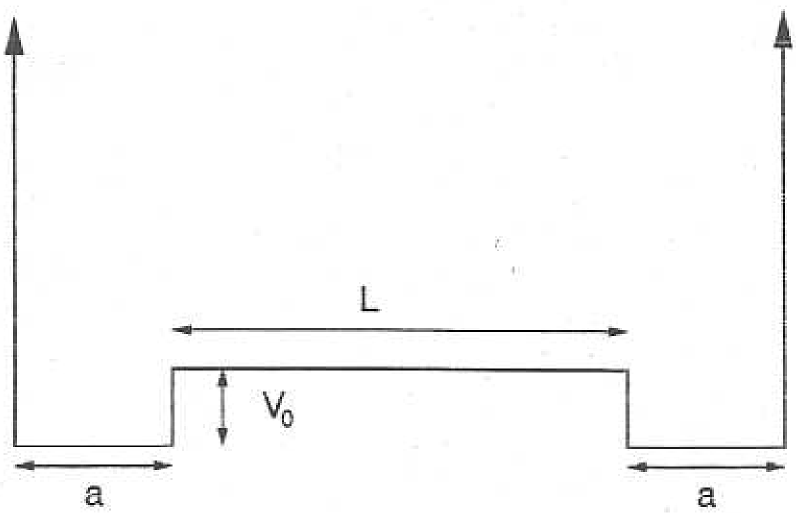
Problem 11

A particle of mass m is confined in a double-well potential as shown below where $a > 0$. For what finite and non-zero values of V_0 is the ground state energy of the system independent of L?



Problem 12

Consider a two dimensional simple harmonic oscillator of mass M and angular frequency ω. Construct a state $|\psi\rangle$ for which $\langle\psi|H|\psi\rangle = 3/2\,\hbar\omega$. Determine the allowed range of $\langle\psi|L|\psi\rangle$ (where L is the angular momentum) for this state.

Problem 13

The Hamiltonian of a certain spin-one system can be brought to the form

$$H = \alpha J_x^2 + \beta J_y^2 + \gamma J_z^2$$

where α, β, γ are real constants and the operators J_i denote the Cartesian components of the angular momentum.

a) Explain why the set of energy eigenvalues is unaltered under exchange of any two of the three parameters α, β, and γ.

b) Determine the energy eigenvalues of the system.

Problem 14

Confined in one-dimension, a particle of mass m moves in the force field of an infinite square well potential $V(x)$,

$$V(x) = \begin{cases} 0 & |x| < a \\ \infty & |x| > a \,. \end{cases}$$

The system is in its ground state for $t<0$. At $t=0$, a force $F(t)$ is applied along the positive x-direction where

$$F(t) = \begin{cases} F_0 & 0 < t < \tau \\ 0 & t > \tau \,. \end{cases}$$

To order F_0^2 what is the probability of finding the system in the state n^{th} excited state for $t > \tau$?

Problem 15

Consider three spinless particles A, B, C. Let the mass of A be M, the mass of B be $M-m$, and the mass of C be $M-2m$, where $m << M$. Particle A can decay under the action of a perturbing Hamiltonian H' by either $A \to B+\gamma$ or $A \to C+\gamma$, where γ represents a single photon. If $<A|H'|B>=<A|H'|C>$, find the relative probability of the two decays.

Problem 16

Consider atoms A that can bind together to form a diatomic molecule A_2

$$2A \leftrightarrow A_2.$$

The binding energy of the molecule is Δ. Assume that the atoms and the diatomic molecules can be treated as ideal, indistinguishable, classical point particles (i.e., ignore any rotational, vibrational, or electronic excitations). Suppose that there are initially N atoms A and no molecules A_2 confined to a cubic box of volume V. What will be the ratio of the number of atoms A to the number of molecules A_2 when the system is in equilibrium at a temperature T?

Problem 17

Consider a point particle of mass m attached to a harmonic spring with spring constant $k = m\omega_0^2$. The system is in equilibrium at a temperature T.

a) Assume that the particle behaves classically. What is the root-mean-squared fluctuation of the particle about its average position?

b) Assume that the particle must be treated quantum mechanically. What now is the root-mean-squared fluctuation of the particle about its average position?

c) Show that your answer in b) reduces to your answer in a) in the appropriate limit.

Problem 18

One kilomole of ^{4}He is confined to a cubic container of size $\ell = 30$ cm.

a) In which temperature range are quantum effects important for thermodynamic properties of this system?

b) Assuming the ^{4}He gas to be ideal, estimate its Bose-Einstein condensation temperature.

c) How many of the atoms are in the condensate at the temperature of 1K?

You may use:

Avogadro's number is $6.022x10^{23}$
Mass of ^{4}He atom: $m = 6.65x10^{-27}\,kg$

Boltzmann const. $k_B = 1.381x10^{-23}\,J \cdot K^{-1}$
Planck const. $h = 6.626x10^{-34}\,\mathrm{J \cdot s}$

$$\frac{2}{\sqrt{\pi}}\int_0^{\infty}\frac{x^{1/2}dx}{e^x - 1} = \zeta(3/2) = 2.612$$