

1 Spontaneous Parametric Downconversion

Spontaneous parametric downconversion is the process by which one photon splits into two photons. During the process momentum and energy are conserved. The conservation requirements result in a very “entangled” field. Entanglement means that measurement of one of the two photons has an unalterable effect on the other photon. In the regime of weak downconversion, the state of light from a nonlinear crystal pumped by a strong laser beam has the form [1, 2, 3]

$$|\psi\rangle_{1,2} = |\text{vac}\rangle + \int A(\mathbf{p}_1, \mathbf{p}_2) |\mathbf{p}_1, \mathbf{p}_2\rangle d\mathbf{p}_1 d\mathbf{p}_2 \quad (1)$$

where $|\text{vac}\rangle$ denotes the vacuum state and the two-photon amplitude A is

$$A(\mathbf{p}_1, \mathbf{p}_2) = \chi E_p(\mathbf{p}_1 + \mathbf{p}_2) \frac{\exp(i\Delta k_z L) - 1}{i\Delta k_z}. \quad (2)$$

Here χ is the coefficient of the nonlinear interaction, E_p is the amplitude of the plane-wave component of the pump with transverse momentum $\mathbf{p}_1 + \mathbf{p}_2$, L is the length of the nonlinear medium, and $\Delta k_z = k_{p,z} - k_{1,z} - k_{2,z}$ (where $\mathbf{k} = \mathbf{p}/\hbar$) is the longitudinal wavevector mismatch, which generally increases with transverse momentum and limits the angular spread of signal and idler photons. The vacuum component of the state makes no contribution to photon counting measurements and may be ignored.

1.1 Momentum Conservation and Correlation

Consider the momentum conservation represented in Fig. 1. A blue pump photon has a trajectory shown as a blue line. The blue photon is downconverted into (i.e., absorbed and reemitted as) two red photons. Because of momentum conservation, measuring the momentum of one photon means that the momentum of the other photon is known. Before measurement, both photons are diverging cones and can be separated with a polarizing beam splitter (not shown). An example of a possible measurement outcome is shown with two red arrows. Note that the two arrows are symmetric about the pump demonstrating momentum conservation. However, even if one knows the precise momentum of one of the photons, the momentum of the other photon is only known up to the momentum uncertainty of the annihilated pump photon momentum. The momentum uncertainty is depicted by the circle of diameter Δk (this momentum correlation uncertainty represents a lower bound on the image resolution or pixelization in the far

field). This is analogous to the classic problem in freshman physics in which a projectile explodes into two equal masses. If one knows the coordinates and momentum of the center of mass and of one of the pieces of the projectile, the coordinates and momentum of the other mass can be uniquely determined. However, unlike the classical scenario, there is an uncertainty to the center of mass direction. The uncertainty in the pump photon direction can be made very small by making the pump beam diameter very large.

The momenta of the two photons created by parametric down conversion can be measured in the far field. One can either place detectors at infinity inside the cones of emission or one can image the photons in the focus of a lens. A simple unfolded ray diagram is shown in Fig. 2. The figure shows two parallel rays converging to the same point in the focal plane of a lens. Owing to the momentum entanglement the other photon of the entangled pair is measured at the antisymmetric point relative to the optic axis in the focal plane of the other lens. Thus, if one places a detector in both focal planes one will measure photons with nearly perfectly anticorrelated positions relative to the optic axis.

1.2 Position Correlation

When one chooses to measure in the image plane, one is interested in the location of the origin of each photon (not their propagation directions). Essentially this means that the face of the Beta Barium Borate crystal is imaged. Photons are created in pairs in a nonlinear crystal inside a diameter determined by the phase matching conditions of the crystal. Critical phase matching occurs when the momenta of the two downconverted photons are equal to the momentum of the destroyed pump photon in the longitudinal direction. As the signal and idler deviate from the critical longitudinal direction the probability of creating a pair of photons is reduced. The effective cross-sectional area of the phasematching is called the two-photon “birthplace”. The effective birthplace diameter, for a standard crystal, is a few 10’s of microns, which is much smaller than the pump photon diameter. A pictorial representation is shown in Fig. 3

The face of the crystal is $1 \rightarrow 1$ imaged similarly to Fig. 2 with exception that the lenses are two focal lengths from the crystal and the respective image planes are then two additional focal lengths from the lenses. Unlike the momentum distribution, the position correlations are symmetric relative to the optical axis.

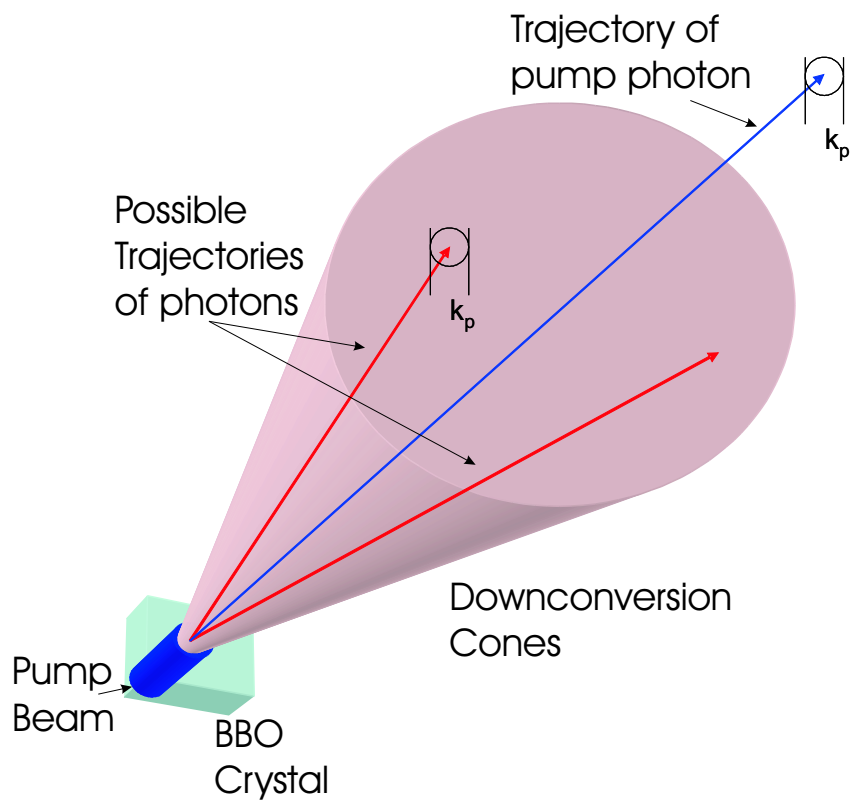


Figure 1: Momentum entanglement from spontaneous parametric down conversion. A blue pump photon has a trajectory shown in blue. The blue photon is downconverted into two red photons. Because of momentum conservation, measuring the momentum of one photon means that the momentum of the other photon is known. Before measurement, both photons are diverging cones and can be separated with a polarizing beam splitter (not shown). An example of a possible measurement outcome is shown with two red arrows. Even if one knows the precise momentum or direction of one photon, the momentum of the other photon is known up to the momentum uncertainty of the annihilated pump photon momentum. The momentum uncertainty is depicted by the circle Δk .

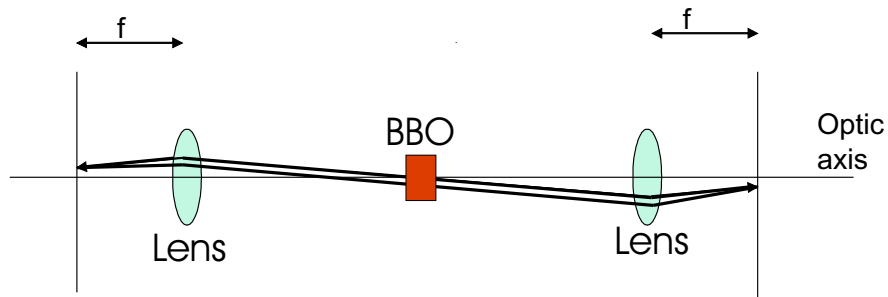


Figure 2: A simple unfolded ray diagram for measuring momentum of each photon. The figure shows two parallel rays converging to the same point in the focal plane of a lens. Owing to the momentum entanglement the far field measurement of one photon is correlated to the far field measurement of the other photon such that the two photons will be measured at antisymmetric points relative to the optic axis.

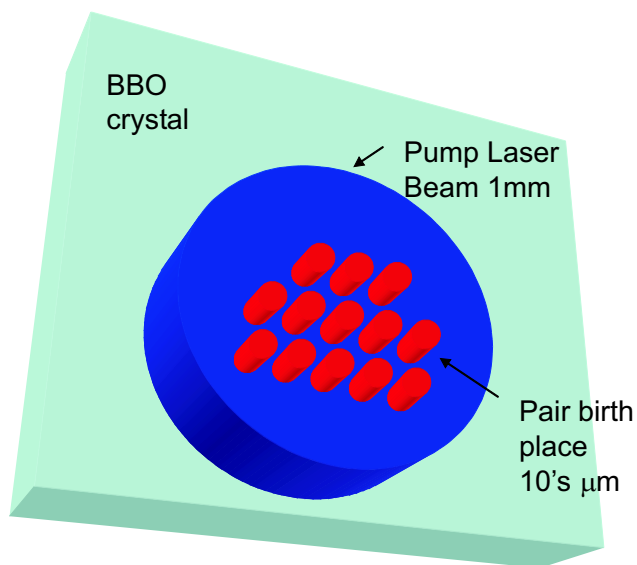


Figure 3: A representation of the birthplace generation of a two photon. Red circles are shown that indicate that the birthplace diameter is much smaller than the pump diameter. However, it should be realized that there is a continuous distribution of two photon probability. It just happens that when a measurement is performed which gives the observer knowledge about the origin of one photon, then the origin of the other photon is known to within a probability distribution characterized by the birthplace diameter.

References

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