Physics 411 - Homework 3
Kepler orbits
Due Sep. 24, 2009

Problem 1: Suppose that you have an attractive central force with a potential

$$V(r) = -\frac{\alpha}{r^2}. \tag{1}$$

Suppose further that the angular momentum and/or the mass of the particle that moves under the potential's influence is tuned so that the effective 1D potential vanishes, $V'(r) = 0$. What does this mean physically? Sketch the trajectory of the particle.

Problem 2: (a) Show that the motion of a particle in the potential field

$$V(r) = -\frac{k}{r} + \frac{h}{r^2} \tag{2}$$

is the same as that of the motion under the Kepler potential alone when expressed in terms of a coordinate system rotating or precessing around the center of force.

(b) For negative total energy show that if the additional term is very small compared to the Kepler potential, then the angular speed of the precession of the elliptical orbit is

$$\dot{\Omega} = \frac{2\pi m h}{\ell^2 \tau}. \tag{3}$$

(c) The perihelion of Mercury is observed to precess (after corrections for the perturbations of the other planets) at the rate of $40''$ of arc per century. Show that this precession could be accounted for non-relativistically if the dimensionless quantity $\eta = h/ka$ were as small as 7×10^{-8}. (The eccentricity of Mercury's orbit is 0.206, and its period is 0.24 year.) The notation used above is the same as that used in class.

Problem 3: In the previous problem the correction to the Kepler potential can be thought of as a correction to the centrifugal barrier in the effective potential, as in problem 1. Why is it that changing this barrier causes a precession, while changing ℓ does not?