Franson Interference: Space-like separated method for determining time-time correlations of entangled photons

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Since the early 70's, several groups have tried to measure the time-correlation of downconverted photons. Hong, Ou and Mandel (HOM) showed that two photon interference gave an indirect method for measuring the correlations. However, the HOM method is a local measurement, because it requires both photons of the pair to be present at the same space-time location. Here, it will be shown that fourth order interference in a Franson interferometer can also be used to determine the time-time correlations of parametric downconversion. In contrast to the HOM interference, Franson interference can performed with the two photons of the pair at two space-like separated locations.

A simplified longitudinal wavefunction, which neglects transverse fields and polarization is given by

$$|\Psi_0\rangle = \int\int dtdtA(t_1,t_2)a_1^\dagger(t)a_3^\dagger(t)|0\rangle \tag{1}$$

where $A(t,t')$ is the nonseparable temporal amplitude correlation function, $a_1^\dagger(t)a_2^\dagger(t')$ are photon creation operators for modes 1 and 2 respectively and $|0\rangle$ is the vacuum state.

The photons from the EPR source are then each sent to imbalanced Mach-Zehnder interferometers as shown in Fig. 1. The imbalanced interferometers are designed such that there is no single-photon interference. Typically, only 100 micron imbalances are needed to insure this. There are also two more constraints. In order to see the fourth order temporal interference, the imbalance must be long enough so that it is possible to postselect out events in photon 1 took the short path and photon 2 took the long path and vice versa. These measurement outcomes are rejected. This latter requirement necessitates the imbalance to be larger than the detector response time, which is on the order of 300 ps. The last constraint is that the imbalance must be shorter than the pump coherence time, which in our experiment is approximately 1 μs. Typically, the imbalance is approximately 2 to 3 ns, which satisfies all the constraints.

Now consider the evolution of the two-photon wavepacket through the Franson interferometer. The beamsplitters, which will be assumed to be 50/50 beamsplitters, have the matrix form

$$B = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}. \tag{2}$$

The field operators at the detectors, after passing through the interferometer, are given by (it is assumed that the trasmitted mode through the beam splitter preserves the mode)

$$a_1^\dagger(t) \to \frac{1}{2}(a_1^\dagger(t-\tau_1) - e^{i\phi_{12}} a_1^\dagger(t-\tau_2)) \tag{3}$$

and

$$a_3^\dagger(t') \to \frac{1}{2}(a_3^\dagger(t'-\tau_3) - e^{i\phi_{34}} a_3^\dagger(t'-\tau_4)) \tag{4}$$

where the τ_i are the relative times of the amplitudes to arrive at the detectors and the phase factors ϕ_{12}, ϕ_{34} are the relative phases picked up in the lower and upper interferometers respectively. It should be noted that field amplitudes are also in the other outputs of the interferometers, but we are only interested in the fields at detectors 1 and 3. The wavefunction is then given by

$$|\Psi_1\rangle = \frac{1}{4}\int\int dt_1 dt_1' A(t,t')(a_1^\dagger(t-\tau_1) - e^{i\phi_{12}} a_1^\dagger(t-\tau_2))(a_3^\dagger(t'-\tau_3) - e^{i\phi_{34}} a_3^\dagger(t'-\tau_4))|0\rangle \tag{5}$$

As stated earlier, the long short and short long coincidences are filtered out using postselection of events. Thus, the two-photon events in which one photon propagated along path 1 and the other photon propagated along path 4 or the events along paths 2 and 3 are filtered out. The wavefunction can then be further simplified

$$|\Psi_2\rangle = \frac{1}{4}\int\int dt_1 dt_1' A(t,t')(a_1^\dagger(t-\tau_1) a_3^\dagger(t'-\tau_3) + e^{i(\phi_{12}+\phi_{34})} a_1^\dagger(t-\tau_2) a_3^\dagger(t'-\tau_4)|0\rangle \tag{6}$$

The physical intuition behind this wavefunction can be understood in terms of fourth order quantum interference. Essentially, this wavefunction states that there is interference between the two amplitudes that travel along the short paths of the interferometer with the amplitudes that travel along the long paths of the interferometer. Since the pump coherence length is much longer than the imbalance in the interferometers, the pump photon is destroyed at an infinite number of different space-time points and two of those events *can* be made indistinguishable in the interferometer.

The fourth order coherence between the two detectors can be mathematically represented by

$$P_{1,3} \propto \langle \Psi_2 | a_1^\dagger(t_1) a_2^\dagger(t_1-\tau) a_2(t_1-\tau) a_1(t_1) | \Psi_2 \rangle \tag{7}$$

where the narrow bandwidth approximation has been assumed (i.e., the bandwidth of the downconversion is much smaller than the radial frequency of the light). An observation of this coherence function shows that there are four terms of eight operators. The effort is to remove the operators from the function by normally ordering the operators. When normally ordered $a|0\rangle = 0$ which means that the operators can be removed from the calculation. The ordering is achieved using the equal time boson commutator relation $[a(t), a^\dagger(t')] = \delta(t-t')$. After a somewhat lengthy, but straightforward calculation, the fourth order coherence is found to be

$$\begin{aligned} P_{1,3} \;\propto\; & |A(\tau_1-\tau_3-\tau)|^2 + |A(\tau_2-\tau_4-\tau)|^2 + \\ +\; & e^{i(\phi_{12}+\phi_{34})} A^*(\tau_1-\tau_3-\tau)A(\tau_2-\tau_4-\tau) + e^{-i(\phi_{12}+\phi_{34})} A^*(\tau_2-\tau_4-\tau)A(\tau_1-\tau_3-\tau) \end{aligned} \tag{8}$$

Now, we integrate over the detector response time τ. Since the detector response time is typically much larger than the correlation time, the limits of integration can be assumed to be taken over all time. Also, without loss of generality, but in an effort to simplify the mathematics as well as the physics, we define two relative time widths. The first time width $\Delta\tau = \frac{(\tau_1-\tau_3)+(\tau_2-\tau_4)}{2}$ and $\delta\tau = \frac{(\tau_1-\tau_3)-(\tau_2-\tau_4)}{2}$. Lastly, we will assume that the correlation function is real. Making these assumptions, the two-photon detection rate is then

$$R_{1,3} \propto 1 + \cos\phi \left(\frac{\int d\tau A(\Delta\tau-\tau-\delta\tau)A(\Delta\tau-\tau+\delta\tau)}{\int d\tau |A(\Delta\tau-\tau)|^2} \right) \tag{9}$$

where $\phi = \phi_{12}+\phi_{34}$. This rate function is identical to the HOM fourth order cross coherence function with the exception of the $\cos\phi$ term. Thus, the two-photon envelope function of the Franson interferometer can be used just as the HOM interference pattern, to determine the relative time-time correlations of the photons from downconversion even though the detector response times are several orders of magnitude less sensitive to time measurements.

The rate equation in eqn. 9 is a general statement about the relative time correlations of photon emissions. For downconversion, the temporal correlation function is primarily attributed to the gaussian spectral passbands of interference filters. We therefore find $A(t,t') = e^{-(t-t')^2\Delta^2/2}$, where Δ is the spectral bandwidth of the filters. The rate equation

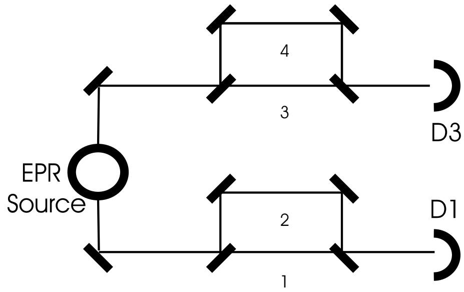

FIG. 1: Fig 1. A Franson interferometer. Two photons travel in opposite directions from an EPR source. Each photon then enters an imbalanced Mach-Zehnder interferometer, such that there is no single photon interference. The temporal pathlengths through the arms of the interferometers are denoted τ_i. Detectors in paths 1 and 3 measure the outputs of the interferometers and the coincidences between the detectors are recorded.

is then simplified:

$$R_{1,3} = 1 + \cos\phi e^{-\delta\tau^2\Delta^2}. \tag{10}$$

Hence, when $\delta\tau = 0$ or in other words, the difference in path lengths of the two Mach-Zehnder interferometers is the same, 100% visibility fringes will result as has been demonstrated in numerous experiments. When $\delta\tau >> 1/\Delta$ there will be no fringes. It should be noted that this function is identical to the HOM degree of second order coherence with only the addition of the $\cos\phi$ term. Thus, instead of the "HOM dip", there is a "Franson" envelope.

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