We propose precision measurements of ultra-small angular velocities of a mirror within a modified Sagnac interferometer, where the counter-propagating beams are spatially separated, using the recently proposed technique of almost-balanced weak values amplification (ABWV) [Phys. Rev. Lett. 116, 100803 (2016)]. The separation between the two beams provides additional amplification with respect to using collinear beams in a Sagnac interferometer. Within the same setup, the weak-value amplification technique is also performed for comparison. Much higher amplification factors can be obtained using the almost-balanced weak values technique, with the best one achieved in our experiments being as high as $1.2 \times 10^7$. In addition, the amplification factor monotonically increases with decreasing of the post-selection phase for the ABWV case in our experiments, which is not the case for weak-value amplification (WVA) at small post-selection phases. Both techniques consist of measuring the angular velocity. The sensitivity of the ABWV technique is $\sim 38 \text{ nrad/s per averaged pulse}$ for a repetition rate of 1 Hz and $\sim 33 \text{ nrad/s per averaged pulse}$ for the WVA technique.

Anomalous amplification of a homodyne signal via almost-balanced weak values

WEI-TAO LIU,1,2,3,* JULIÁN MARTÍNEZ-RINCÓN,1 GERARDO I. VIZA,1 AND JOHN C. HOWELL1,4,5

1Department of Physics and Astronomy & Center for Coherence and Quantum Optics, University of Rochester, Rochester, New York 14627, USA
2College of Science, National University of Defense Technology, Changsha 410073, China
3Interdisciplinary Center of Quantum Information, National University of Defense Technology, Changsha, 410073, China
4Institute of Optics, University of Rochester, Rochester, New York 14627, USA
5Institute for Quantum Studies, Chapman University, Orange, California 92866, USA

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been done to show how WVA works using a Sagnac interferometer [13–15] and paying no attention to the rotation velocity of the mirror. Regarding technical noise, efforts to clarify the advantages of the WVA [8, 9] compared to the standard methods are made. At the same time, increasing the efficiency of the WVA technique with the help of entanglement [26] or power recycling is also discussed [27, 28]. However, in those experiments, one of the outputs is spatially overlapped with the input laser, while the ABWV requests to detect both outputs. To achieve this, a modified Sagnac interferometer is employed where the counter-propagating beams in the interferometer are spatially separated. Therefore, the outputs are spatially separated. Therefore, the outputs are detected and the ABWV amplification can be performed.

The counter-propagating beams are spatially separated, introducing a phase shift due to the tilt of the rotating mirror. Both ports are tracked and by blocking or unblocking one of them, measurements using the WVA or ABWV technique can be performed.

A piezoelectric actuator (Thorlabs PE4) is adapted to one of the mirrors to induce the time-dependent tilt, \((\phi + \omega_0 t)\), in the horizontal direction. The origin of \(\phi = 0\) for WVA is determined by the mirror’s position where destructive interference occurs and where the two outputs are entirely balanced for the case of ABWV. The angle \(\phi\) controls the peak output intensity on both ports of the interferometer, and the constant angular velocity \(\omega_0\) is the parameter to be estimated during the experiments. The tilt induced by the laser pulse and the ABWV is defined by the two arms of the interferometer where \(k_0 = 2\pi/\lambda\) and \(L = L_1 \cos \theta_1 - L_2 \cos \theta_2\) with labels 1 and 2 referring to the counter-propagating beams. \(L_s\) is the distance from the pivot in the mirror mount to the place where the 7th beam hits the mirror, and \(\theta_s\) is the incident angle of the 7th beam with respect to the mirror’s normal. For example, if the interferometer was a perfect 45°, 45°, 90° triangle (which it was not), these angles would be \(\theta_1 \approx \theta_2 \approx 22.5°, \) and \(L \approx L \cos (22.5°)\), where \(L = L_1 - L_2\) is the distance between both beams on the surface of the piezo-driven mirror. The effective value of \(L = 5.64 \pm 0.03\) mm was independently measured by applying a 300 mV peak-to-peak 1-Hz signal on the piezo actuator and using the known piezo response of \(a = 3.12 \text{ mrad/V} (\text{Note that the time shift from Eq. (3) can be written as } \Delta t = V_{pp} f_r^3/3 V_0\) and the post-selection phase as \(2k_0 L_0 V_0\), where \(V_0\) is the voltage applied in the piezo giving a fixed value \(\phi_0\), i.e., \(\phi = \phi_0 V_0.\) We measured the time shift and the post-selection phase for 480 pulses from which the average quantity \(\Delta t = (1.759 \pm 0.004) \times 10^{-8}\) m/V was estimated. During the experiment, the angle \(\phi\) was set such that \(\omega_0 \tau \ll \phi\) and \(2k_0 L_0 \phi_0 \ll 1\) satisfy the weak-value approximation. A 60% duty-cycle triangle ramp with peak-to-peak voltage \(V_{pp}\) and frequency \(f_r\) was applied to the piezo actuator, so the angular velocity of the mirror during the positive ramp takes the form of \(\omega_0 = 5aV_{pp} f_r^3/3.\)

The output beams of both ports of the interferometer were directed to two of the four detectors in a quadrant cell photodetector (Newport 2921), which outputs signals equivalent to the sum and difference intensities. An additional fixed tilt of the mirror (M1) is introduced to roughly set the value of \(\phi\) while monitoring the intensity distribution of the outputs. By blocking or unblocking one of the two optical ports and controlling the angle \(\phi\), the system resembled either the WVA or the ABWV technique. The difference and sum electrical signals were sent to two, connected-in-series, low-noise voltage preamplifiers (Standard Research Systems SR560) before being recorded using an oscilloscope and a computer. These preamplifiers were set as 12 dB/oct rolloff low-pass filters at 30 Hz. The experiments were performed at 1 Hz with no frequency filtering close to the working frequency.

According to the theory of ABWV amplification [24], the sum and difference signals are given by

\[
I_+ (t) = I_0 e^{-(\omega_0 t)^2/2\sigma^2},
\]

\[
I_- (t) = \frac{I_0}{4} \left[ |1 + e^{i[2k_0 L_0 \phi(\phi + \omega_0 t) + \Delta t]}|^2 - |1 - e^{i[2k_0 L_0 \phi(\phi + \omega_0 t) + \Delta t]}|^2 \right],
\]

\[
\approx I_0 \sin(2k_0 L_0 \phi) e^{-(\omega_0 t^2/2\sigma^2)}.
\]
That is, the differencing signal shows a time shift of $\omega_0^2/\phi$ with respect to the sum signal, including an amplification factor of $1/\phi$.

For the case of WVA, the intensity at the dark port is

$$I_{WVA} = I_0 \sin^2(k_0 L \phi) e^{-(\omega_0^2 / \phi^2)/2\tau^2}, \quad (4)$$

where there is a time shift of $2\omega_0^2/\phi$ compared to the input signal. Here $\sin^2(k_0 L \phi)$ is the probability of post-selection, i.e., only $N \sin^2(k_0 L \phi)$ photons are detected in the dark port for WVA, with $N$ being the number of input photons to the interferometer.

The outputs of the preamplifiers are then measured on an oscilloscope for data collection. Data from 60-s sets (each containing 60 pulses) from the oscilloscope are recorded on the computer for post-processing. Each collection window consists of three million points, giving a time resolution of 20 µs. Each collected pulse is numerically fit to a Gaussian distribution such that peak values, characteristic lengths of pulses $\tau$, and time shifts are obtained. The results of time shifts are shown in Fig. 2 where each point shows the result of statistics over a set of 60 pulses. Although $\tau$ can be characterized according to the AOM modulation signal, we estimate $\tau$ from the detected results to show that ABWV does not need prior knowledge of the input states and it is free of possible systematic errors in preparing input states.

Comparing Eqs. (3) and (4), we notice that the ABWV technique provides higher detectable intensities, since the quadratic form in WVA, $\sin^2(k_0 L \phi)$, is replaced with a larger linear response $\sin(2k_0 L \phi) \sim 2k_0 L \phi$. For ABWV, the sum and difference signals are both analyzed. By fitting the measured intensities to Eqs. (1) and (3), the phase $2k_0 L \phi$ and the time shift can be determined. The measurement results for $\omega_0$ can then be extracted. For WVA, prior information about the input is necessary to obtain the post-selection phase and the time shift, which can be obtained by setting the phase to constructive interference and detecting the bright port separately. Results from both techniques are shown in Fig. 3. Similar accuracy is observed within each technique’s observed well-behaved intervals. These intervals are $\phi \in [83 \text{ nrad}, 2.5 \text{ µrad}]$ for ABWV and $\phi \in [4 \text{ µrad}, 9 \text{ µrad}]$ for WVA.

From the results, with ABWV we can perform measurements with smaller angles $\phi$ than with WVA by more than one order of magnitude. This allows one to achieve larger amplification factors—thus, larger time shifts (Fig. 2). Also, the ABWV technique does not show the undesired reversed tendency of WVA for the time shift at small angles $\phi$, which also allows for larger amplification. The WVA technique breaks down at around $\phi \sim 4 \text{ µrad}$, while ABWV behaves well down to $\phi \sim 83 \text{ nrad}$. For WVA, the strong falling off behavior is mainly caused by imperfections of optics [29]. These imperfections can be eliminated by using the homodyne-like differencing response. For the ABWV technique, the two outputs are set almost balanced, which combines the advantages of both techniques: elimination of common mode noise because of homodyne detection and amplification inversely proportional to the post-selection angle as in the WVA technique. At the same time, intensity of possible unexpected background noise induced by imperfect optics is much smaller than the intensity of each output. Therefore, the relative error (intensity of background noise over detected intensity) for the ABWV case is much smaller than that of WVA. In this situation, ABWV is less sensitive to imperfections of the interferometer than WVA.

It should be noted that the largest amplification factor achieved in our experiments is $1/83 \text{ nrad} \sim 1.2 \times 10^7$. As a contrast, the amplification factors are usually at the level of $10^2$ for the WVA technique using a regular Sagnac interferometer, with the best post-selection phases being around tens of mrad [9,29]. For weak-value experiments, the amplification factor is usually equal to the reciprocal of the post-selection phase. In our case, from Eq. (3), the post-selection phase is $2k_0 L \phi$, which is $\sim 7 \text{ mrad}$ with $\phi = 83 \text{ nrad}$. However, the amplification factor is $1/\phi$ instead of the reciprocal of the post-selection phase. A significant enhancement in the amplification factor, by a factor of $2k_0 L \sim 9 \times 10^4$, is induced using the modified Sagnac interferometer. The rotation angle of the mirror is transferred into the phase difference between two
beams being magnified at the same time because of the spatial separation of two beams.

From the results shown in Fig. 3, for our ABWV case, the mean value of measured $\omega_0$ is 164 nrad/s. The sensitivity we obtained is $\sim 4.9$ nrad/s after averaging over 60 pulses with a time-constant of $\tau = 85.45$ ms at a repetition rate of $f_r = 1$ Hz. Therefore, the sensitivity of our experiments is $\sim 38$ nrad/s per averaged pulse. For the WVA case, the mean value of measured $\omega_0$ is 163 nrad/s for the well-behaved region with the sensitivity being $\sim 33$ nrad/s per averaged pulse. This shows that the ABWV technique provides similar measurement sensitivity with WVA. The difference between measured $\omega_0$ and the expected value is mainly caused by the response of the piezo and the piezo driver. The expected value is estimated from the manual, while the actual response of piezo during the experiment is different from the estimated response from the manual. We also observed a slight drift over time in the output of the piezo driver. This could be fixed using a better driver and careful calibrations. It should be noted that both techniques show similar accuracy in their respective regions. At the same time, it can be inferred that the dynamic range of ABWV is limited. It is mainly caused by the approximation we used to obtain Eq. (3) assuming $2k_{0}/L \phi \ll 1$. The dynamic range of most WVA experiments is also limited since a similar approximation is required. In practice, we are usually interested in measuring a small signal in a very precise way; the ABWV technique works well since it offers a higher amplification factor when $\phi$ is smaller.

The time delay between the signals arriving at the two detectors of the quadrant cell photoreceiver might introduce errors in practice. In our experiments, the distance is controlled with mirrors outside of the interferometer (not shown in Fig. 1) such that the path difference is smaller than 1 cm. Therefore, the time delay is much smaller than the time resolution of our equipment, which will not affect the accuracy of our measurements.

In conclusion, we experimentally demonstrated precision measurements of the angular velocity of a mirror within a modified Sagnac interferometer, using the ABWV technique. The separation between two counter-propagating beams provides an additional amplification factor, which can be useful for precision measurement of parameters related to angular rotations. The measurements of a constant angular velocity were performed using two techniques for comparison, WVA and ABWV, within the same experimental setup. It was observed that ABWV can achieve much larger amplification. Also, we can infer that the ABWV technique is less sensitive to imperfection of interference than the WVA, and the reversed tendency of WVA—when the post-selection phase is smaller than a certain value—does not show up in our experiments. In addition, both sum and differing signals can be obtained from the detection results; thus, no prior information of the input pulses is requested, which makes the estimation free of systematic errors. For our specific experiments, we saw the maximum amplification for the ABWV technique 24 times [30] larger than the best result for the WVA technique. We believe the ABWV technique will find interesting applications on precision metrology within and outside optical systems.

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**REFERENCES AND NOTES**

30. The amplification factor is $1/\phi$ in case of ABWV and $2/\phi$ for WVA, respectively. The best observed $\phi$ is 83 nrad for ABWV and 4 μrad for WVA. Therefore the maximum amplification from ABWV is 4 μrad / (83 nrad * 2) ~ 24 times larger than that of WVA.