Physics 411 - Homework 2
Lagrangian & Hamiltonian basics, Central force problems
Due Sep. 17, 2009

Problem 1: Consider a 3D space measured with spherical coordinates (r, θ, ϕ). A particle moves under the influence of the potential $V(\mathbf{r}) = (1/2)k\,\mathbf{r}\cdot\mathbf{r}$, where k is a positive constant.

(a) Find the generalized momenta p_r, p_θ, p_ϕ.

(b) What is the Lagrangian in these coordinates?

(c) Find the Lagrangian equations of motion.

(d) Find the Hamiltonian in these coordinates.

(e) Find the Hamiltonian equations of motion.

(f) What are the conserved quantities?

(g) Are the solutions the ones you expected from elementary methods?

Problem 2: A particle moves in a central force field given by the screened Coulomb potential:

$$V(r) = -k\frac{e^{-ar}}{r}, \tag{1}$$

where k and a are positive constants. Using the method of the equilivant one-dimensional potential discuss the nature of the motion, stating the ranges of ℓ and E appropriate to each type of motion. When are circular orbits possible? Find the period of small radial oscillations about the circular motion.