PHYSICAL REVIEW A **70**, 010303(R) (2004)

Hong-Ou-Mandel cloning: Quantum copying without an ancilla

Irfan Ali Khan and John C. Howell
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627, USA
(Received 24 June 2003; revised manuscript received 6 January 2004; published 19 July 2004)

In this paper we report an experimental realization of an ancilla-free $1 \rightarrow 2$ phase-covariant quantum cloner. The cloner is realized by interfering a linearly polarized photon, which we wish to clone with a circularly polarized photon at a beam splitter. The two-photon effect can be understood in light of Hong-Ou-Mandel interference. The fidelity of the cloner was measured as 0.829 ± 0.008 for the 0/90 basis and 0.835 ± 0.006 for the 45/135 basis, which is in good agreement with the theoretical prediction of 5/6 fidelity. The experimental scheme is straightforward and has a high cloning success rate.

DOI: 10.1103/PhysRevA.70.010303 PACS number(s): 03.67.−a, 42.50.Dv

In 1982 Wooters and Zurek [1] proposed the "no cloning" theorem in order to solve the interesting and searching question of superluminal communication posed by Herbert [2] earlier that same year. The field of quantum cloning has since experienced immense interest and growth, owing mostly to the fact that the theorem should be more accurately labelled the "no-perfect-cloning" theorem. The inability to perfectly clone initiated the development of quantum cryptography [3–6], and it has even been shown to be useful in quantum computing [7]. Theoretical aspects of quantum cloning have been studied for some time now; first in discrete [8–11], then recently in continuous variable [12,13] quantum systems. Success in experimental aspects and realizations of cloning, however, have only come relatively recently [14–17].

Buzek and Hillery [9] proposed the first theoretical model of a universal quantum cloning machine (UQCM). In their original work, the universal quantum cloning machine takes a two-state particle in any arbitrary, unknown state and copies all possible states equally well. In other words, the fidelity (a measure of the quality of the copying procedure) is independent of the unknown input state. The best possible copying, or optimal fidelity, was derived by Buzek and Hillery to be 5/6 for a 1 to 2 copying procedure. This was later derived via constructive proof by Bruss *et al.* [18].

In pursuing the universal cloner, Buzek and Hillery discovered that it was necessary to have an additional "ancilla bit" in order to realize the universal cloner. This ancilla bit is sometimes referred to as the "machine" state or anticlone in the literature. However, in many instances, such as optimal eavesdropping on a Bennett-Brassard 1984 protocol (BB84) cryptochannel, it is only necessary to clone arbitrary *linearly* polarized states instead of any possible polarized states (e.g., any elliptically polarized state). Restricting the cloning to only linearly polarized states dramatically simplifies the cloning requirements. As we show experimentally, cloning only linearly polarized photons does not require an ancilla bit. This type of restrictive cloning is referred to as phase covariant cloning [19–25]. On a more formal level, phase covariant cloning is the study of restricted copying in which the symmetric cloning only occurs on a great circle of the Bloch sphere. In this work, the great circle is the linear polarization equator of the Bloch sphere. Further, the cloner presented here is nonperturbative, which ultimately means that the cloning procedure will occur with much higher repetition rate than in previous experiments.

In this Rapid Communication, the phase covariant cloner is achieved by interfering a linearly polarized photon we wish to clone with a circularly polarized photon at a beam splitter (as seen in Fig. 1). From a practical point of view, a circularly polarized photon has a 50% chance of being transmitted through (or absorbed in) a polarizer *regardless of the orientation of the polarizer*. This means that there is no preferred orientation of a circularly polarized photon in a linearly polarized basis. With this simple fact, interfering a linearly polarized photon with a circularly polarized photon then causes a "stimulated" two photon effect when both photons are measured with the same linear polarization orientation and in the same spatio-temporal mode and a "noise"-like term when the photons are measured with orthogonal linear polarizations. The cloner interference can be understood in light of the famous Hong-Ou-Mandel two-photon interference effect [26]. Interestingly, no additional ancilla photon is needed to achieve the same fidelity as the UQCM for the phase-covariant conditions of this cloner.

Recall that if two bosons are made indistinguishable in every quantum variable, they occupy the same quantum state and get a corresponding boson mode occupation enhancement. Therefore, if two photons (which are bosons) are made to be spatially and temporally indistinguishable at a beam splitter while having the same spectral and polarization orientation characteristics, they must both leave the same output port of the beam splitter. This two-photon behavior has been labelled Hong-Ou-Mandel interference [26] and has played a

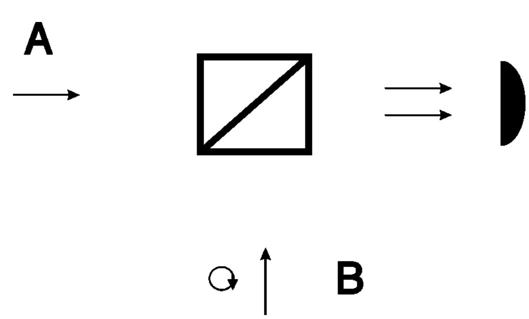

FIG. 1. A linearly polarized photon in mode A interferes with a circularly polarized photon in mode B at a beam splitter. Phase-covariant cloning occurs when both photons are measured in the same output port of the beam splitter.

vital role in many remarkable experiments in quantum information such as teleportation [27], and dense coding [28]. However, the two photons will not interfere if they have orthogonal polarizations, even if all the other characteristics are the same (e.g., spectral, spatial, and temporal) because they are distinguishable. Therefore, the two photons will behave independently at the beam splitter.

Consider the outcome when a circularly polarized photon interferes with a linearly polarized photon. For the moment, we are only interested in one output port of the 50-50 beam splitter. Also, we will assume that the two photons are distinguishable in some quantum variable such as temporal mode overlap. In other words, the photons arrive at the beam splitter at different times and are therefore distinguishable. Lastly, assume that the linearly polarized photon in mode A is horizontally polarized and that the photons will be measured in the horizontal–vertical basis. The wave function for the linearly polarized photon in the output port is $|\Psi\rangle_1 = |1,0\rangle_1$, where the subscript 1 labels the linearly polarized photon and the ket $|1,0\rangle_1$ denotes that there is one horizontally polarized photon and zero vertically polarized photons. As a note, we will not worry about the normalization of the wave function. The wave function in the same output port for the circularly polarized photon is given by $|\Psi\rangle_2 = |1,0\rangle_2 + i|0,1\rangle_2$. Without worrying about normalization, the circularly polarized wave function denotes that the circularly polarized photon can be decomposed into an equal amplitude superposition with both a horizontal and vertical component. The two-photon wave function is then given by the tensor product of the two individual wave functions

$$|\Psi\rangle = |\Psi\rangle_1 \otimes |\Psi\rangle_2 = |1,0\rangle_1 \otimes (|1,0\rangle_2 + i|0,1\rangle_2). \quad (1)$$

As it is, this two-photon wave function is not very interesting. However, if one applies a quantum eraser to erase any distinguishable space-time information between the two photons in the same output port, the two-photon wave function then becomes

$$|\Psi\rangle = \sqrt{2}|2,0\rangle + i|1,1\rangle, \quad (2)$$

where it should be noted that the $\sqrt{2}$ is now in front of the first ket. This factor is a result of the boson mode enhancement and leads to the stimulated enhancement needed for cloning. The other ket is the noise term which has an analog to spontaneous emission in an orthogonal mode of a linear amplifier. If on the other hand, the linearly polarized photon (to be cloned) is vertically polarized, the two-photon wave function is given by

$$|\Psi\rangle = |1,1\rangle + i\sqrt{2}|0,2\rangle. \quad (3)$$

Using the sum-frequency technique proposed by Simon *et al.* [14,29] the fidelity is computed by adding all the contributions with the same polarization as the incoming linearly polarized photon and dividing by all the contributions. From the wavefunction it can be seen that there is twice the probability of measuring both photons in the same polarization mode as to measure one photon in each polarization mode. The fidelity is then computed to be

$$F = \frac{2 \times 2 + 1 \times 1}{2 \times 2 + 1 \times 2} = \frac{5}{6}, \quad (4)$$

which is the same as the optimal universal cloning fidelity.

As asserted, the cloning should be independent of the linear polarization of the incoming photon. Suppose the incoming linearly polarized photon is horizontal in a *new* primed basis (a basis which can be achieved by rotating the linear analyzers). The wave function is written as $|\Psi\rangle'_1 = |1,0\rangle'_1$, where the $'$ denotes that the wave function is written in the primed basis. The important aspect of this phase-covariant cloner is that the circularly polarized photon can be written as $|\Psi\rangle_2 = |1,0\rangle'_2 + i|0,1\rangle'_2$ in the new basis. Thus, applying the quantum eraser, the two-photon wave function in the primed basis is given by

$$\langle\Psi\rangle = \sqrt{2}|2,0\rangle' + i|1,1\rangle', \quad (5)$$

which yields the same cloning fidelity as the unprimed basis. We arrive at the very important conclusion that the cloning is independent of the linear polarization basis.

A more careful analysis of the input-output relations of the beam splitter reveal that ideally the probability that both photons will be measured in the same output port is 75%. It should be kept in mind that due to symmetry cloning occurs with equal probability in both exit ports of the beam splitter. This statistically means $3/4$ of the time a cloning event will occur if a circularly polarized photon enters one input port at the same time that a linearly polarized photon enters the other input port of a 50-50 beam splitter. Thus, two-photon postselection is needed to observe the cloning. Ideally, this implies that single photons on demand can be cloned with single photons on demand with high success probability. This very high success rate can be contrasted with stimulated emission in a crystal, where the perturbative (meaning that there is a small probability that an entangled pair will be created when a signal photon enters the crystal) three-photon postselection success is very low. However, this latter system could be improved dramatically if one and only one pair of entangled photons can be created on demand.

We report on an experimental demonstration of the phase covariant cloner using collinear type-II parametric down-conversion (a schematic of the experiment is shown in Fig. 2). The spontaneously emitted pair of photons, having orthogonal polarization, are separated at a polarizing beam splitter. The signal photon (the linearly polarized photon) is rotated into its linear polarization state using a half wave plate. The cloning photon (the circularly polarized photon) is made circularly polarized by a quarter wave plate. They are then made to recombine at a 50-50 beam splitter.

The photons were generated by using a 390 nm laser (Toptica TA 100 DL series 780 nm source driving the Toptica series SG100 frequency doubling system) to pump a 2 mm BBO crystal. The down–converted photons centered at 780 nm were then separated out from the 390 nm pump using a UV grade fused-silica prism. Interference filters of 10 nm bandwidth are used to increase the coherence length of the downconverted photons to approximately 60 microns, and to reduce background noise.

Owing to the symmetry of the 50-50 beam splitter, mea-

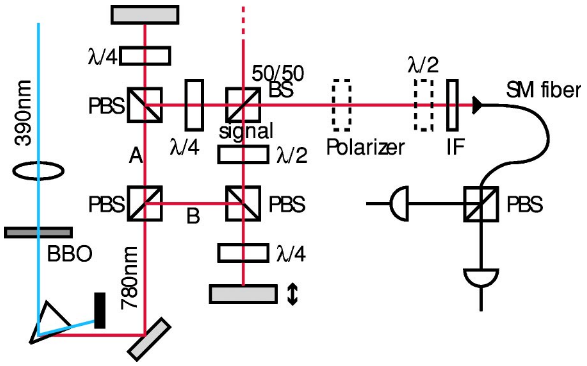

FIG. 2. Schematic of the experimental setup. The down–converted photons are separated at the first PBS. The photon from arm A is circularly polarized when it is incident on the 50-50 beam splitter. HOM interference occurs when the path lengths of arms A and B are matched, leading to cloning of the linearly polarized photon from arm B after postselection.

surements were made in only one output port. The first experiment was to insert a horizontally polarized photon to be cloned. The two photon measurements are then horizontal-horizontal (H-H) or horizontal-vertical (H-V). As can be seen from the results in Fig. 3 we observe an enhancement in the H-H polarized pairs when the path lengths are matched without affecting the H-V pairs. The results were performed in two different nonorthogonal bases to confirm that the cloner works equally well in any linear basis. For any one basis, we ideally expect the measured H-H coincidences to be twice those of the H-V coincidences. However, long term laser instability affected count rates in between measurement runs. Even with this in mind, the qualitative information presented in the two measurements is critical. Using a theoretical Gaussian fit for the H-H correlations the coherence length was estimated to be 75 microns, in good agreement with our initial expectations.

We now calculate the fidelity of the cloning from the peak and base values of the H-H coincidences. Examining the statistics at the peak of the H-H coincidences, we get

$$F = \frac{2R_{H\text{-}H} + R_{H\text{-}V}}{2R_{H\text{-}H} + 2R_{H\text{-}V}}, \tag{6}$$

where $R_{H\text{-}H}$ is the rate of H-H coincidences, and $R_{H\text{-}V}$ is the rate of H-V coincidences. Using this we get a fidelity of 0.829 ± 0.008 for the 0/90 basis and 0.835 ± 0.006 for the 45/135 basis, which is in good agreement with the 5/6 fidelity as predicted earlier in this paper. It should be noted that to obtain the fidelity in Eq. (6), the data was normalized by making the baselines equal. Owing to our inability to measure two identical photons (temporal, spatial, spectral, and polarization), beam splitter cascading [30] was required. This leads to a lower baseline for the H-H coincidences than the H-V coincidences. The baselines would have been the same with a photon number resolving detector.

The HOM cloner represents a significant advance in cloning success rate. For example, in the demonstration of the UQCM via stimulated emission by Lamas-Linares *et al.* [16] the cloning success rate was approximately 10^{-5}. In the the UQCM experiment, two major problems limited the success rate. While the cloner we have reported here has a high cloning success rate, there are still technological issues of concern. First, the collection and detection efficiency of the photons is still quite low ($\approx 10\%$). This could be greatly improved with more efficient detectors and improved collection efficiency optics. One can envision a fiber based source of single photons on demand, which would dramatically improve the collection efficiency.

While it is unlikely that a HOM cloner will be a standard tool in the quantum key distribution eavesdropping, it does point out a potential weakness of only using two nonorthogonal bases for key distribution. As Bruss showed, three mutually unbiased bases provide additional security for which the HOM cloner is not symmetric [31]. Further, one can think of a myriad of ways to thwart any cloning machine as an eavesdropping tool such as creating spectral or temporal jitter to the signal photons.

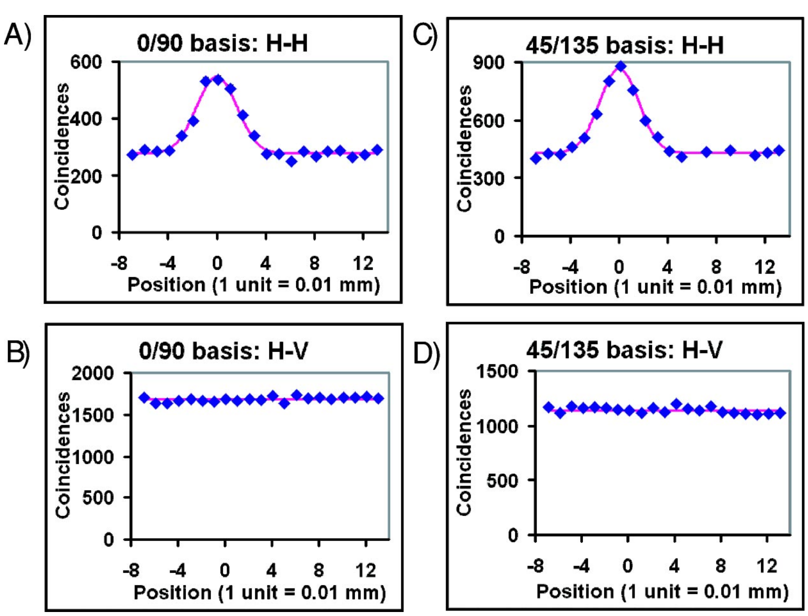

FIG. 3. The figures show the results of measuring same port coincidences with respect to path length mismatch for the four possible polarization combinations. The *position* axes are only for scale, and have a systematic offset of ~0.08 mm. Boson mode enhancement only occurs in A and C where both photons have the same polarizations, thus leading to the cloning effect.

Lastly, the fidelity of the HOM cloner is $5/6=83.33\%$, which is slightly smaller than the optimal predicted fidelity of a phase covariant cloner of 85.4%. For example, Fiurasek recently proposed an all-optical optimal cloner [22]. However, the experimental complexity is much greater and the maximum success probability of a cloning event is only $1/3$. Thus, in our experiment fidelity is sacrificed at the expense of higher cloning success rate and experimental simplicity.

We have demonstrated the first experimental ancilla-free phase-covariant quantum cloner by restricting the cloning to the linearly polarized photons (equator of the Bloch sphere). The experimental results of the HOM cloner agree well with theoretical predictions. Interestingly, the cloning device uses only linear optics and interference. All previous demonstrations have used seeded amplifiers (weak optical parametric amplifier [16] or a fiber amplifier [17]) to demonstrate the cloning effect.

We would like to thank V. Buzek and Mark Hillery for helpful comments and several readings of various manuscripts. J.C.H. gratefully acknowledges support from the NSF, the Research Corporation, and the University of Rochester.

[1] W.K. Wootters and W.H. Zurek, Nature (London) **299**, 802 (1982).
[2] N. Herbert, Found. Phys. **12**, 1171 (1982).
[3] C.H. Bennett and G. Brassard, in *Proceedings of the IEEE International Conference on Computers, Systems and Signal Processing, Bangalore, India* (IEEE, New York, 1984), p. 175.
[4] C.H. Bennett, Phys. Rev. Lett. **68**, 3121 (1992).
[5] D. Bruss and C. Macchiavello, Phys. Rev. Lett. **88**, 127901 (2002).
[6] N.J. Cerf *et al.*, Phys. Rev. Lett. **88**, 127902 (2002).
[7] E.F. Galvao and Lucien Hardy, Phys. Rev. A **62**, 022301 (2000).
[8] L. Mandel, Nature (London) **304**, 188 (1983).
[9] V. Buzek and M. Hillery, Phys. Rev. A **54**, 1844 (1996).
[10] N. Gisin and S. Massar, Phys. Rev. Lett. **79**, 2153 (1997).
[11] R. Werner, Phys. Rev. A **58**, 1827 (1998).
[12] N.J. Cerf, A. Ipe, and X. Rottenberg, Phys. Rev. Lett. **85**, 1754 (2000).
[13] S.L. Braunstein *et al.*, Phys. Rev. Lett. **86**, 4938 (2001).
[14] C. Simon, G. Weihs, and A. Zeilinger, Phys. Rev. Lett. **84**, 2993 (1999).
[15] F. De Martini *et al.*, Opt. Commun. **179**, 581 (2000).
[16] A. Lamas-Linares *et al.*, Science **296**, 712 (2002).
[17] S. Fasel *et al.*, Phys. Rev. Lett. **89**, 107901 (2002).
[18] D. Bruss, A. Ekert, and C. Macchiavello, Phys. Rev. Lett. **81**, 2598 (1998).
[19] D. Bruss, D. P. DiVincenzo, A. Ekert, C. A. Fuchs, C. Macchiavello, and J. A. Smolin, Phys. Rev. A **57**, 2368 (1998).
[20] D. Bruss, M. Cinchetti, G. M. D'Ariano, and C. Macchiavello, Phys. Rev. A **62**, 012302 (2000).
[21] V. Karimipour and A. T. Rezakhani, Phys. Rev. A **66**, 052111 (2002).
[22] J. Fiurášek, Phys. Rev. A **67**, 052314 (2003).
[23] L.-M. Duan and G.-C. Guo, Phys. Rev. Lett. **80**, 4999 (1998).
[24] H. Fan, K. Matsumoto, X.-B. Wang, and M. Wadati, Phys. Rev. A **65**, 012304 (2002).
[25] G. M. D'Ariano and C. Macchiavello, Phys. Rev. A **67**, 042306 (2003).
[26] C. K. Hong, Z. Y. Ou, and L. Mandel, Phys. Rev. Lett. **59**, 2044 (1987).
[27] D. Bouwmeester *et al.*, Nature (London) **390**, 575 (1997).
[28] K. Mattle, H. Weinfurter, P. G. Kwiat, and A. Zeilinger, Phys. Rev. Lett. **76**, 4656 (1996).
[29] I. Ali Khan and J.C. Howell, Quantum Inf. Comput. **4**, 114 (2004).
[30] J.C. Howell, A. Lamas-Linares, and D. Bouwmeester, Phys. Rev. Lett. **88**, 030401 (2002).
[31] D. Bruss, Phys. Rev. Lett. **81**, 3018 (1998)