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ANTHROPOCENE-GENERALIZED

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ABSTRACT

We present a framework for studying generic behaviors possible in the interaction between a resource harvesting technological civilization (an *exo-civilization*) and the planetary environment in which it evolves. Using methods from dynamical systems theory, we introduce and analyze a suite of simple equations modeling a population which consumes resources for the purpose of running a technological civilization and the feedback those resources drive on the state of the host planet. The feedbacks can drive the planet away from the initial state the civilization originated in and into domains that are detrimental to its sustainability. Our models conceptualize the problem primarily in terms of feedbacks from the resource use onto the coupled planetary systems. In addition we also model the population growth advantages gained via the harvesting of these resources. We present three models of increasing complexity: (1) Civilization-planetary interaction with a single resource; (2) Civilization-planetary interaction with two resources each of which has a different level of planetary system feedback; (3) Civilization-planetary interaction with two resources and nonlinear planetary feedback (i.e. runaways). All three models show distinct classes of exo-civilization trajectories. We find smooth entries into long-term, "sustainable" steady-states. We also find population booms followed by various levels of "die-off". Finally we also observe rapid "collapse" trajectories for which the population approaches $n = 0$. Our results are part of a program for developing an "Astrobiology of the Anthropocene" in which questions of sustainability, centered on the coupled Earth-system can be seen in their proper astronomical/planetary context. We conclude by discussing the implications of our results for both the coupled Earth system and for the consideration of exo-civilizations across cosmic history.

Keywords: Anthropocene; Astrobiology; Civilization; Dynamical System Theory; Exoplanets; Population dynamics

1. INTRODUCTION

In Frank et al. (2018) we ...

2. THE COUPLED MODEL

2.1. *Variables, Constants and Units*

$\boxed{N = \textit{Global Population } (\times 10^6 \textit{ ppl})}$

- N_0 = Initial Global Population
- N_{max} = Carrying Capacity
- A_0/B_0 = Initial Per-Capita Birth/Death Rates
- A/B = Current Per-Capita Birth/Death Rates

$\boxed{P = \textit{Global Carbon Dioxide Partial Pressures}}$

- P_0 = Initial Carbon Dioxide Partial Pressures, (Note: $x\ ppm * \left(\frac{1Bar}{10^6 ppm}\right) = y\ bar$)
- ΔP = A proportionality factor between the birth rate and changes in pCO_2. (higher values correspond to less technologically efficient civilizations, ie: must burn more fossil fuels in order to increase the birth rate)
- C = Annual Per-Capita Carbon Footprint

$\boxed{T = \textit{Average Global Temperature}}$

- T_0 = Equilibrium (initial) Temperature, calculated with the energy balance model
- ΔT = Temperature Range in which humans can survive (higher values correspond to lower fragility)
- $D = \frac{dT}{dP}$ = Temperature-CO2 climate sensitivity

2.2. *Outline of Coupled Model*

First, let the energy balance model reach an equilibrium between incoming and outgoing radiation, this gives us the equilibrium temperature. The model continues by setting the initial temperature to this equilibrium value, as well as setting the birth and death rates to their initial values. The main loop now begins, where each loop represents one year.(Note: made population have a minimum of 1 million people, to avoid values of 10^{-100})

i) Call: $\frac{dT}{dt} = EBM(P) = \frac{\psi(1-A) - I + \nabla\cdot(\kappa\nabla T)}{C_v}$

ii) $$A = A_0\left[1 + \frac{P - P_0}{\Delta P}\right]$$

iii) $$B = B_0\left[1 + \left(\frac{T - T_0}{\Delta T}\right)^2\right]$$

iv) Call: $\dfrac{dN}{dt} = min(AN,\ B_0 N_{max}) - BN$

- If $dNdt > 0$: set variable peakTime = currentTime
- If $dNdt < 0$: set variable peaked = True

v) Call: $\dfrac{dP}{dt} = CN$

a) If (peaked=True) and (currentTime - peakTime ≥ 500), then end program

b) Else, go back to the first step.

(Note: Ran model for 20 generations after the population has peaked, where a generation is defined as $t_{gen} \approx t_{growth} = \frac{1}{\alpha_{birth,0}} = 25\ years$)

2.3. *Example: Modeling Earth* *($t_0 = 1820,\ P_0 = 284,\ N_0 = 1,129$)*

- $N_{max} = 10$ billion people
- $A_0 = 0.04\ yr^{-1}$
- $B_0 = 0.036\ yr^{-1}$
- $\Delta T = 5K$
- $\Delta P = 200\ ppm$
- $C = 0.000275\ \frac{ppm}{10^6 ppl*yr}$

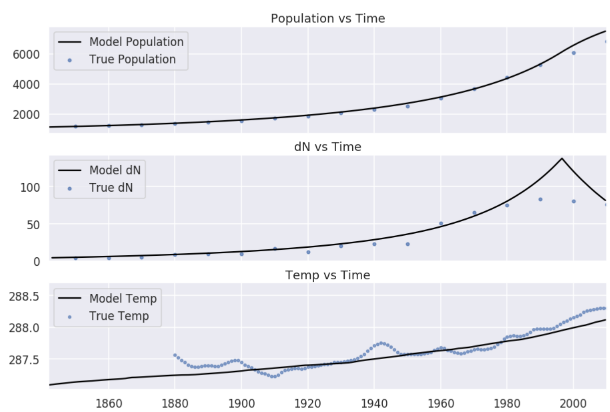


Figure 1. : Model Output (solid black line) vs Real Global Data (dotted line)

3. EXPERIMENT #1: CONSTANT COMPOSITION ($P_0 = 284\ PPM = 2.84 \times 10^{-4}\ BAR$)

3.1. *Habitable Zone*

In this experiment, we define the habitable zone by the range of distances that will result in temperatures above freezing and below boiling.

$$273.15\ K < T_{habitable} < 373.15\ K \qquad (1)$$

$$\boxed{0.94\ AU < a_{habitable} < 1.02\ AU} \qquad (2)$$

3.2. *Linear Regressions of Temp vs pCO_2*

For 5 different distances, I first ran the model, uncoupled, with initial $pCO2 = 284ppm$. The resultant equilibrium temperature for this $pCO2$ is saved as a variable called initialTemp. I then **decremented** the initial pCO_2 by $5ppm$ and re-ran the model until the absolute value of the difference between the equilibrium temperature and the initial temperature was greater than 2. I then set the initial $pCO2$ back to $284ppm$, and continued by **incrementing** the initial pCO_2 by $5ppm$ and re-running the model until the absolute value of the difference between the equilibrium temperature and the initial temperature was greater than 2. At this point, I changed distances and repeated the same process. After all distances have been looped, I ran a linear regression (using scipy.stats.linregress) for the data from each distance to find the relationship between changes in pCO2 and changes in global temperature.

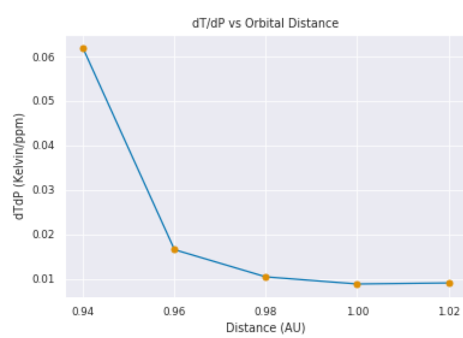
$$T = 6.178 * 10^{-2}\left(\frac{P}{ppm}\right) + 322 \qquad \textbf{\textit{(0.94 AU)}}$$

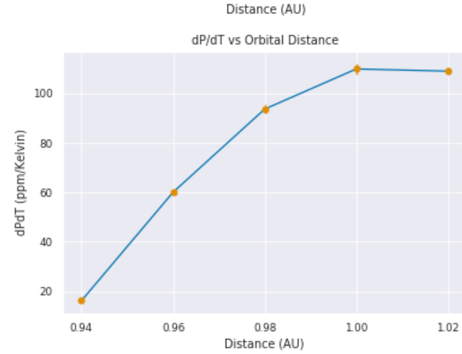
$$T = 1.655 * 10^{-2}\left(\frac{P}{ppm}\right) + 309 \qquad \textbf{\textit{(0.96 AU)}}$$

$$T = 1.044 * 10^{-2}\left(\frac{P}{ppm}\right) + 296 \qquad \textbf{\textit{(0.98 AU)}}$$

$$T = 8.838 * 10^{-3}\left(\frac{P}{ppm}\right) + 284 \qquad \textbf{\textit{(1.0 AU)}}$$

$$T = 9.067 * 10^{-3}\left(\frac{P}{ppm}\right) + 273 \qquad \textbf{\textit{(1.02 AU)}}$$





4. EXPERIMENT #2: CONSTANT TEMPERATURE ($T_0 \approx 287.09K = 57.09\ ^OF$)

4.1. *Habitable Zone*

In this experiment, we define the habitable zone by the range of distances that have temperatures approximately equal to the equilibrium temperature for our current planet, $287.09K$, with corresponding $pCO2's$ greater than 10ppm and less than 10^5ppm. (Which corresponds to an atmosphere that is composed of 10% CO2). The way I found this zone was by first setting the initial pco2 to $10ppm$ and the distance to $1AU$, then continually decrementing the distance by $0.001AU$ until the temperature was greater or equal to 287.09, the distance for which this occurs is the minimum distance. I then set the initial pco2 to 10^5ppm and the distance to $1AU$, and continually incremented the distance by $0.001AU$ until the temperature was less than or equal to 287.09, the distance for which this occurs is the maximum distance. (Note: The minimum allowed value for pCO2 in our EBM is 10ppm)

$$10\ ppm < pCO2_{habitable} < 10^5\ ppm \tag{3}$$

$$\boxed{0.975\ AU < a_{habitable} < 1.105\ AU} \tag{4}$$

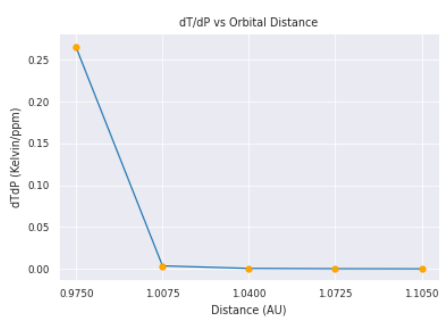
$$T = 2.649 * 10^{-1}\left(\frac{P}{ppm}\right) + 285 \quad (0.975\ AU:\ 21\ ppm)$$

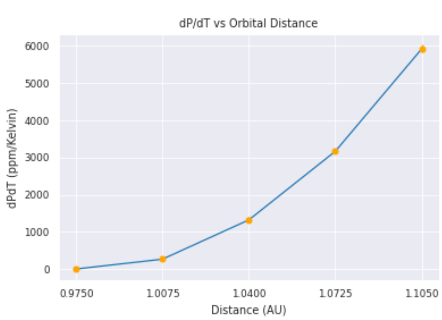
$$T = 3.689 * 10^{-3}\left(\frac{P}{ppm}\right) + 283 \quad (1.0075\ AU:\ 1,055\ ppm)$$

$$T = 6.846 * 10^{-4}\left(\frac{P}{ppm}\right) + 278 \quad (1.04\ AU:\ 13,150\ ppm)$$

$$T = 3.130 * 10^{-4}\left(\frac{P}{ppm}\right) + 274 \quad (1.0725\ AU:\ 42,286\ ppm)$$

$$T = 1.678 * 10^{-4}\left(\frac{P}{ppm}\right) + 271 \quad (1.105\ AU:\ 93,567\ ppm)$$





5. DIMENSIONLESS PARAMETER (γ)

First, we define the timescales:

$$t_{growth} \equiv \frac{1}{A_0} \tag{5}$$

$$t_{climate} \equiv \frac{\Delta T}{CN_{max}D} \tag{6}$$

Then we can write our dimensionless parameter like:

$$\boxed{\begin{aligned} \gamma \equiv \frac{t_{growth}}{t_{climate}} &= \frac{DCN_{max}}{A_0 \Delta T} && (7)\\ &= \frac{\textit{Timescale for Population Growth}}{\textit{Timescale for Climate to Change}} && (8)\end{aligned}}$$

- $\gamma << 1 \implies$ Climate will change on timescales much longer than the average generation. Corresponds to a civilization having a low risk for an Anthropocene.
- $\gamma = 1 \implies$ Climate will change within one generation.
- $\gamma >> 1 \implies$ Climate will change on timescales much shorter than the average generation. Corresponds to a civilization having a high risk for an Anthropocene.

$$\boxed{\begin{aligned} N_A \equiv \frac{N_{max}}{\gamma} &= \frac{A_0 \Delta T}{DC} && (9)\\ &= \textit{number of people required to force} && (10)\\ &\quad \textit{the climate out of equilibrium in} && (11)\\ &\quad \textit{a generation} && (12)\\ &= \textit{"Anthropogenic Population"} && (13)\end{aligned}}$$

6. ANALYTIC MODEL

With natural birth rate A_0, natural death rate B_0, per capita emission rate C, and temperature-CO2 climate sensitivity D, birth rate temperature sensitivity ΔT, CO2 emission birth rate advantage range ΔP, and carrying capacity $N_{\max}$ we can model the population N, CO2 concentration P, and temperature T using

$$\begin{aligned} \frac{dN}{dt} &= \min\left[A_0 N\left(1 + \frac{P - P_0}{\Delta P}\right),\ B_0 N_{\max}\right] \\ &\quad - B_0 N\left[1 + \left(\frac{T - T_0}{\Delta T}\right)^2\right] \\ \frac{dP}{dt} &= CN \\ \frac{dT}{dt} &= \frac{dT}{dP}\frac{dP}{dt} = D(T)CN \end{aligned}$$

where

$$D(T) = \frac{dT}{dP} \tag{14}$$

$$\approx \frac{T - T_0}{P - P_0} \tag{15}$$

If we assume that the climate sensitivity $D(T)$ is constant, then we have

$$T = T_0 + D\left(P - P_0\right) \tag{16}$$

This allows us to reduce the system of equations to

$$\begin{aligned}\frac{dN}{dt} &= \min\left[A_0 N\left(1 + \frac{T - T_0}{D\Delta P}\right), B_0 N_{\max}\right] \\ &- B_0 N\left[1 + \left(\frac{T - T_0}{\Delta T}\right)^2\right] \\ \frac{dT}{dt} &= DCN\end{aligned}$$

Now dividing both equations by A_0, and the first equation by $N_{\max}$ and the second equation by ΔT we arrive at

$$\begin{aligned}\frac{d\eta}{d\tau} &= \min\left[\eta\left(1 + \theta\epsilon\right), \beta\right] - \beta\eta(1 + \epsilon^2) \\ \frac{d\epsilon}{d\tau} &= \gamma\eta\end{aligned}$$

where

$\eta =$	$\frac{N}{N_{\max}}$	Normalized population
$\tau =$	$A_0 t$	Normalized time
$\beta =$	$\frac{B_0}{A_0}$	Normalized natural death rate
$\epsilon =$	$\frac{T - T_0}{\Delta T}$	Normalized temperature
$\theta =$	$\frac{\Delta T}{D\Delta P}$	Normalized Birth rate acceleration
$\gamma =$	$\frac{DCN_{\max}}{A_0\Delta T}$	Normalized forcing

Note that in the absence of any means to reduce the CO2 in the atmosphere - and therefore the temperature - there is no equilibrium for the temperature except for the trivial one $\eta = 0$. Also, because of the min function, the population will peak in one of two scenarios.

6.1. *High carrying capacity ($\gamma >> 1$)*

If $\gamma >> 1$, the carrying capacity is so large, that anthropogenic forcing of the climate can push the temperature far enough out of equilibrium to stop the population growth. In this limit, the population growth rate $\eta\left(1 + \theta\epsilon\right)$ does not exceed the maximum growth rate β, and we can solve for how far the climate is pushed out of equilibrium when the population peaks.

$$\begin{aligned}\frac{d\eta}{d\tau} &= \left[(1 + \theta\epsilon) - \beta(1 + \epsilon^2)\right]\eta = 0 \\ \rightarrow \epsilon &= \frac{\theta + 2\sqrt{\beta - \beta^2 + \left(\frac{\theta}{2}\right)^2}}{2\beta}\end{aligned}$$

as well as the time scale for the population to decline after the peak.

$$\begin{aligned}\frac{d^2\eta}{d\tau^2} &= \frac{d\eta}{d\tau}\left[(1 + \theta\epsilon) - \beta(1 + \epsilon^2)\right] \\ &+ \eta\left[(1 + \theta\frac{d\epsilon}{d\tau}) - \beta(1 + 2\epsilon\frac{d\epsilon}{d\tau})\right] \\ &= \eta\left[1 - \beta - 2\gamma\eta\sqrt{\beta - \beta^2 + \left(\frac{\theta}{2}\right)^2}\right] \\ \rightarrow \tau_{\text{coll}} &= -\sqrt{\frac{\eta}{\frac{d^2\eta}{d\tau^2}}} \\ &= -\left[1 - \beta - 2\gamma\eta\sqrt{\beta - \beta^2 + \left(\frac{\theta}{2}\right)^2}\right]^{-1/2}\end{aligned}$$

Note the weak dependence on θ and β means that $\tau_{\text{coll}} \sim 1$ which means that $T_{\text{fall}} \sim \frac{1}{A_0}$ so the collapse will be on a generation time scale.

6.2. *Low carrying capacity ($\gamma << 1$)*

On the other hand, if the forcing γ is small, the population can reach the carrying capacity without impacting the climate, so we expect $\eta \rightarrow 1$ while $\epsilon \sim 0$. In this case we can calculate the time to push the climate to critical $\epsilon = 1$ using

$$\frac{d\epsilon}{d\tau} = \gamma\eta$$

And at the carrying capacity, $\eta = 1$, so the time scale for $\epsilon \rightarrow 1$ is just $\tau_{\text{coll}} = \frac{1}{\gamma}$.

Combining the two we have

$$\tau_{\text{coll}} = \max\left[1, \frac{1}{\gamma}\right] \tag{17}$$

7. EBM MODEL

For different radii and temperature, we can determine the climate sensitivity using the EBM model of Jacob... which solves the following sets of equations

8. FULLY COUPLED MODEL

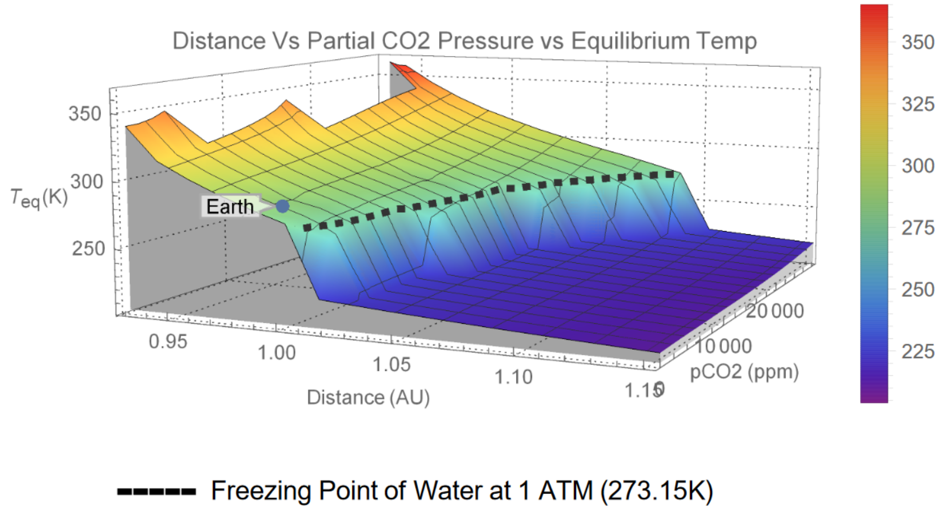
We also coupled the EBM model, with the ODE's including population feedback and then measured the free fall time after the population peaked.

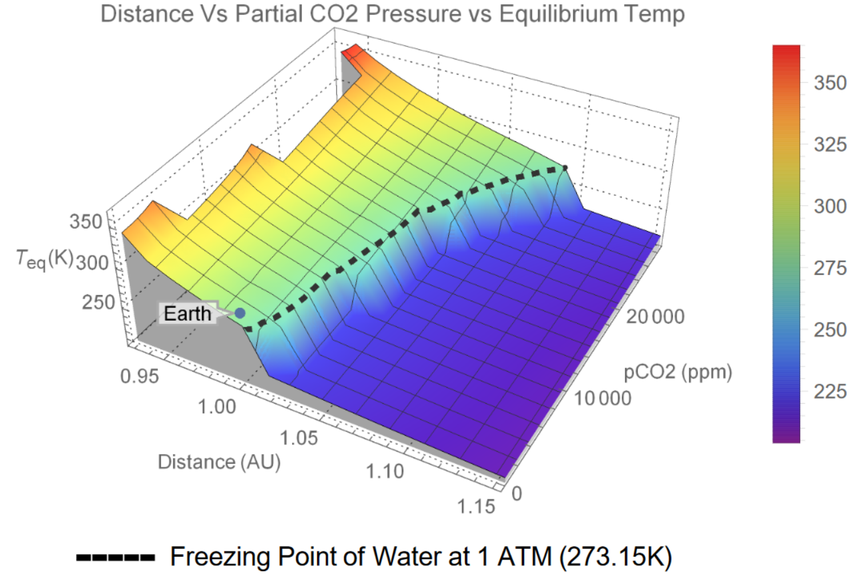
9. RESULTS

Figure ... shows the results from the coupled ODE's along with the semi-analytic predictions using the climate sensitivity.

REFERENCES

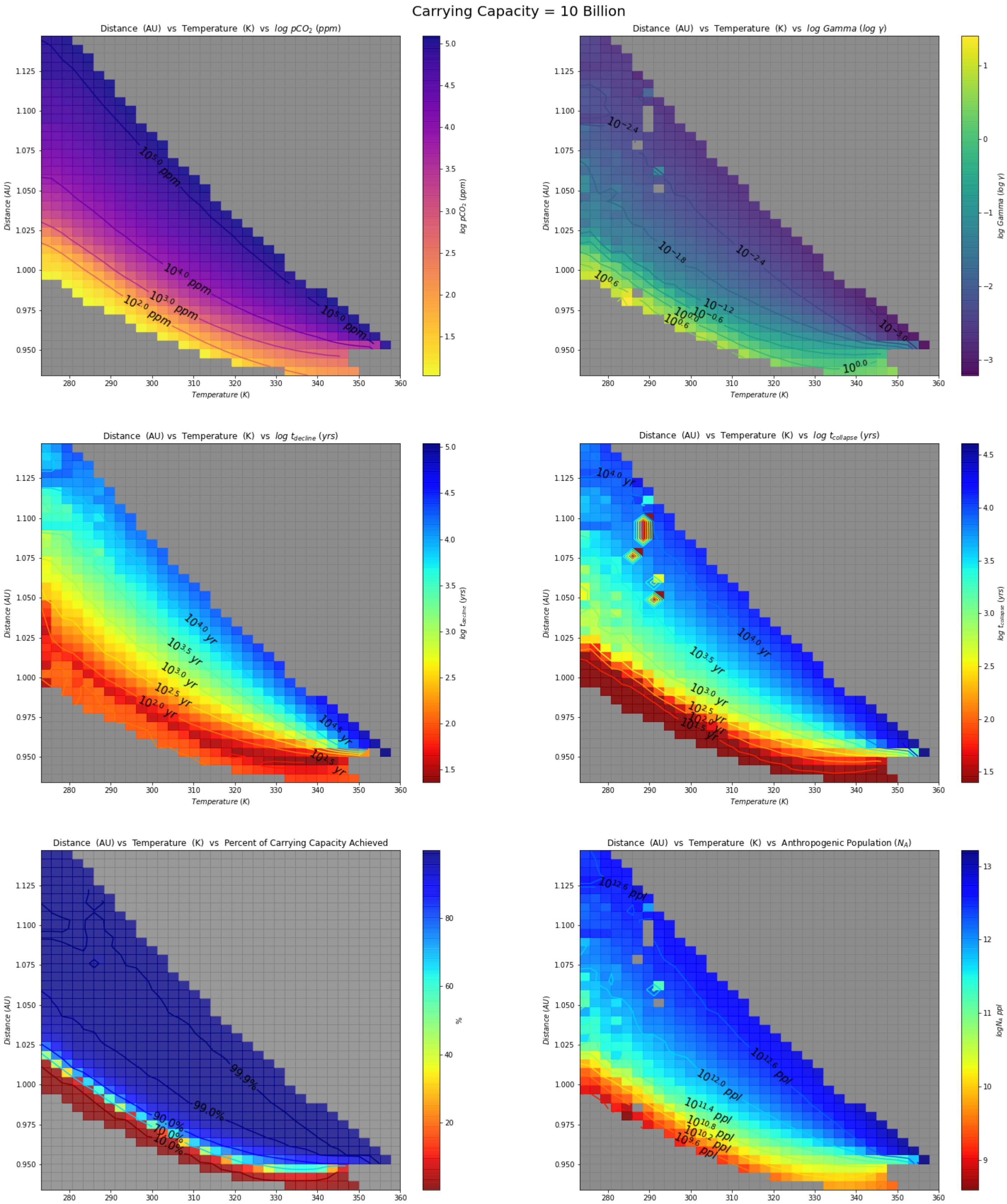
Frank A., Carroll-Nellenback J., Alberti M., Kleidon A., 2018, Astrobiology, 18, 503

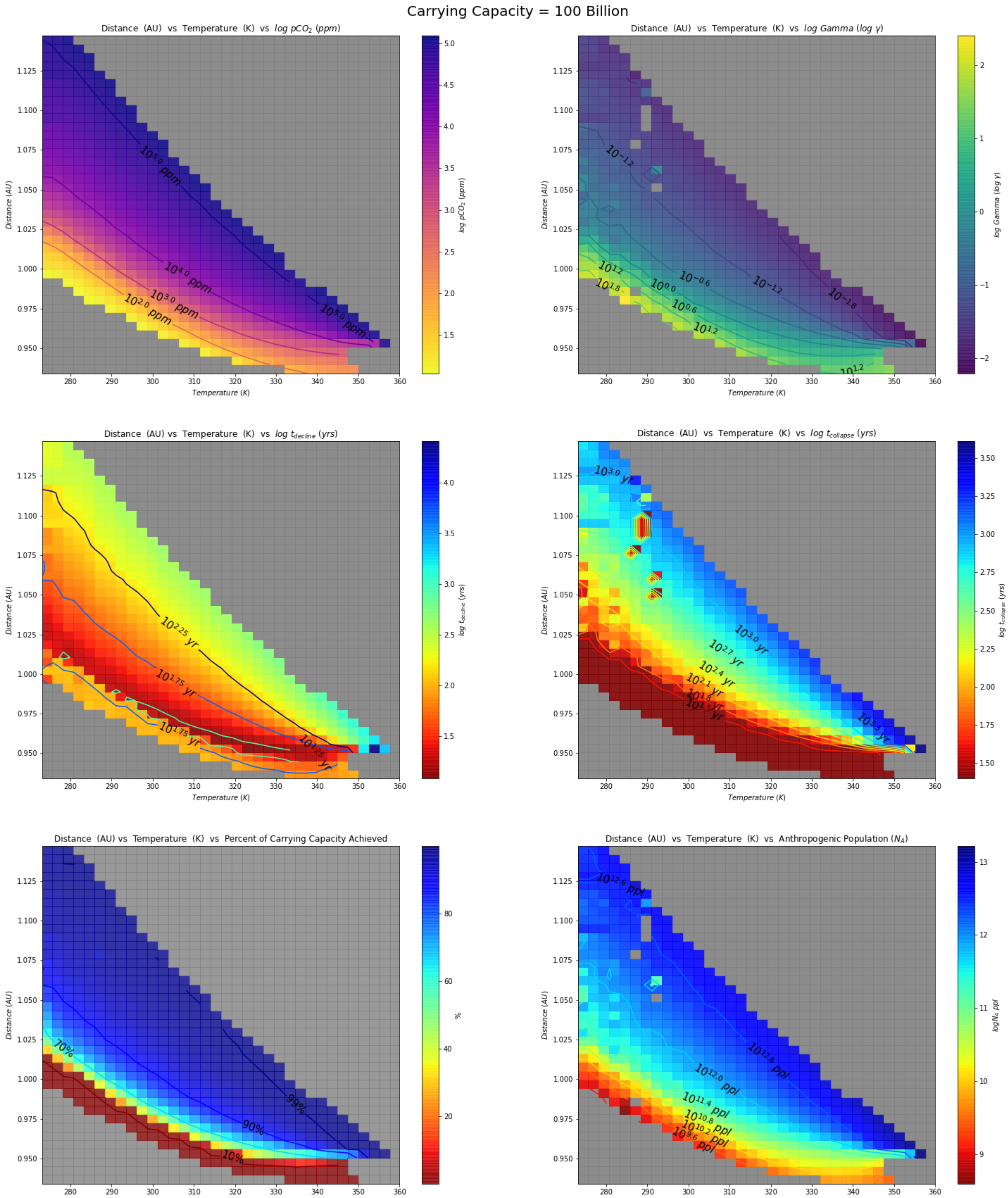


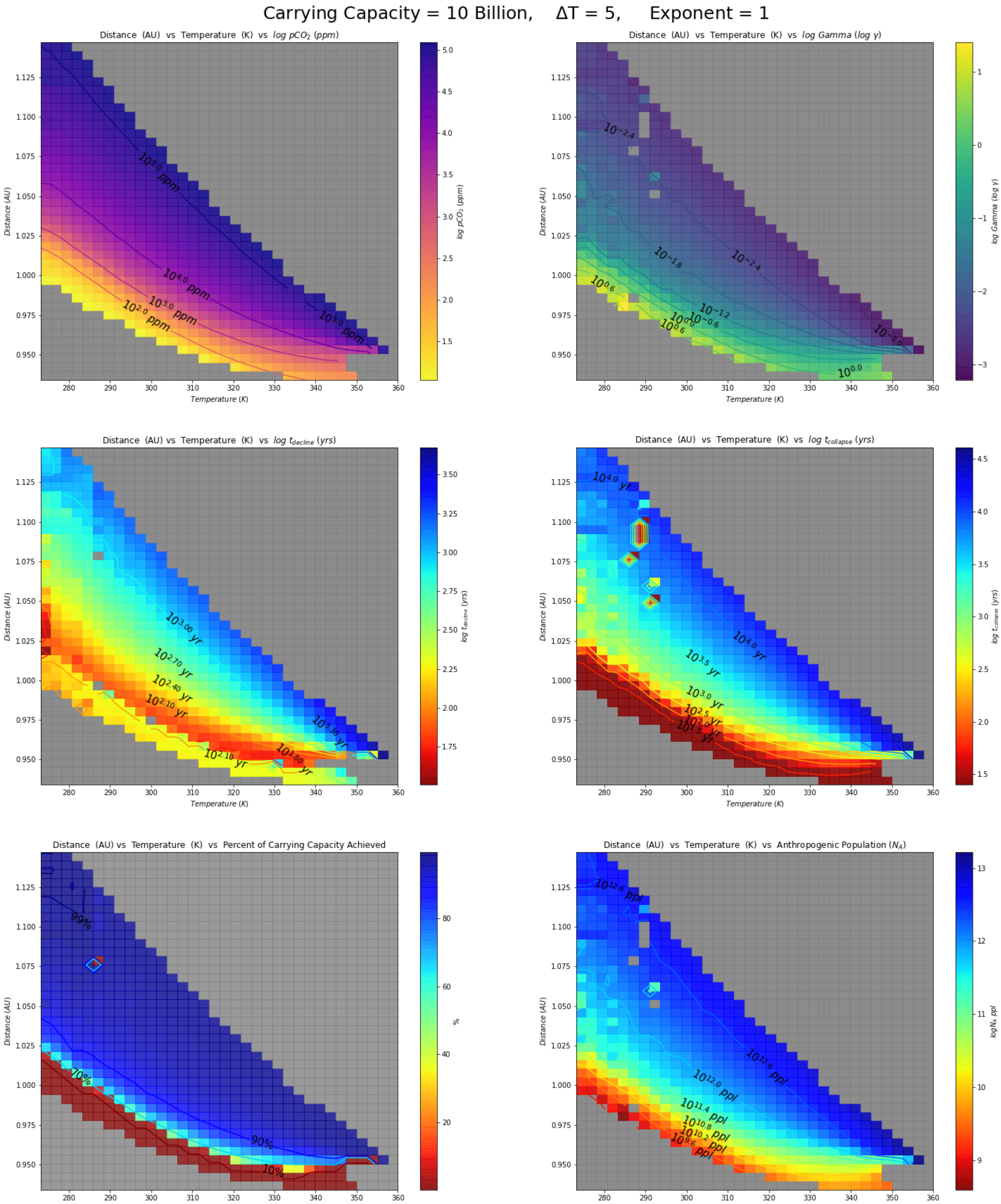


10. PLOTS OF PARAMETER SWEEPS

- For each distance and temperature (40 values of each), I found the value of $pCO2$ that would make that distance have that temperature, this then became the **initial partial CO2 pressure** for when I run the model coupled, it is denoted in the plots below
- I then ran the model at $pCO_2's$ of 1% higher and lower, and from that found an approximation for dTdP, this let me calculate our non-dimensional **gamma**
- The **collapse time**, $t_{collapse}$, is given by equation 17, and is the analytically derived time scale for the population to decline after the peak
- I then ran the model, coupled, until 25 generations had passed after the population peaked. The timescale for the population to drop by 10% of its peak is a number I called the **decline time**, $t_{decline}$
- I then found the amount of people that had died within two generations of the peak (50 years), and called this number **popDeath**
- Additionally, I calculated the ratio of the peak population reached by the civilization to the carrying capacity, and called this **Percent of Carrying Capacity Achieved**
- The last variable plotted below is N_A, which is the **Anthropogenic Population** of equation 9, which is analytically derived as a function of my approximated $dTdP$. It describes the number of people required to force the climate out of equilibrium in a single generation (25 years)

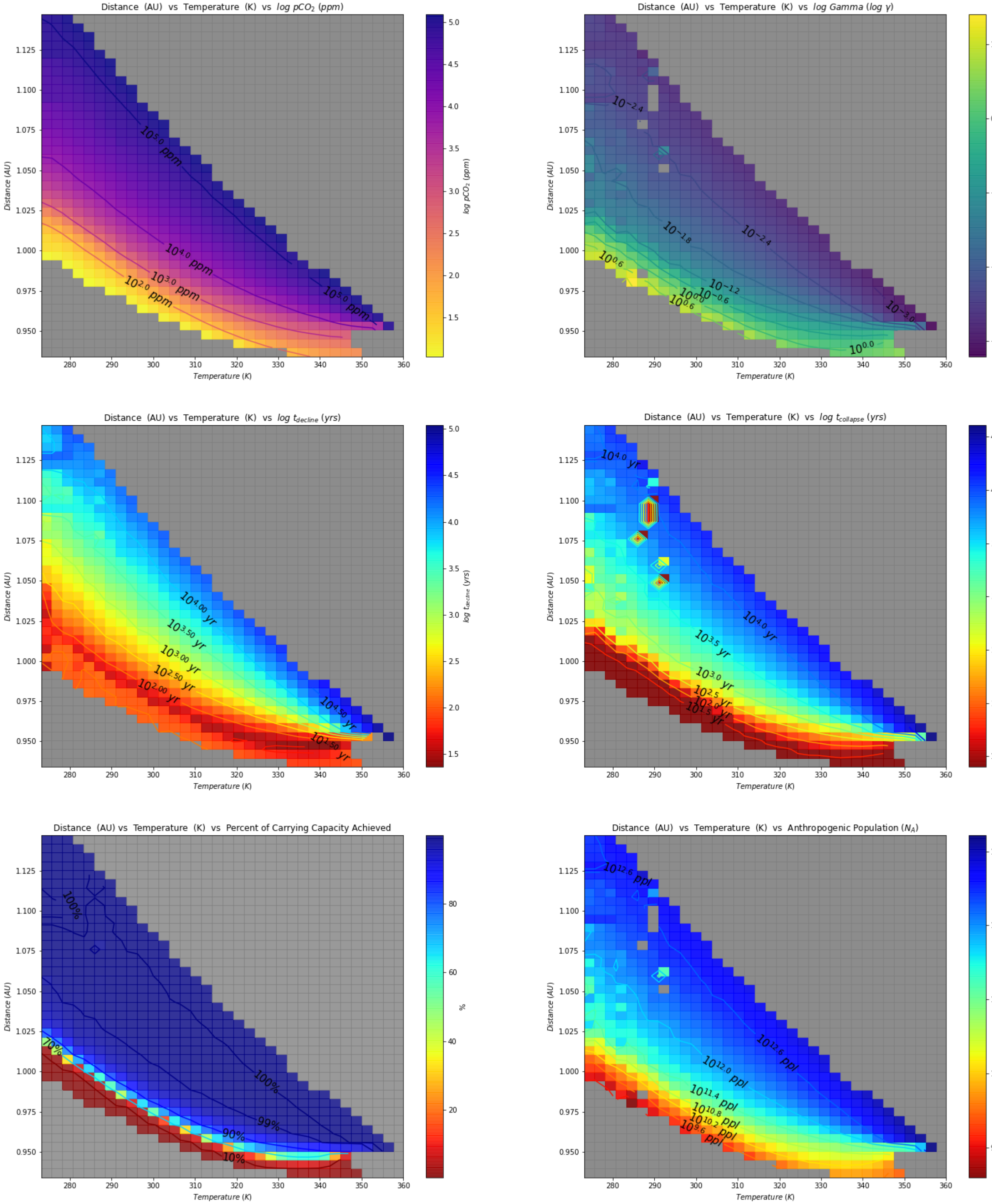

Carrying Capacity = 10 Billion
Distance (AU) vs Temperature (K) vs log pCO_2 (ppm)
Distance (AU)
Temperature (K)
log pCO_2 (ppm)
$10^{5.0}$ ppm
$10^{4.0}$ ppm
$10^{3.0}$ ppm
$10^{2.0}$ ppm
Distance (AU) vs Temperature (K) vs log Gamma (log γ)
log Gamma (log γ)
$10^{-2.4}$
$10^{-1.8}$
$10^{-1.2}$
$10^{-0.6}$
$10^{0.6}$
$10^{0.0}$
$10^{-3.0}$
Distance (AU) vs Temperature (K) vs log $t_{decline}$ (yrs)
log $t_{decline}$ (yrs)
$10^{4.0}$ yr
$10^{3.5}$ yr
$10^{3.0}$ yr
$10^{2.5}$ yr
$10^{2.0}$ yr
$10^{4.5}$ yr
$10^{1.5}$ yr
Distance (AU) vs Temperature (K) vs log $t_{collapse}$ (yrs)
log $t_{collapse}$ (yrs)
$10^{4.0}$ yr
$10^{3.5}$ yr
$10^{3.0}$ yr
$10^{2.5}$ yr
$10^{2.0}$ yr
$10^{1.5}$ yr
Distance (AU) vs Temperature (K) vs Percent of Carrying Capacity Achieved
%
99.9%
99.0%
90.0%
70.0%
10.0%
Distance (AU) vs Temperature (K) vs Anthropogenic Population (N_A)
logN_A ppl
$10^{12.6}$ ppl
$10^{12.0}$ ppl
$10^{11.4}$ ppl
$10^{10.8}$ ppl
$10^{10.2}$ ppl
$10^{9.6}$ ppl


Carrying Capacity = 100 Billion
Distance (AU) vs Temperature (K) vs log pCO2 (ppm)
Distance (AU)
Temperature (K)
log pCO2 (ppm)
10^5.0 ppm
10^4.0 ppm
10^3.0 ppm
10^2.0 ppm
Distance (AU) vs Temperature (K) vs log Gamma (log γ)
log Gamma (log γ)
Distance (AU) vs Temperature (K) vs log t_decline (yrs)
log t_decline (yrs)
10^2.25 yr
10^1.75 yr
Distance (AU) vs Temperature (K) vs log t_collapse (yrs)
log t_collapse (yrs)
10^3.0 yr
10^2.7 yr
10^2.4 yr
10^2.1 yr
10^3.3 yr
Distance (AU) vs Temperature (K) vs Percent of Carrying Capacity Achieved
%
70%
99%
90%
10%
Distance (AU) vs Temperature (K) vs Anthropogenic Population (N_A)
log N_A ppl
10^12.6 ppl
10^12.0 ppl
10^11.4 ppl
10^10.8 ppl
10^10.2 ppl
10^9.6 ppl

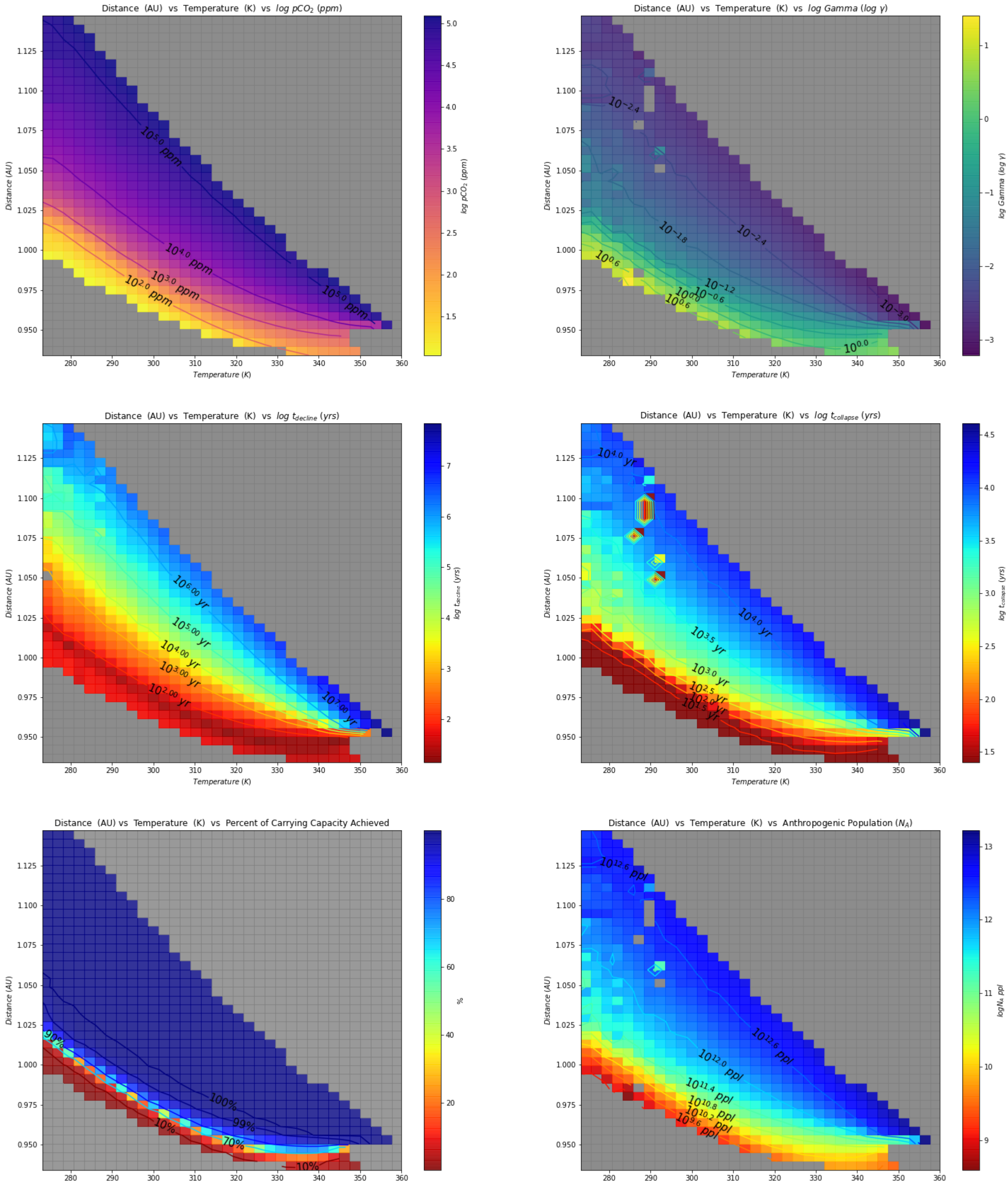


Carrying Capacity = 10 Billion, ΔT = 5, Exponent = 1
Distance (AU) vs Temperature (K) vs log pCO2 (ppm)
Distance (AU) vs Temperature (K) vs log Gamma (log γ)
Distance (AU) vs Temperature (K) vs log t_decline (yrs)
Distance (AU) vs Temperature (K) vs log t_collapse (yrs)
Distance (AU) vs Temperature (K) vs Percent of Carrying Capacity Achieved
Distance (AU) vs Temperature (K) vs Anthropogenic Population (N_A)
Distance (AU)
Temperature (K)

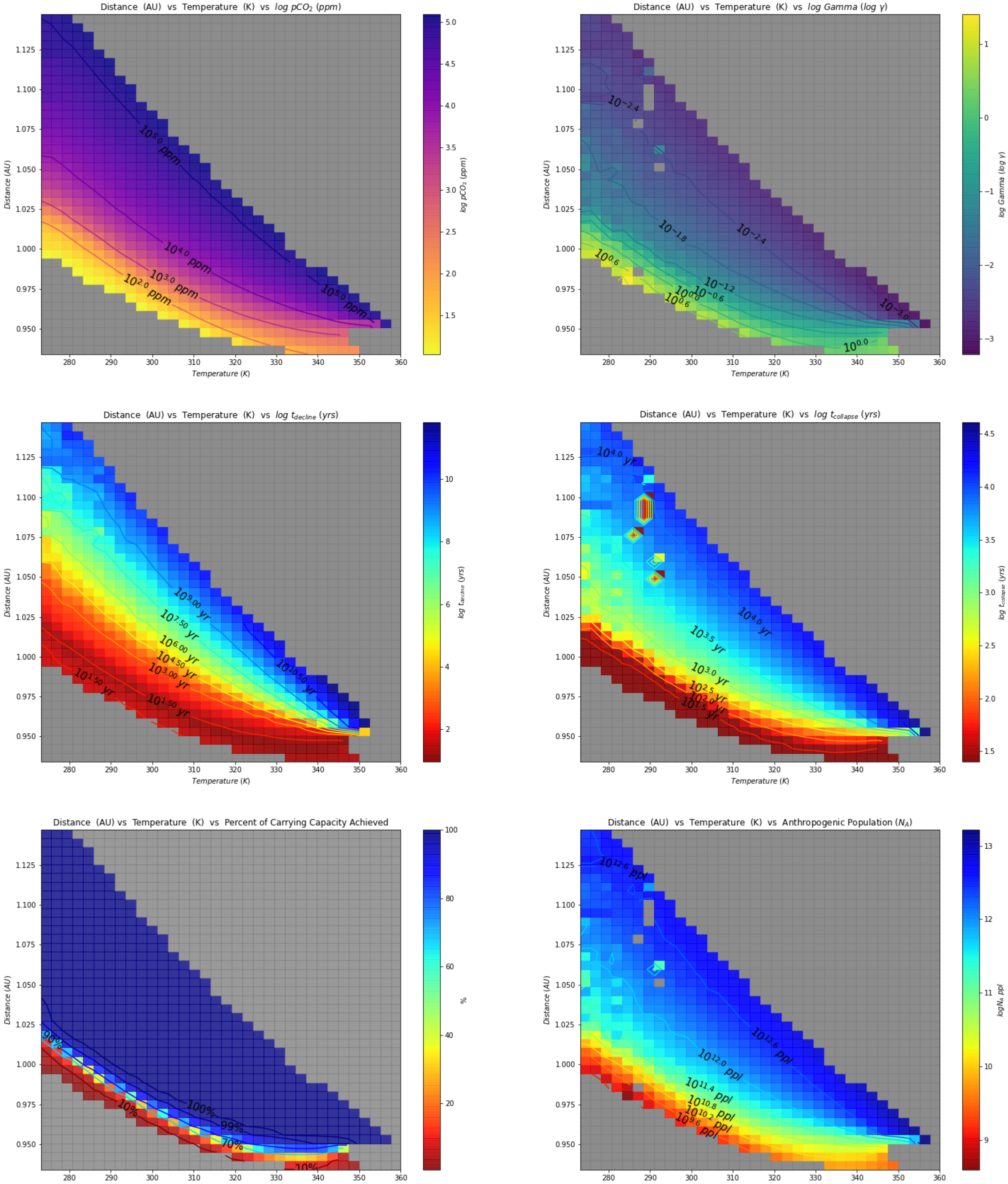
Carrying Capacity = 10 Billion,   ΔT = 5,   Exponent = 2

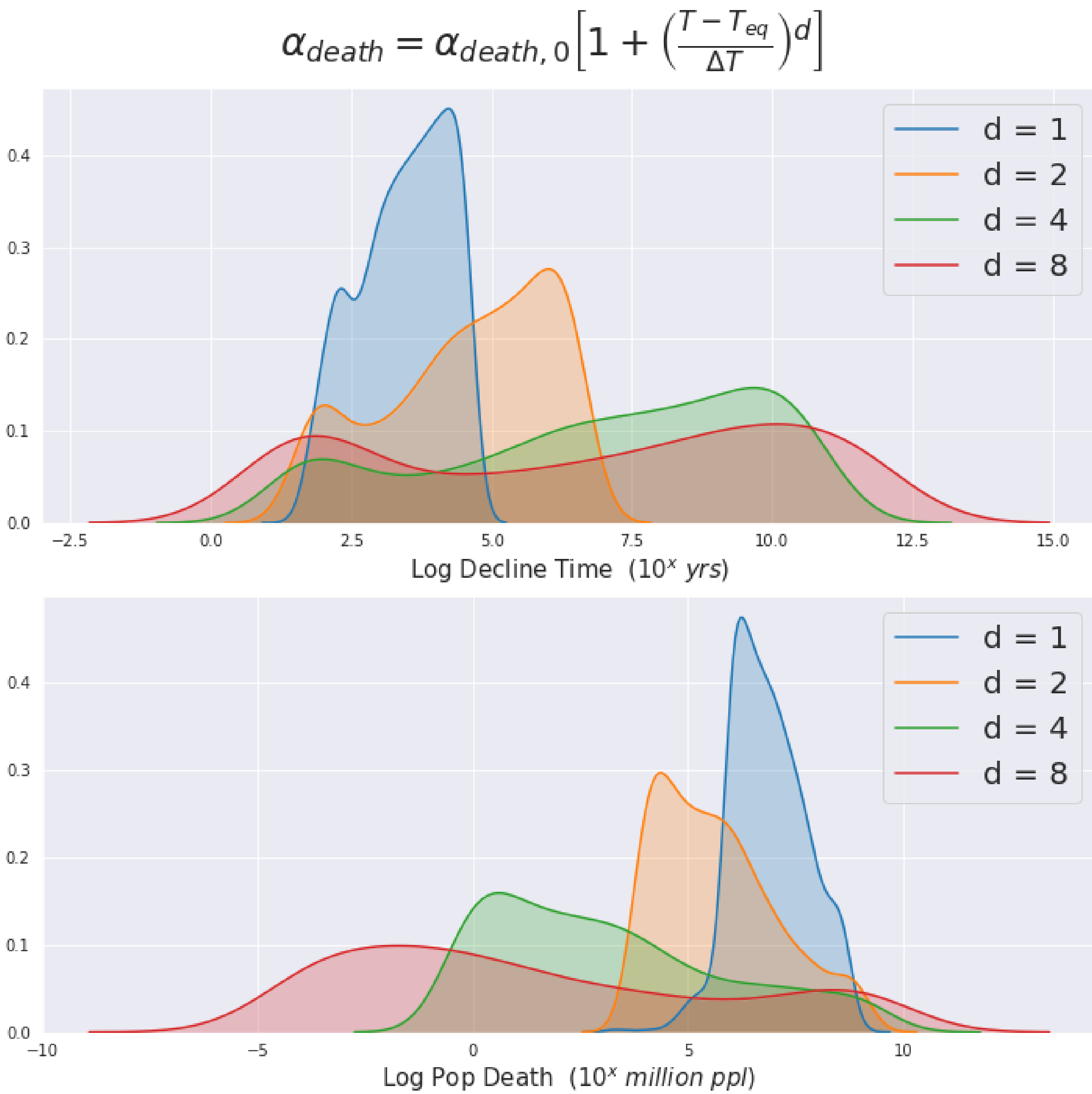


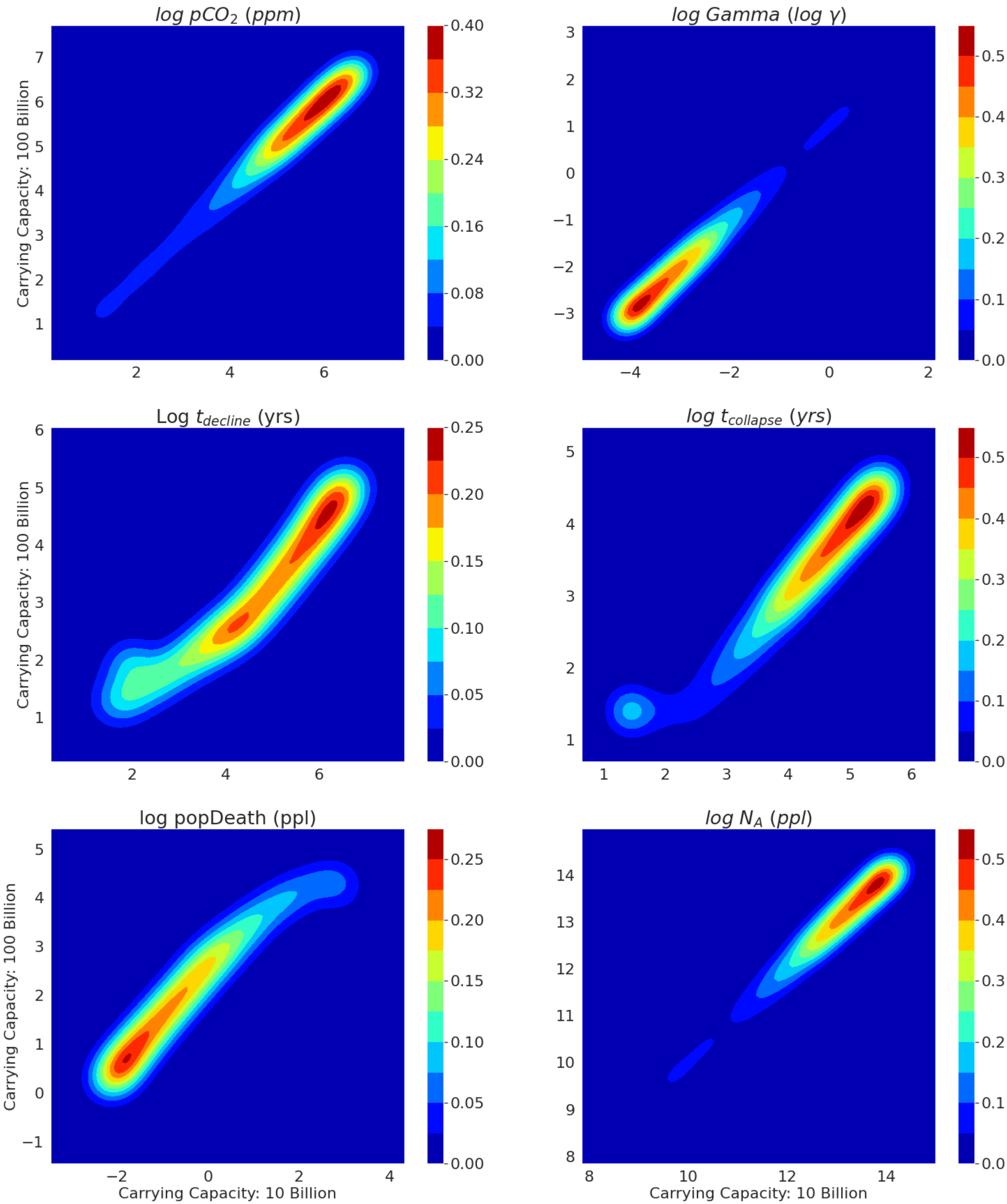
Carrying Capacity = 10 Billion, ΔT = 5, Exponent = 4



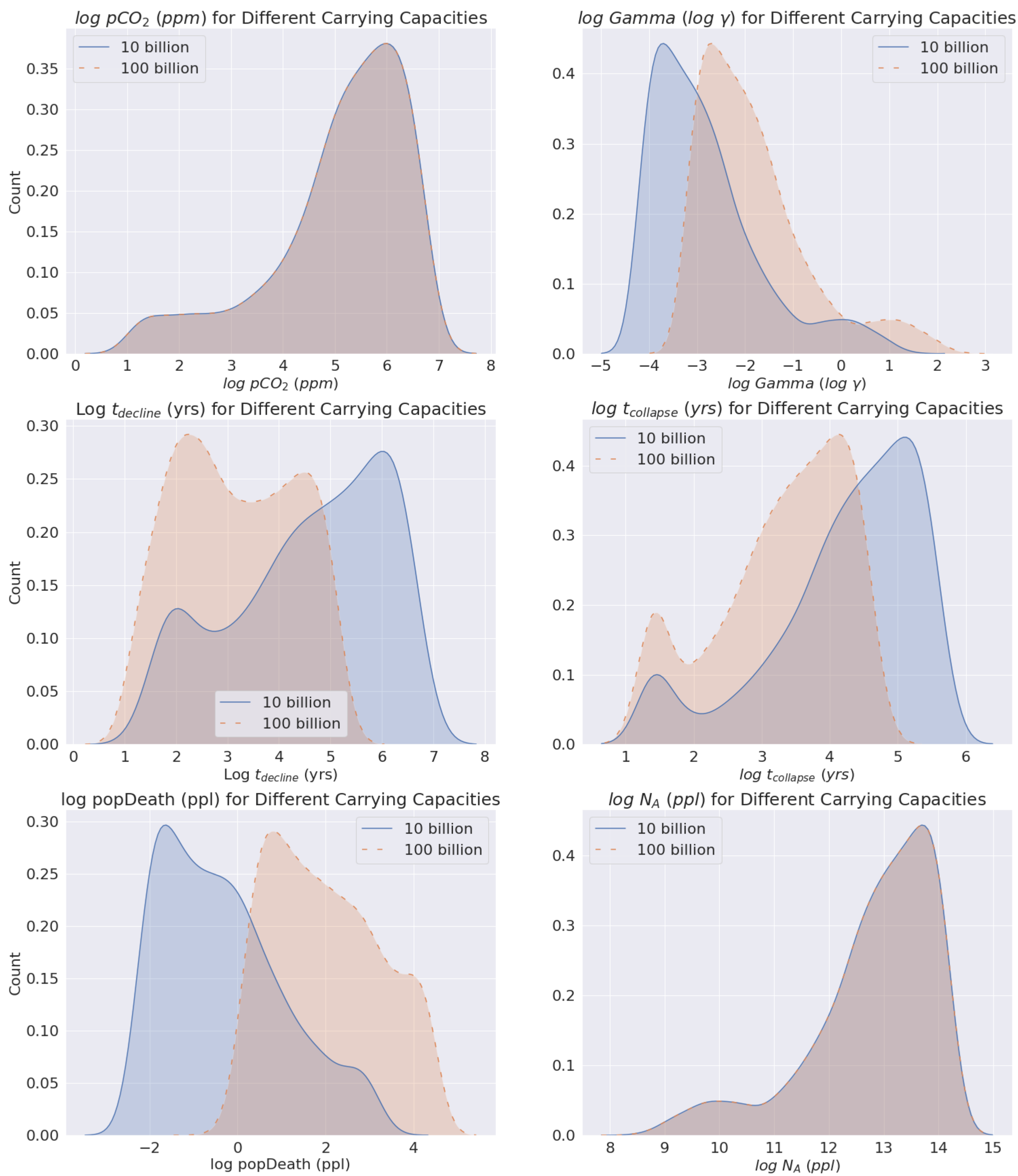
Carrying Capacity = 10 Billion,   ΔT = 5,   Exponent = 8

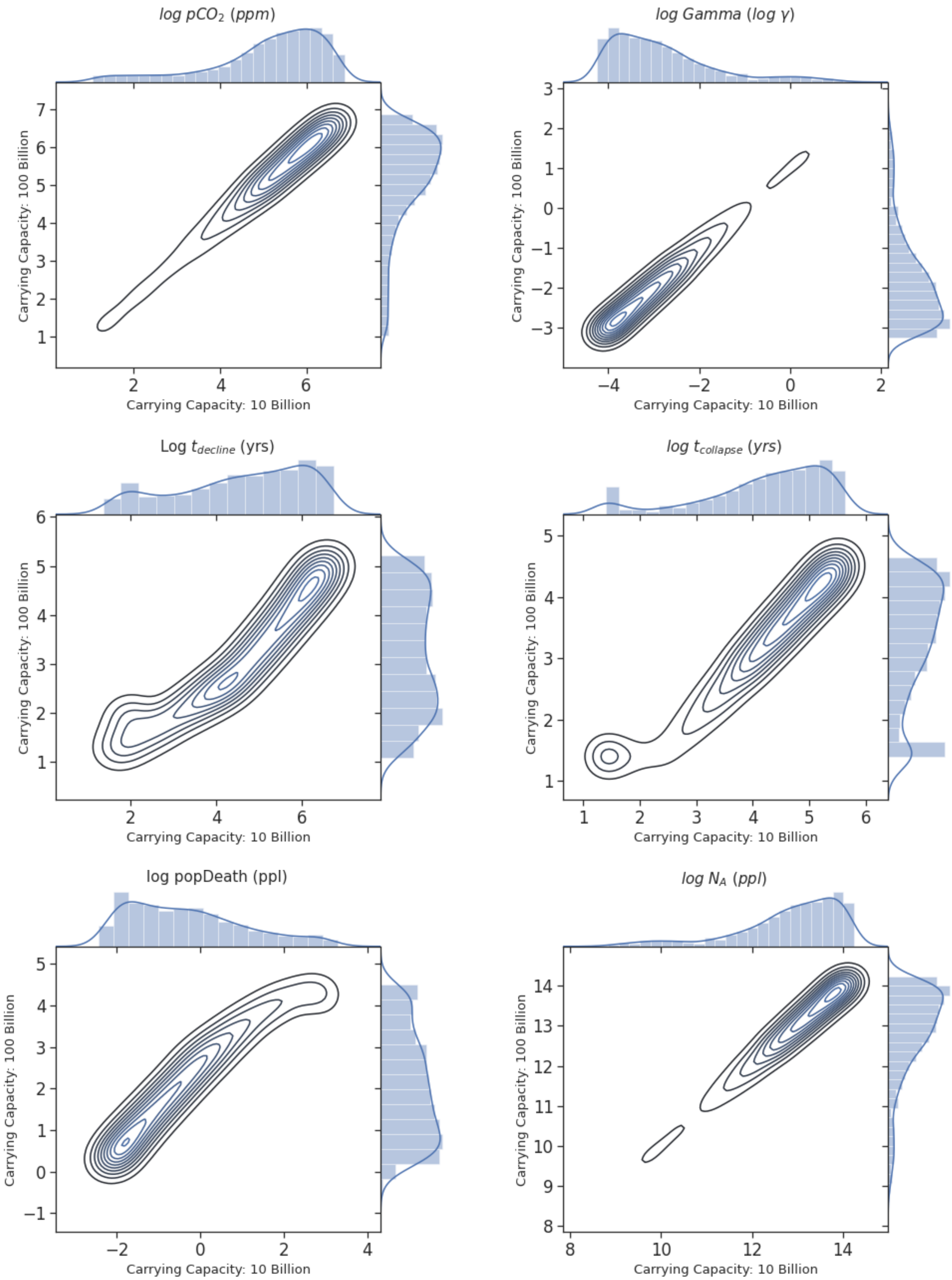



$\alpha_{death} = \alpha_{death,0}\left[1 + \left(\frac{T - T_{eq}}{\Delta T}\right)^d\right]$
d = 1
d = 2
d = 4
d = 8
0.0
0.1
0.2
0.3
0.4
−2.5
0.0
2.5
5.0
7.5
10.0
12.5
15.0
Log Decline Time (10^x yrs)
d = 1
d = 2
d = 4
d = 8
0.0
0.1
0.2
0.3
0.4
−10
−5
0
5
10
Log Pop Death (10^x million ppl)



log pCO_2 (ppm)
log Gamma (log γ)
Log $t_{decline}$ (yrs)
log $t_{collapse}$ (yrs)
log popDeath (ppl)
log N_A (ppl)
Carrying Capacity: 100 Billion
Carrying Capacity: 10 Billion


log pCO2 (ppm) for Different Carrying Capacities
10 billion
100 billion
Count
log pCO2 (ppm)
log Gamma (log γ) for Different Carrying Capacities
10 billion
100 billion
log Gamma (log γ)
Log t_decline (yrs) for Different Carrying Capacities
10 billion
100 billion
Count
Log t_decline (yrs)
log t_collapse (yrs) for Different Carrying Capacities
10 billion
100 billion
log t_collapse (yrs)
log popDeath (ppl) for Different Carrying Capacities
10 billion
100 billion
Count
log popDeath (ppl)
log N_A (ppl) for Different Carrying Capacities
10 billion
100 billion
log N_A (ppl)


log pCO_2 (ppm)
Carrying Capacity: 100 Billion
Carrying Capacity: 10 Billion
log Gamma (log γ)
Carrying Capacity: 100 Billion
Carrying Capacity: 10 Billion
Log $t_{decline}$ (yrs)
Carrying Capacity: 100 Billion
Carrying Capacity: 10 Billion
log $t_{collapse}$ (yrs)
Carrying Capacity: 100 Billion
Carrying Capacity: 10 Billion
log popDeath (ppl)
Carrying Capacity: 100 Billion
Carrying Capacity: 10 Billion
log N_A (ppl)
Carrying Capacity: 100 Billion
Carrying Capacity: 10 Billion

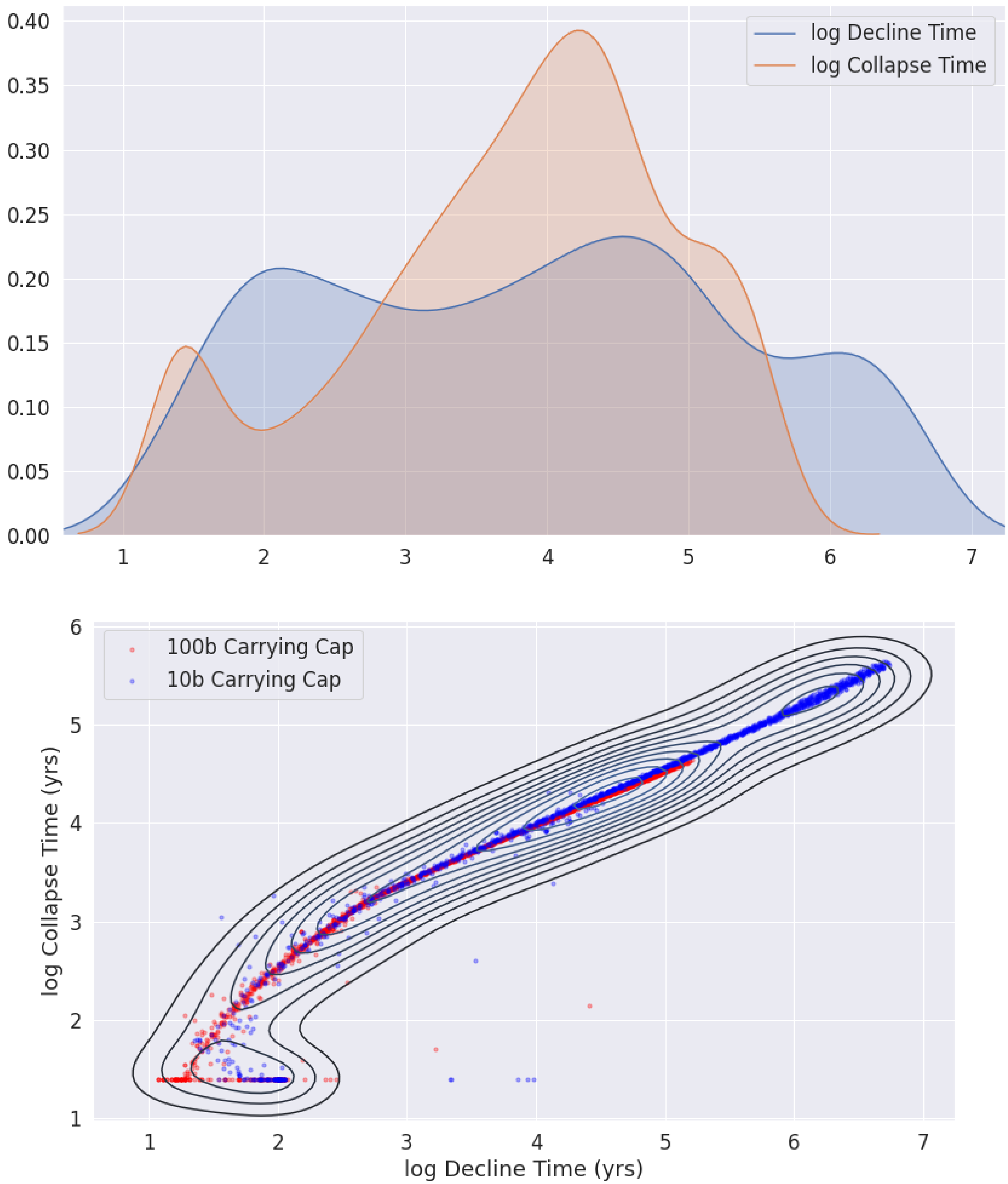


Figure 2. : Analytically Derived Collapse Time vs Numerically Derived Free Fall Time