Planet-Civ Update

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Contents

1 Overview of Model

1.1 Variables, Constants and Units

- T = Average Global Temperature (Kelvin)
  - T_{eq} = Equilibrium (initial) Temperature, calculated with the energy balance model ($Kelvin$)
  - ΔT = Temperature Range in which humans can survive (higher values correspond to lower fragility)
  - D = Orbital Distance (AU)
- P = Global Carbon Dioxide Partial Pressures (ppm)
  - P_0 = Initial Carbon Dioxide Partial Pressures[1] (ppm)
  - ϵ = Annual Per-Capita[2] Carbon Footprint ($\frac{ppm}{10^6 ppl*yr}$)
  - ΔP = A proportionality factor between the birth rate and changes in pCO_2. (higher values correspond to less technologically efficient civilizations, ie: must burn more fossil fuels in order to increase the birth rate)
- N = Global Population ($\times 10^6$ ppl)
  - N_0 = Initial Global Population ($\times 10^6$ ppl)
  - N_{max} = Maximum Allowed Global Population ($\times 10^6$ ppl)
  - $\alpha_{birth,0}/\alpha_{death,0}$ = Initial Per-Capita[2] Birth/Death Rates ($1/yr$)
  - $\alpha_{birth}/\alpha_{death}$ = Current Per-Capita[2] Birth/Death Rates ($1/yr$)

[1] $x\ ppm * \left(\frac{1Bar}{10^6 ppm}\right) = y\ bar$
[2] Per-Capita Meaning Per-Million People

1.2 Dimensionless Parameter (Γ)

First, we define the timescales:

$$t_{growth} = \frac{1}{\alpha_{birth,0}} \tag{1}$$

$$t_{climate} = \frac{\Delta T}{\epsilon N_{max} \frac{dT}{dP}} \tag{2}$$

Then we can write our dimensionless parameter like:

$$\Gamma = \frac{t_{growth}}{t_{climate}} = \frac{\epsilon N_{max} \frac{dT}{dP}}{\alpha_{birth} \Delta T} = \frac{\textit{Timescale for Population Growth}}{\textit{Timescale for Climate to Change}} \tag{3}$$

- $\Gamma << 1 \implies$ Climate will change on timescales much longer than the average generation. Corresponds to a civilization having a low risk for an Anthropocene.
- $\Gamma = 1 \implies$ Climate will change within one generation.
- $\Gamma >> 1 \implies$ Climate will change on timescales much shorter than the average generation. Corresponds to a civilization having a high risk for an Anthropocene.

1.3 Outline of Coupled Model

First, let the energy balance model reach an equilibrium between incoming and outgoing radiation, this gives us the equilibrium temperature. The model continues by setting the initial temperature to this equilibrium value, as well as setting the birth and death rates to their initial values. The main loop now begins, where each loop represents one year.[3]

i) Call[4]: $\frac{dT}{dt} = EBM(P)$

ii) $\alpha_{birth} = \alpha_{birth,0} \left[1 + \frac{P - P_0}{\Delta P}\right]$

iii) $\alpha_{death} = \alpha_{death,0} \left[1 + \left(\frac{T - T_{eq}}{\Delta T}\right)^2\right]$

iv) Call: $\frac{dN}{dt} = min(\alpha_{birth} N,\ \alpha_{death,0} N_{max}) - \alpha_{death} N$
   - If $dNdt > 0$: set variable peakTime = currentTime
   - If $dNdt < 0$: set variable peaked = True

v) Call: $\frac{dP}{dt} = \epsilon N$
   a) If (peaked=True) and (currentTime - peakTime $\geq$ 500), then end program[5]
   b) Else, go back to the first step.

1.4 Example: Modeling Earth ($t_0 = 1820,\ P_0 = 284,\ N_0 = 1,129$)

1.4.1 Input Values

- $N_{max} = 10$ billion people
- $\alpha_{birth,0} = 0.04\ yr^{-1}$
- $\alpha_{death,0} = 0.036\ yr^{-1}$
- $\Delta T = 5K$
- $\Delta P = 200\ ppm$
- $\epsilon = 0.000275\ \frac{ppm}{10^6 ppl*yr}$

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[3] Note: made population have a minimum of 1 million people, to avoid values of 10^{-100}

[4] $EBM(P) = \frac{\psi(1 - A) - I + \nabla \cdot (\kappa \nabla T)}{C_v}$

[5] Run for 20 generations after the population has peaked, where a generation is defined as $t_{gen} \approx t_{growth} = \frac{1}{\alpha_{birth,0}} = 25\ years$

1.4.2 Output Plots

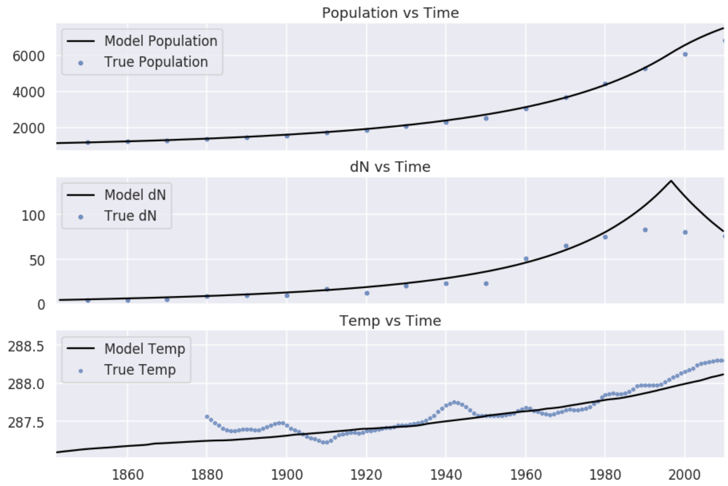


Figure 1: Model Output (solid black line) vs Real Global Data (dotted line)

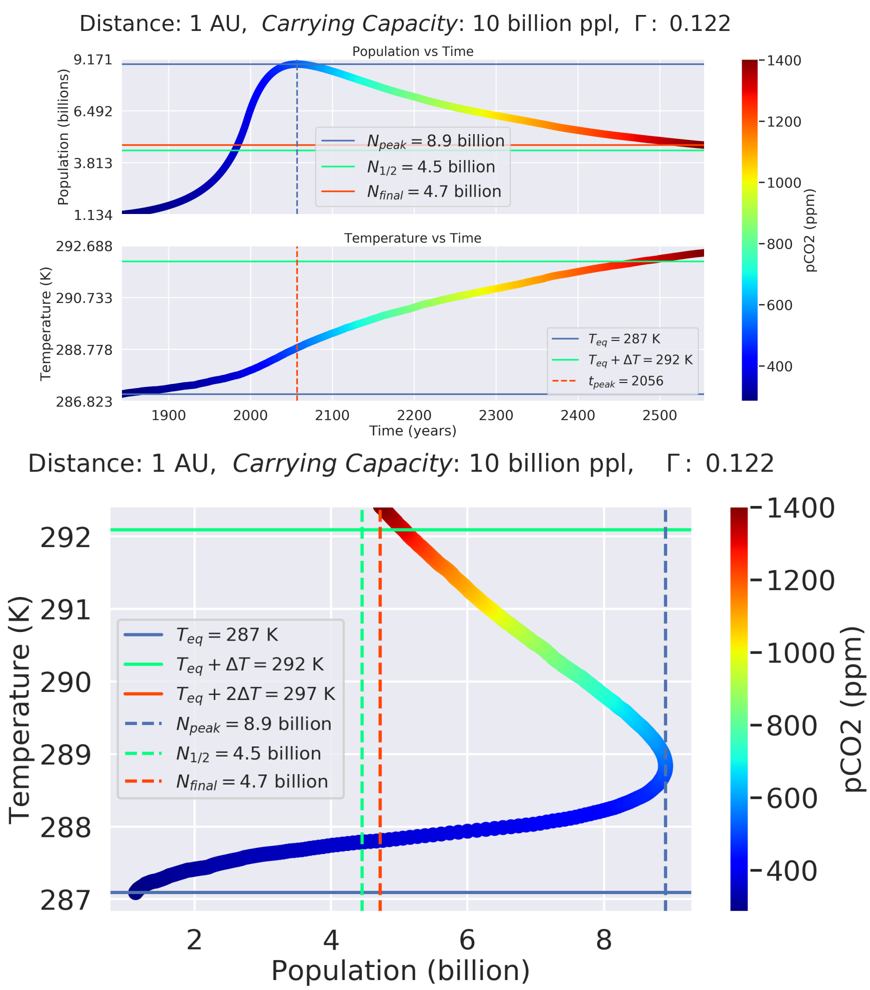


Figure 2: Model Output for 2000 Years

2 Experiment #1: Constant Composition ($P_0 = 284\ ppm = 2.84 \times 10^{-4}\ bar$)

2.1 Habitable Zone

In this experiment, we define the habitable zone by the range of distances that will result in temperatures above freezing and below boiling.[6]

$$273.15\ K < T_{habitable} < 373.15\ K \tag{4}$$

$$\boxed{0.94\ AU < a_{habitable} < 1.02\ AU} \tag{5}$$

2.2 Linear Regressions of Temp vs pCO_2

For 5 different distances, I first ran the model, uncoupled, with initial $pCO2 = 284ppm$. The resultant equilibrium temperature for this $pCO2$ is saved as a variable called initialTemp. I then **decremented** the initial pCO_2 by $5ppm$ and re-ran the model until the absolute value of the difference between the equilibrium temperature and the initial temperature was greater than 2. I then set the initial $pCO2$ back to $284ppm$, and continued by **incrementing** the initial pCO_2 by $5ppm$ and re-running the model until the absolute value of the difference between the equilibrium temperature and the initial temperature was greater than 2. At this point, I changed distances and repeated the same process. After all distances have been looped, I ran a linear regression (using scipy.stats.linregress) for the data from each distance to find the relationship between changes in pCO2 and changes in global temperature.[7]

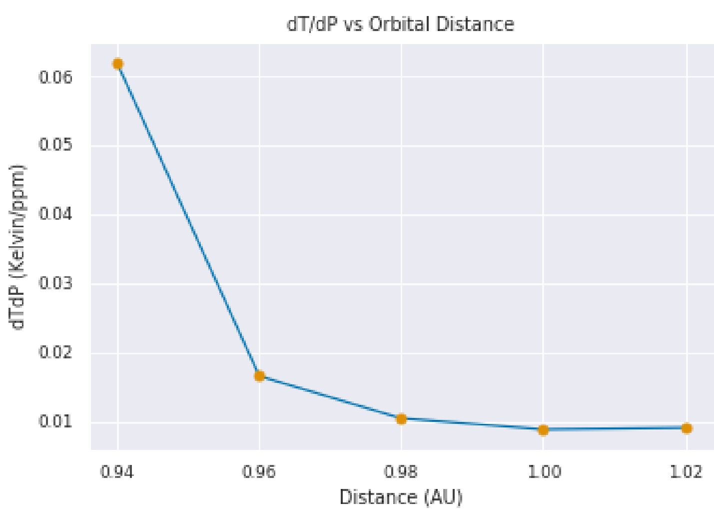
$$T = 6.178 * 10^{-2}\left(\frac{P}{ppm}\right) + 322 \qquad \textbf{\textit{(0.94 AU)}}$$

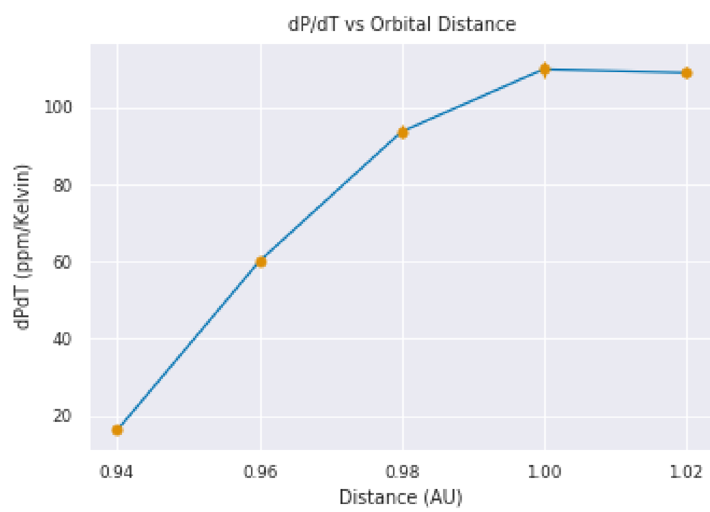
$$T = 1.655 * 10^{-2}\left(\frac{P}{ppm}\right) + 309 \qquad \textbf{\textit{(0.96 AU)}}$$

$$T = 1.044 * 10^{-2}\left(\frac{P}{ppm}\right) + 296 \qquad \textbf{\textit{(0.98 AU)}}$$

$$T = 8.838 * 10^{-3}\left(\frac{P}{ppm}\right) + 284 \qquad \textbf{\textit{(1.0 AU)}}$$

$$T = 9.067 * 10^{-3}\left(\frac{P}{ppm}\right) + 273 \qquad \textbf{\textit{(1.02 AU)}}$$





[6] The lower limit on distance is restricted by our model, which fails to converge when the initial temperature is at or above 330K

[7] For 1.04AU, I couldn't go 2 degrees above or else it would jump, so I did 1 degree on either side.

3 Experiment #2: Constant Temperature ($T_{eq} = 287.09K = 57.09\ ^oF$)

3.1 Habitable Zone

In this experiment, we define the habitable zone by the range of distances that have temperatures approximately equal to the equilibrium temperature for our current planet, $287.09K$, with corresponding $pCO2's$ greater than 10ppm and less than 10^5ppm. (Which corresponds to an atmosphere that is composed of 10% CO2).[8]. The way I found this zone was by first setting the initial pco2 to $10ppm$ and the distance to $1AU$, then continually decrementing the distance by $0.001AU$ until the temperature was greater or equal to 287.09, the distance for which this occurs is the minimum distance. I then set the initial pco2 to 10^5ppm and the distance to $1AU$, and continually incremented the distance by $0.001AU$ until the temperature was less than or equal to 287.09, the distance for which this occurs is the maximum distance.

$$10\ ppm < pCO2_{habitable} < 10^5\ ppm \tag{6}$$

$$\boxed{0.97\ AU\ < a_{habitable} < 1.10\ AU} \tag{7}$$

3.2 Linear Regressions of Temp vs pCO_2

Note: I am going to redo these do mirror the procedure for constant composition. First, I will find the value of $pCO2$ for that makes the planets temp around $287K$, then I will decrement $pCO2$ distance from $278K$ is 2 degrees, and repeat for incrementing.

For 7 different distances, I first found the value of pCO2 which would make that planet's temp be around room temperature, fixing this value as the initial temp for this distance. Then made a loop in which initially the EBM is run with this value of pCO2, but after every loop I incremented this value slightly, then exited the loop once the equilibrium temperature became greater than 10 degrees higher than the initial temperature. After all distances have been looped, I ran a linear regression (using scipy.stats.linregress) for the data from each distance to find the relationship between changes in pCO2 and changes in global temperature.

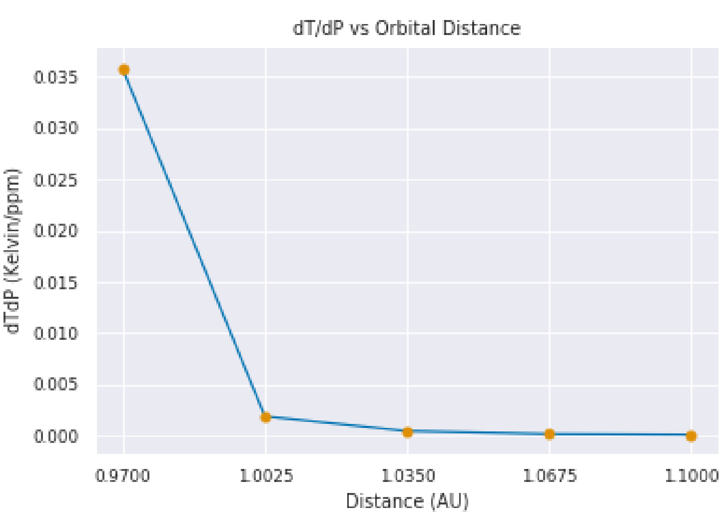
$$T = 3.573 * 10^{-2}\left(\frac{P}{ppm}\right) + 298 \qquad \textbf{\textit{(0.970 AU)}}$$

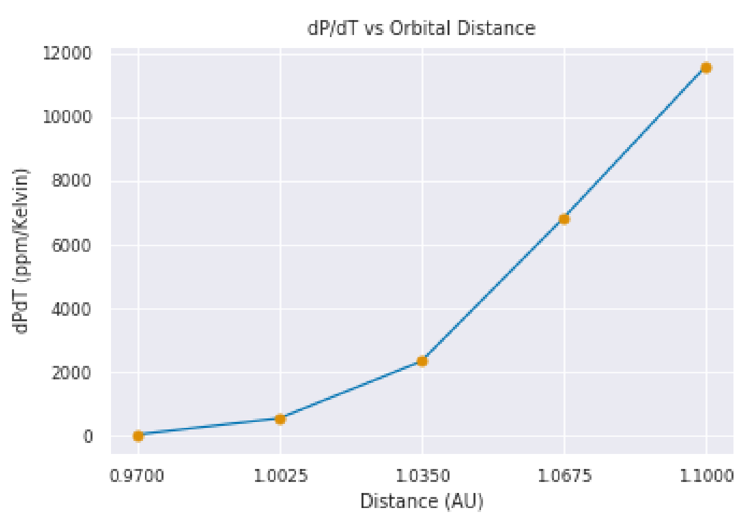
$$T = 1.861 * 10^{-3}\left(\frac{P}{ppm}\right) + 290 \qquad \textbf{\textit{(1.0025 AU)}}$$

$$T = 4.297 * 10^{-4}\left(\frac{P}{ppm}\right) + 285 \qquad \textbf{\textit{(1.035 AU)}}$$

$$T = 1.464 * 10^{-4}\left(\frac{P}{ppm}\right) + 287 \qquad \textbf{\textit{(1.0675 AU)}}$$

$$T = 8.629 * 10^{-5}\left(\frac{P}{ppm}\right) + 284 \qquad \textbf{\textit{(1.1 AU)}}$$





[8] The minimum allowed value for pCO2 in our EBM is 10ppm

4 Analytic Model

With natural birth rate A, natural death rate B, per capita emission rate C, and temperature-CO2 climate sensitivity D, birth rate temperature sensitivity ΔT, CO2 emission birth rate advantage range ΔP, and carrying capacity $N_{\max}$ we can model the population N, CO2 concentration P, and temperature T using

$$\begin{aligned}
\frac{dN}{dt} &= \min\left[AN\left(1+\frac{P-P_0}{\Delta P}\right), BN_{\max}\right] - BN - AN\left(\frac{T-T_0}{\Delta T}\right)^2 \quad (8)\\
\frac{dP}{dt} &= CN \quad (9)\\
\frac{dT}{dt} &= \frac{dT}{dP}\frac{dP}{dt} = D(T)CN \quad (10)\\
& \quad (11)
\end{aligned}$$

where

$$\begin{aligned}
D(T) &= \frac{dT}{dP} \quad (12)\\
& \quad (13)
\end{aligned}$$

If we assume that the climate sensitivity is constant, then we have

$$T = T_0 + D\left(P - P_0\right) \quad (14)$$

This allows us to reduce the system of equations to

$$\begin{aligned}
\frac{dN}{dt} &= \min\left[AN\left(1+\frac{T-T_0}{D\Delta P}\right), BN_{\max}\right] - BN - AN\left(\frac{T-T_0}{\Delta T}\right)^2 \quad (15)\\
\frac{dT}{dt} &= DCN \quad (16)\\
& \quad (17)
\end{aligned}$$

Now dividing both equations by A, and the first equation by $N_{\max}$ and the second equation by ΔT we arrive at

$$\begin{aligned}
\frac{d\eta}{d\tau} &= \min\left[\eta\left(1+\theta\epsilon\right), \beta\right] - \beta\eta - \epsilon^2\eta \quad (18)\\
\frac{d\epsilon}{d\tau} &= \gamma\eta \quad (19)\\
& \quad (20)
\end{aligned}$$

where

$\eta =$	$\dfrac{N}{N_{\max}}$	Normalized population	(21)
$\tau =$	$A_0 t$	Normalized time	(22)
$\beta =$	$\dfrac{B}{A_0}$	Normalized natural death rate	(23)
$\epsilon =$	$\dfrac{T-T_0}{\Delta T}$	Normalized temperature	(24)
$\theta =$	$\dfrac{\Delta T}{D\Delta P}$	Normalized Birth rate acceleration	(25)
$\gamma =$	$\dfrac{DCN_{\max}}{A\Delta T}$	Normalized forcing	(26)

Note that in the absence of any means to reduce the CO2 in the atmosphere - and therefore the temperature, there is no equilibrium for the temperature except for the trivial one $\eta = 0$.

Now because of the min function, the population will peak in one of two scenarios.

First, if the population growth rate never exceeds the maximum growth rate, then we have

$$\frac{d\eta}{d\tau} = \left(1 + \theta\epsilon - \beta - \epsilon^2\right)\eta = 0 \rightarrow \epsilon = \frac{\theta}{2} + \sqrt{\left(\frac{\theta}{2}\right)^2 + (1-\beta)} \tag{27}$$

On the other hand, if the forcing γ is small, the population can reach the maximum growth rate at

$$\frac{d\eta}{d\tau} = \beta - \beta\eta - \epsilon^2\eta = 0 \rightarrow \epsilon = \sqrt{\frac{\beta(1-\eta)}{\eta}} \tag{28}$$

Now it is helpful to look at the second derivative of the population evaluated at these peaks to determine how quickly the population declines.

In the first case, we have

$$\frac{d^2\eta}{d\tau} = \theta\eta\frac{d\epsilon}{d\tau} - 2\eta\epsilon\frac{d\epsilon}{d\tau} = \theta\gamma\eta^2 - \theta\gamma\eta^2 - \eta\sqrt{\theta^2 + (1-\beta)} = -\eta\sqrt{\theta^2 + (1-\beta)} \tag{29}$$

Now the time scale for the population to decrease is less then the natural growth time

$$-\frac{\eta}{\frac{d\eta^2}{d\tau}} = \frac{1}{\theta^2 + (1-\beta)} < \frac{1}{1-\beta} \tag{30}$$

So the collapse will be at least as fast as the growth - and will only accelerate as the temperature continues to increase. But the collapse occurs on the order of a generation.

Now in the second case, if the population can reach the maximum growth rate, then we have

$$\frac{d^2\eta}{d\tau} = -2\epsilon\gamma\eta^2 = -2\gamma\eta^2\sqrt{\frac{\beta(1-\eta)}{\eta}} \tag{31}$$

and the timescale for the population to decline is

$$-\frac{\eta}{\frac{d\eta^2}{d\tau}} = \frac{1}{2\gamma\eta\sqrt{\frac{\beta(1-\eta)}{\eta}}} \tag{32}$$

which can be quite large for $\eta \approx 1$ or go as $\frac{1}{\gamma\sqrt{\beta}}$ for $\eta = \frac{1}{2}$