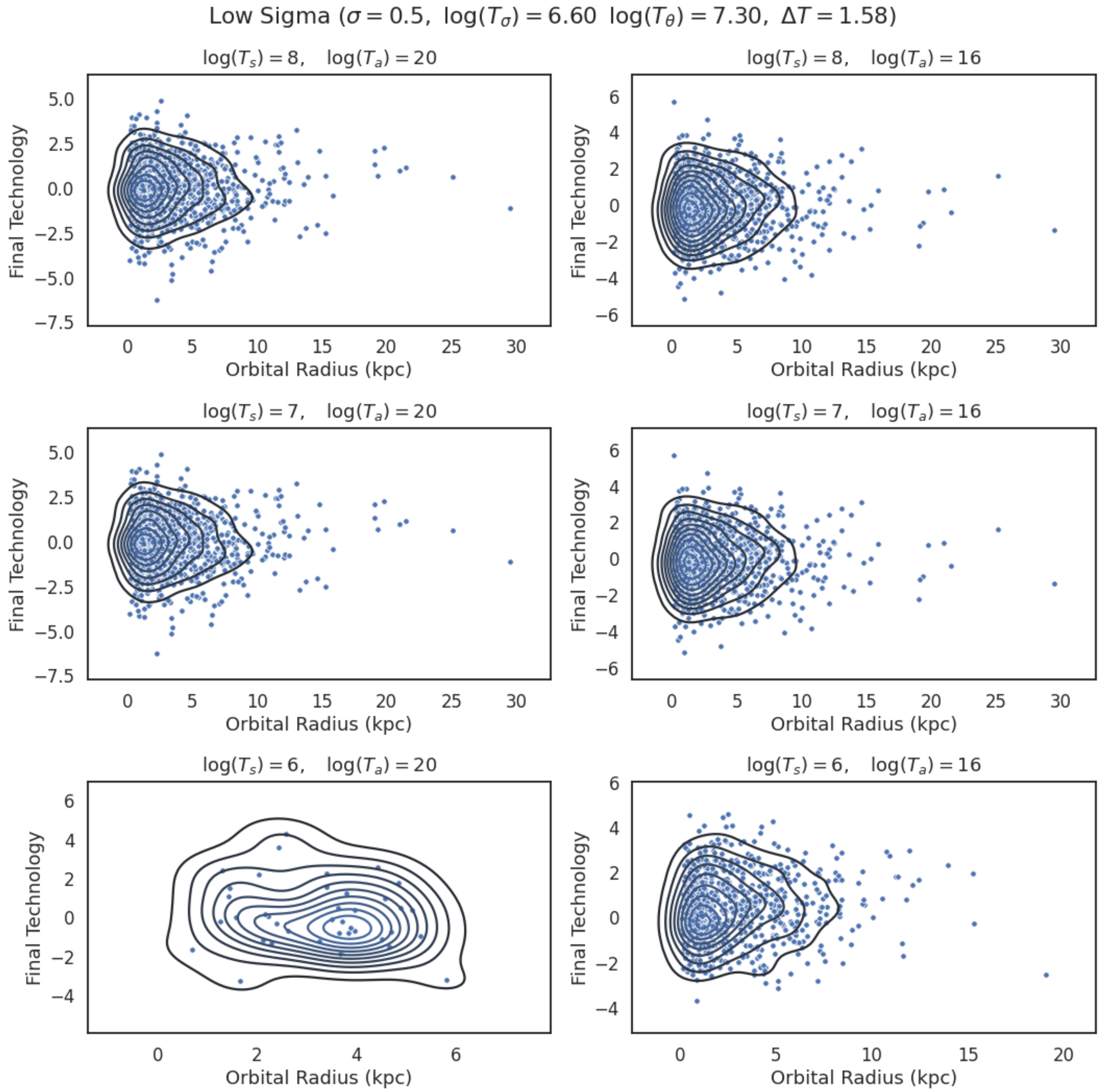


Figure 8: This shows a scatter plot capturing the final configuration of our models (taken at time $t = 100\,Myr$). All settled systems are shown, with their orbital radii on the x-axis and their technological abilities on the $y-axis$. The civilizations of the left column tend to have higher technological abilities as orbitall radii increases.

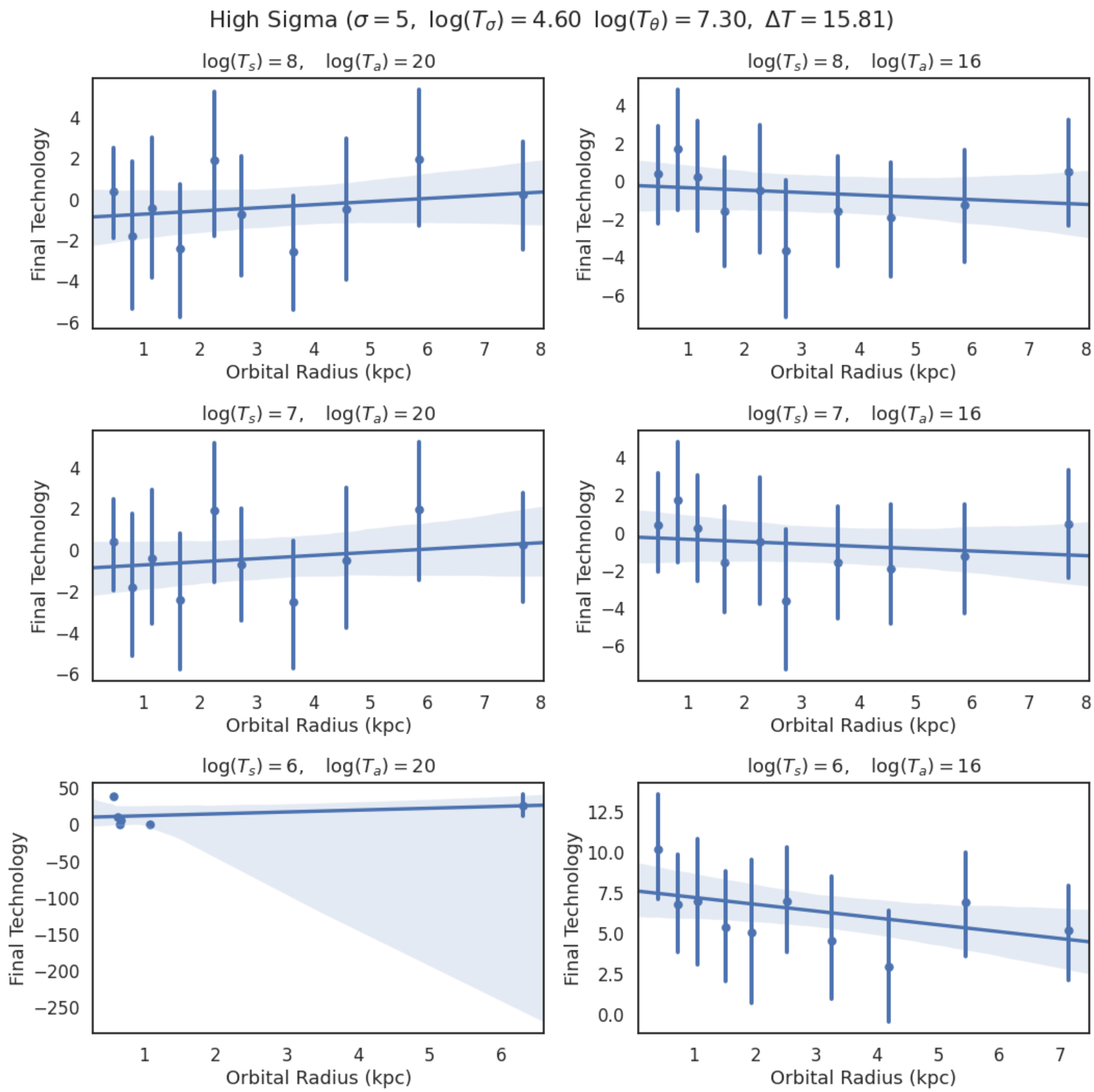


Figure 9: This shows the same snapshot as the previous scatter plot shows. In this plot, a linear regression was calculated using the final tech values and orbital radii of all the settled systems. This linear regression is shown as the blue line, surrounded by a shaded 95% confidence interval. Furthermore, to better visualize this relationship, the scatter points were split into 10 discrete bins, the dot shown tells the mean value of the points in that bin, while the vertical line associated with the point shows the 95% confidence interval for points in that bin.

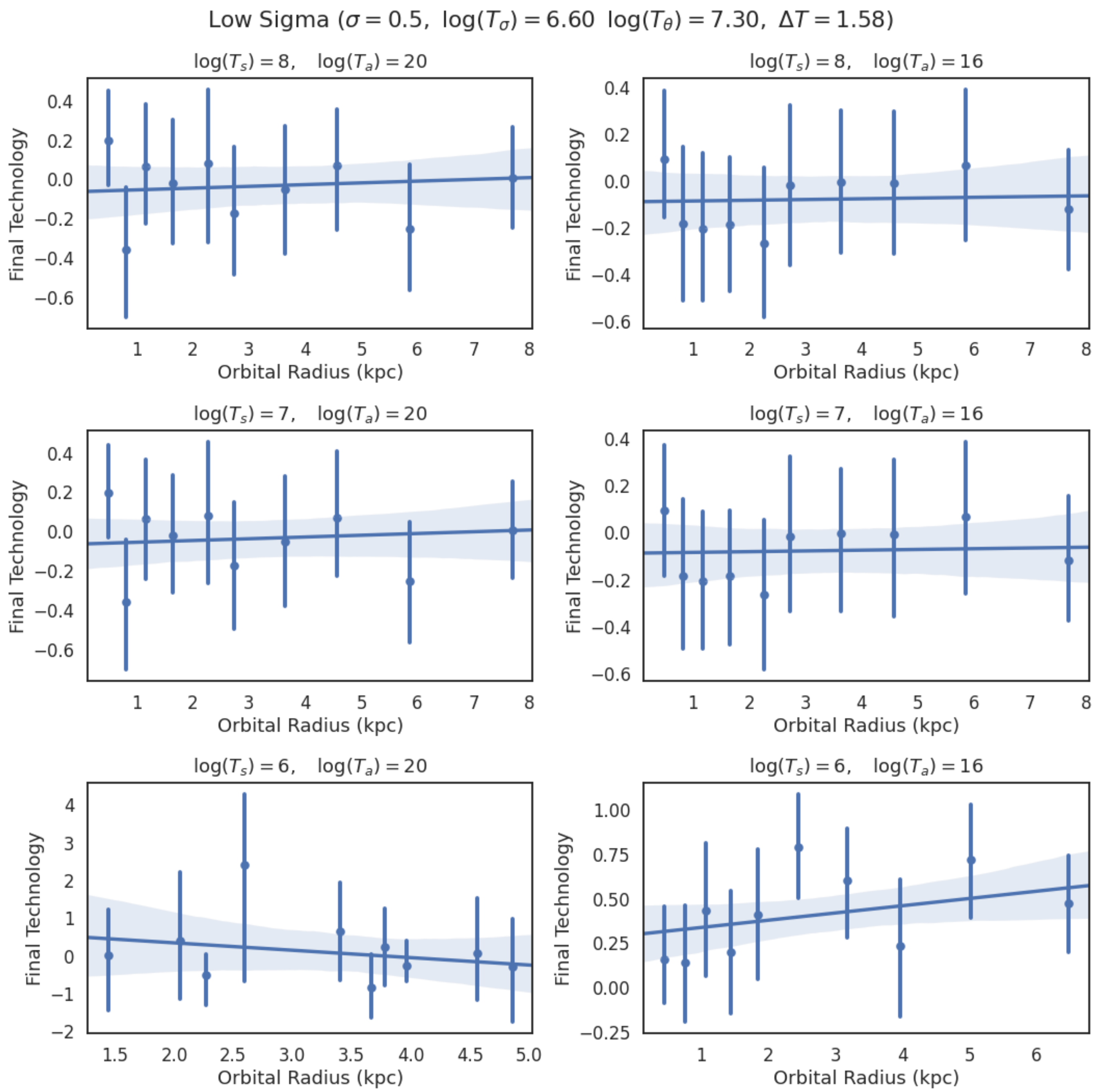


Figure 10: This shows the same snapshot as the previous scatter plot shows. In this plot, a linear regression was calculated using the final tech values and orbital radii of all the settled systems. This linear regression is shown as the blue line, surrounded by a shaded 95% confidence interval. Furthermore, to better visualize this relationship, the scatter points were split into 10 discrete bins, the dot shown tells the mean value of the points in that bin, while the vertical line associated with the point shows the 95% confidence interval for points in that bin.

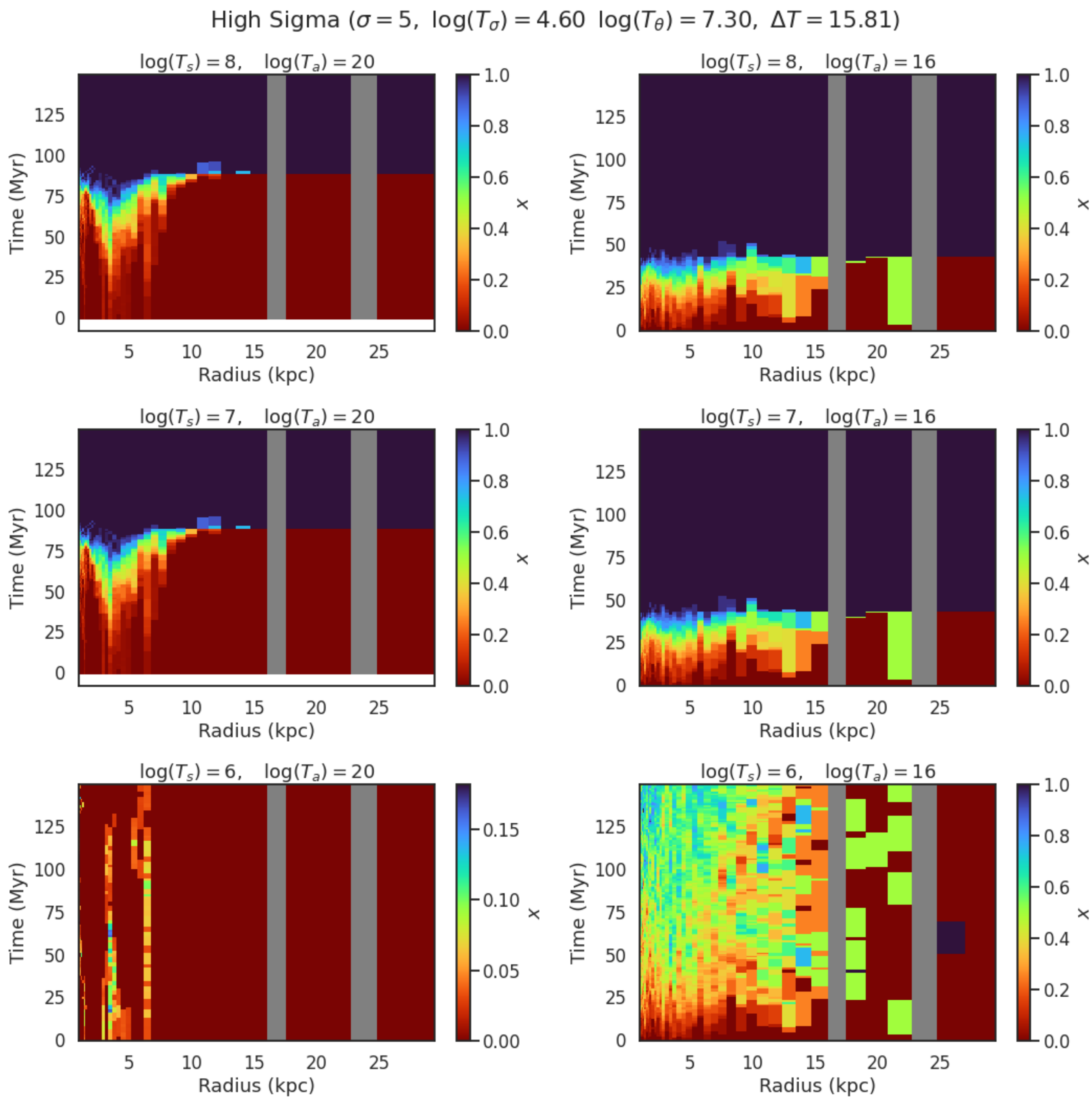


Figure 11: Shown above is a pseudocolor plot showing the temporal and spatial evolution of the model, colored by the settled fraction x (defined as the number of settled systems over the total number of systems). The systems are binned logarithmically by orbital radius into 40 discrete bins. The value of x is then calculated within each individual radius/time bin. The dark red spots on the lower right side of the plot shows areas that have only unsettled systems, so that x is still zero.

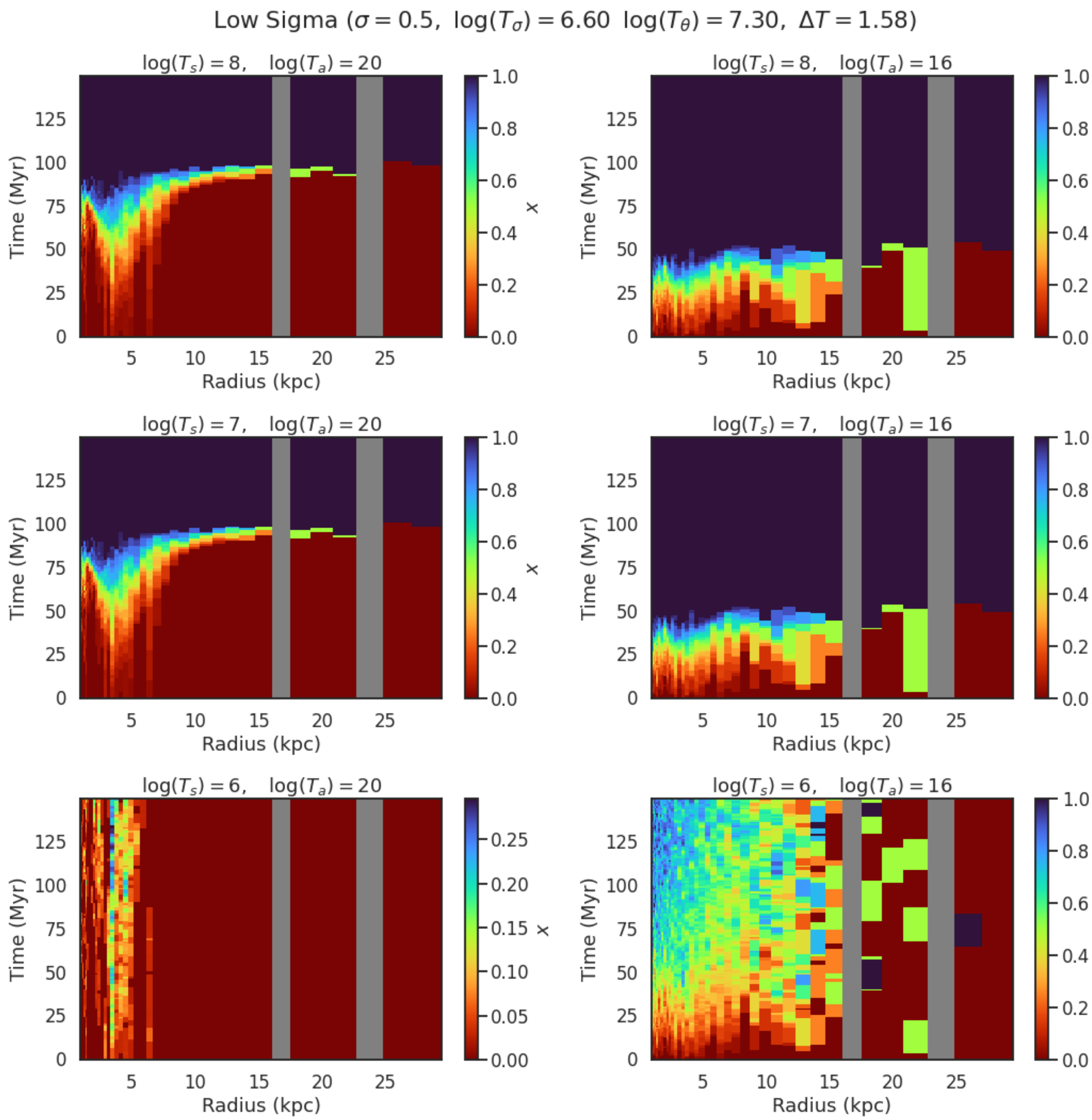


Figure 12: Shown above is a pseudocolor plot showing the temporal and spatial evolution of the model, colored by the settled fraction x (defined as the number of settled systems over the total number of systems). The systems are binned logarithmically by orbital radius into 40 discrete bins. The value of x is then calculated within each individual radius/time bin. The dark red spots on the lower right side of the plot shows areas that have only unsettled systems, so that x is still zero.

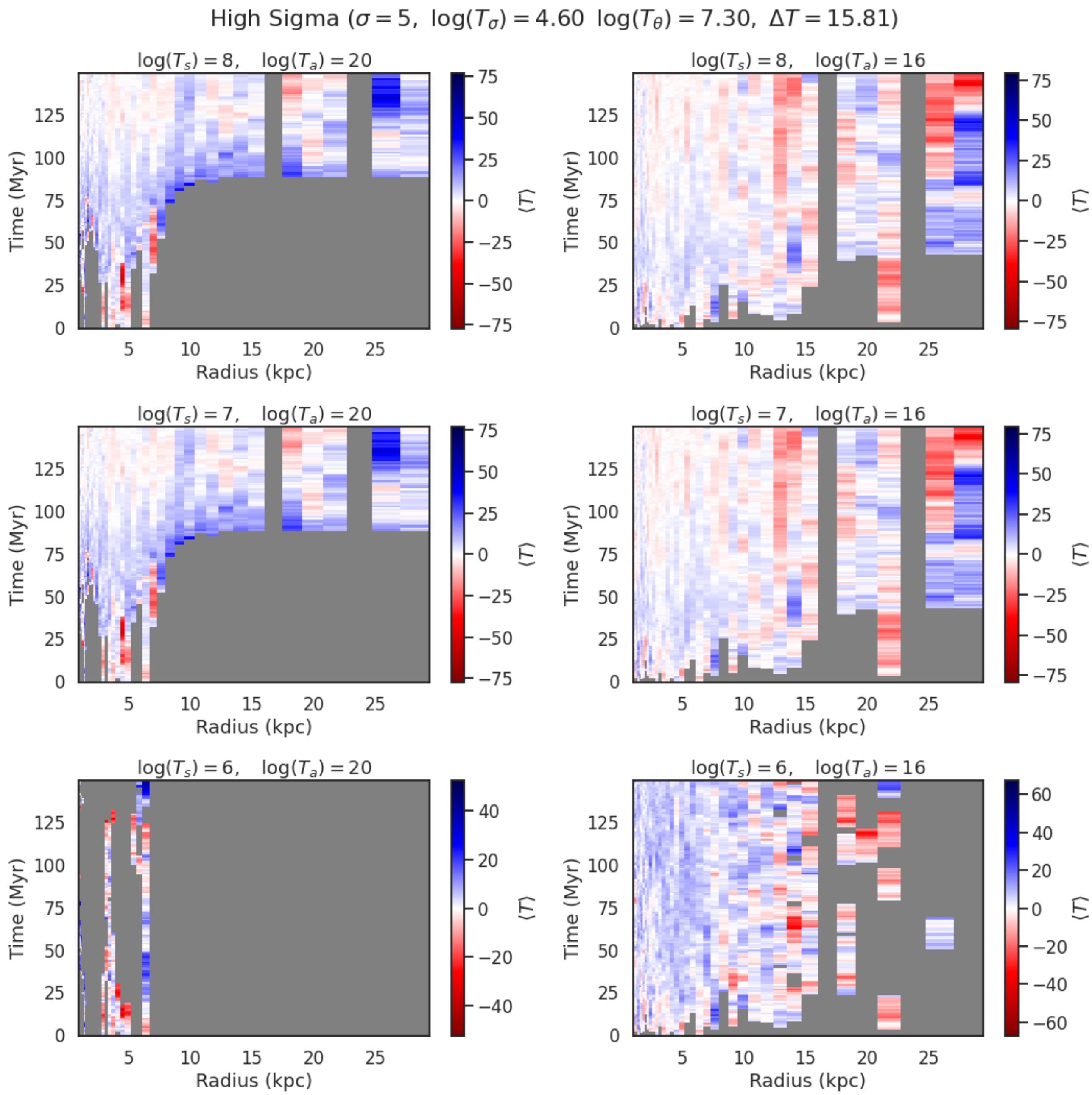


Figure 13: Shown above is a pseudocolor plot showing the temporal and spatial evolution of the model, colored by the average technological abilities of the settled systems. The systems are binned logarithmically by orbital radius into 40 discrete bins. The value of $\langle T \rangle$ is then calculated within each individual radius/time bin. The grey spots in the lower right show bins that do not have any settled systems, so that $\langle T \rangle$ is undefined there. The white spots show areas that have technological abilities close to zero, while the red and blue spots show areas of decreased and increased technological values, respectively.

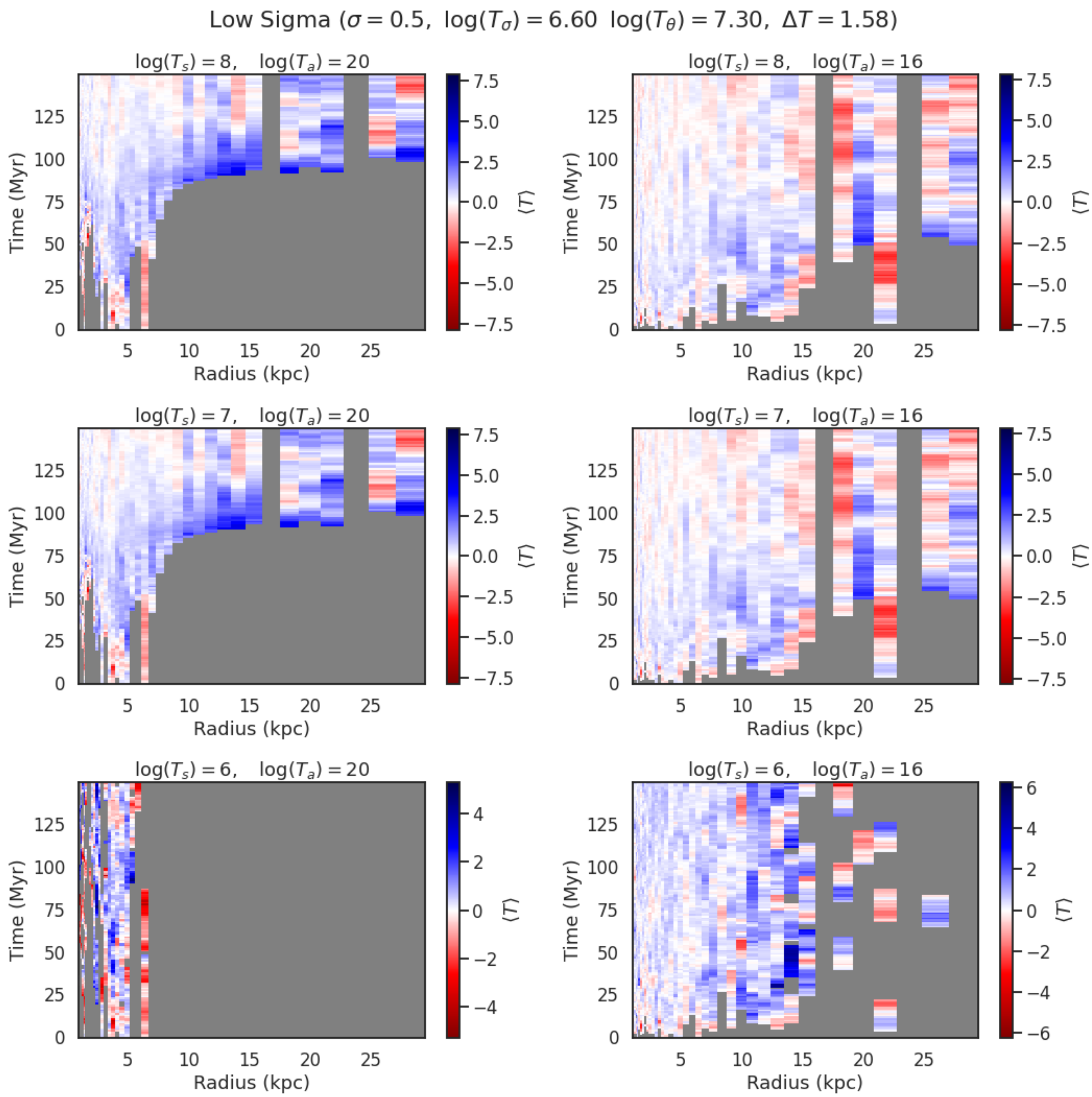


Figure 14: Shown above is a pseudocolor plot showing the temporal and spatial evolution of the model, colored by the average technological abilities of the systems. The systems are binned logarithmically by orbital radius into 40 discrete bins. The value of $\langle T \rangle$ is then calculated within each individual radius/time bin. The white spots show areas that have technological abilities close to zero, while the red and blue spots show areas of decreased and increased technological values, respectively.